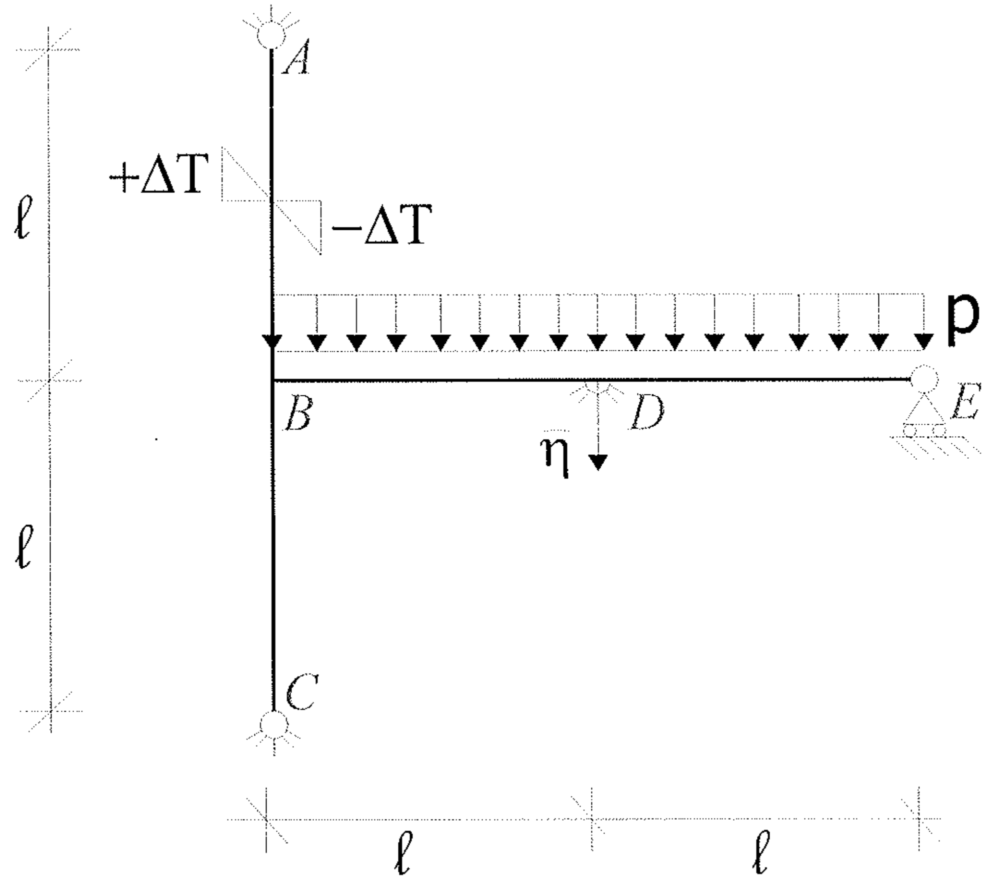
Esame di
FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

PROVA SCRITTA

(Fila 1)

24 – 11 – 2008



$$\frac{\alpha \Delta T}{h} = \frac{5 p \ell^2}{4EJ}; \quad \overline{\eta} = \frac{p \ell^4}{36EJ} \qquad EJ = \cos t \quad EA \to \infty$$

Si richiedono:

- Grafico del Momento flettente (con il valore e la posizione dei massimi)
- Grafico del Taglio
- Grafico dell' Azione assiale
- Deformata qualitativa con posizione dei flessi
- Reazione vincolare verticale nel punto D (valore e verso)

SOLUZIONE

Sistema risolvente

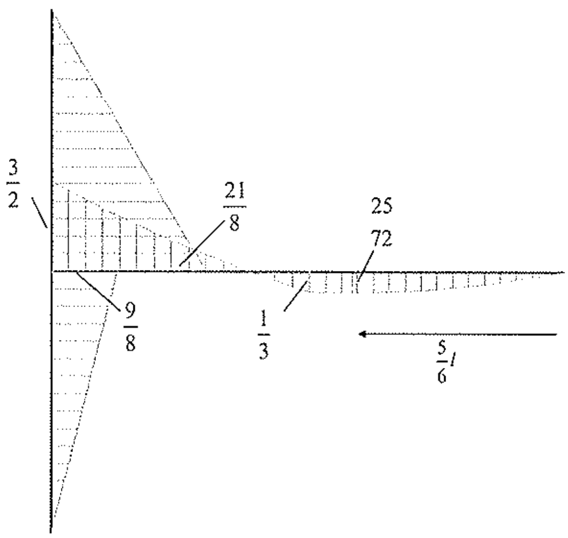
$$\begin{cases} \dfrac{10EJ}{\ell}\varphi_B + \dfrac{2EJ}{\ell}\varphi_D + \dfrac{7}{2}p\ell^2 = 0 \\ \dfrac{2EJ}{\ell}\varphi_B + \dfrac{7EJ}{\ell}\varphi_D - \dfrac{1}{8}p\ell^2 = 0 \end{cases}$$

Soluzioni

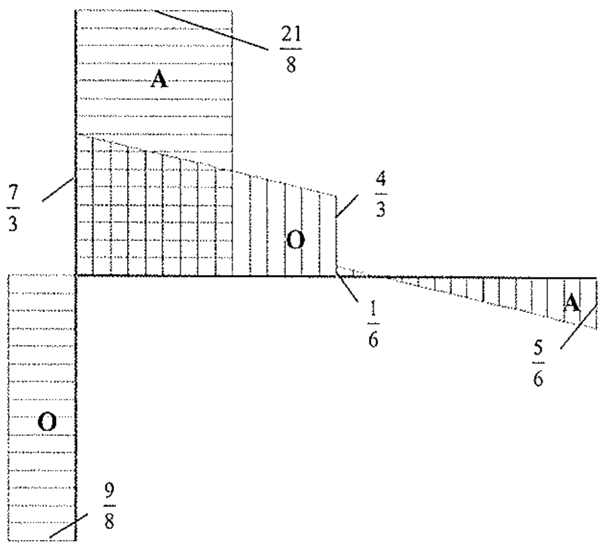
$$\begin{cases} \varphi_B = -\dfrac{3p\ell^3}{8EJ} \\ \varphi_D = \dfrac{p\ell^3}{8EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata

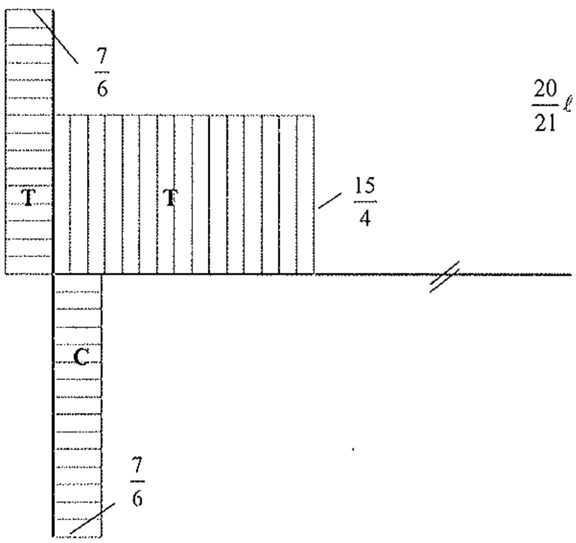
$M\ (\cdot p\ell^2)$



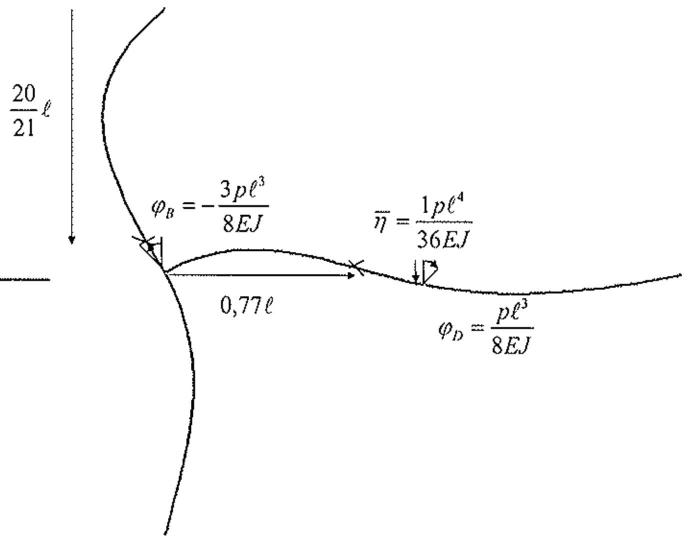
$T\ (\cdot p\ell)$



$N\ (\cdot p\ell)$



Deformata



La reazione vincolare verticale nel punto D vale $V_D = \dfrac{7}{6}p\ell$ verso il basso

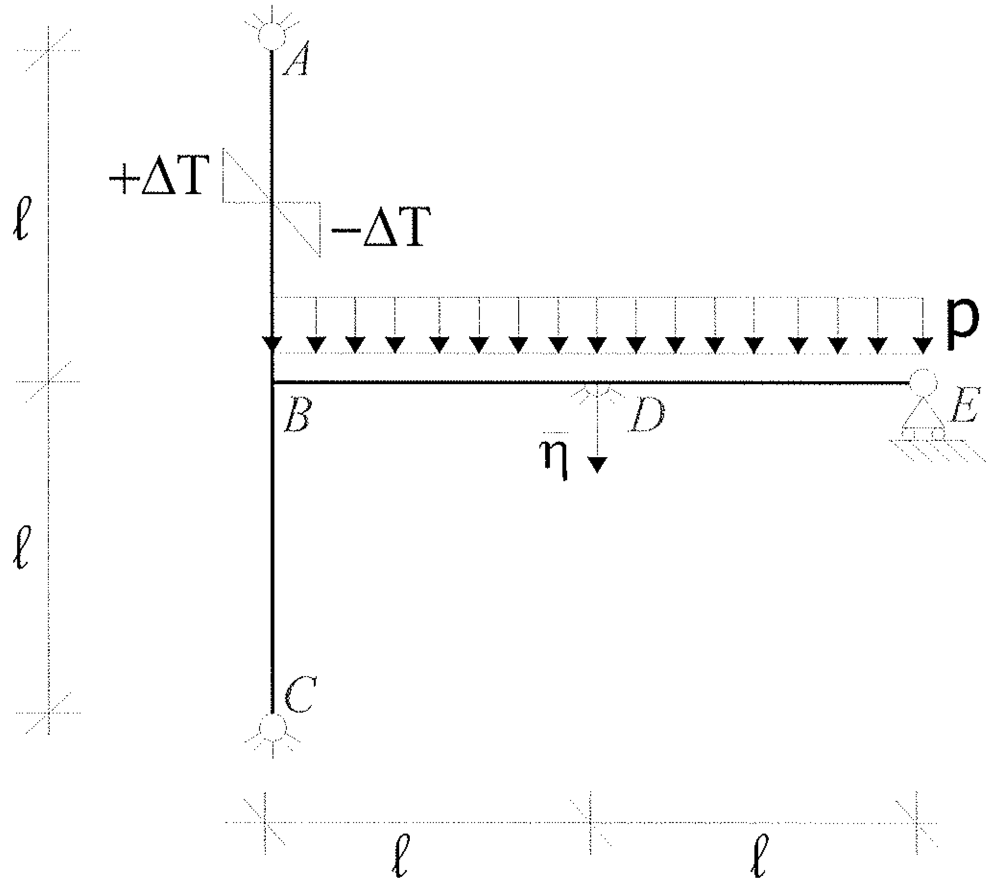
Esame di
FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

PROVA SCRITTA

(Fila 2)

24 – 11 – 2008



$$\frac{\alpha\Delta T}{h} = \frac{p\ell^2}{8EJ}; \quad \bar{\eta} = \frac{p\ell^4}{4EJ} \quad EJ = \cos t \;\; EA \to \infty$$

Si richiedono:

- Grafico del <u>Momento flettente</u> (con il valore e la posizione dei massimi)
- Grafico del <u>Taglio</u>
- Grafico dell' <u>Azione assiale</u>
- Deformata qualitativa con posizione dei flessi
- Reazione vincolare verticale nel punto D (valore e verso)

SOLUZIONE

Sistema risolvente

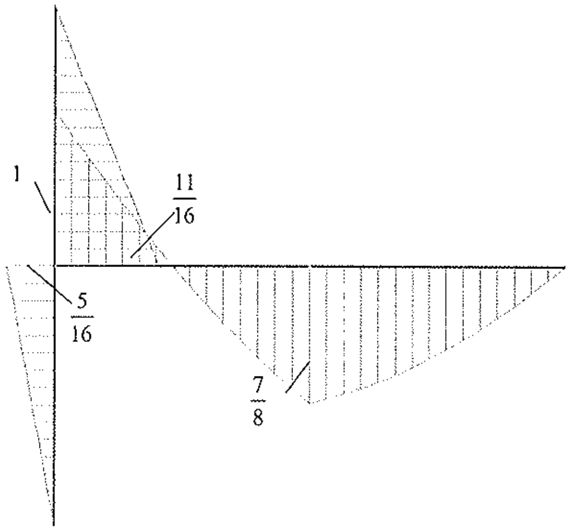
$$\begin{cases} \dfrac{10EJ}{\ell}\varphi_B + \dfrac{2EJ}{\ell}\varphi_D - \dfrac{29}{24}p\ell^2 = 0 \\ \dfrac{2EJ}{\ell}\varphi_B + \dfrac{7EJ}{\ell}\varphi_D - \dfrac{19}{24}p\ell^2 = 0 \end{cases}$$

Soluzioni

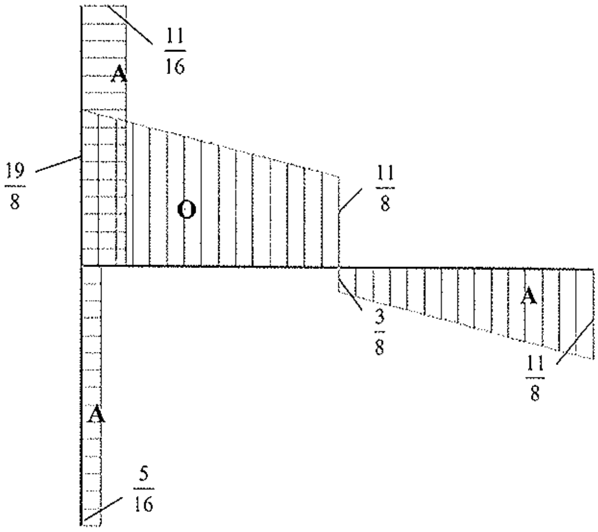
$$\begin{cases} \varphi_B = \dfrac{5p\ell^3}{48EJ} \\ \varphi_D = \dfrac{p\ell^3}{12EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata

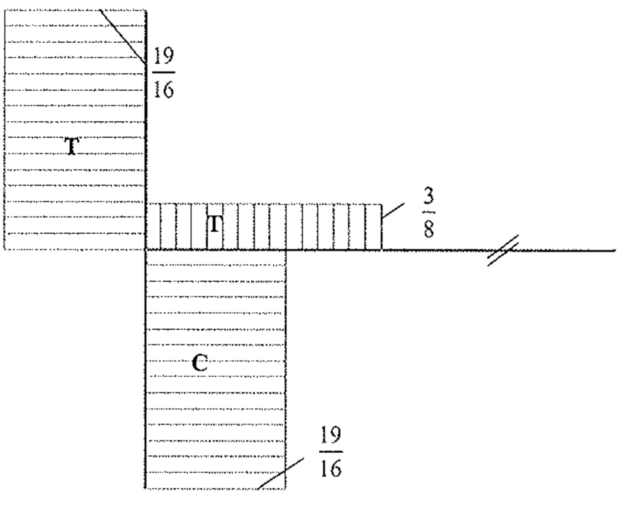
$M\ (\cdot p\ell^2)$



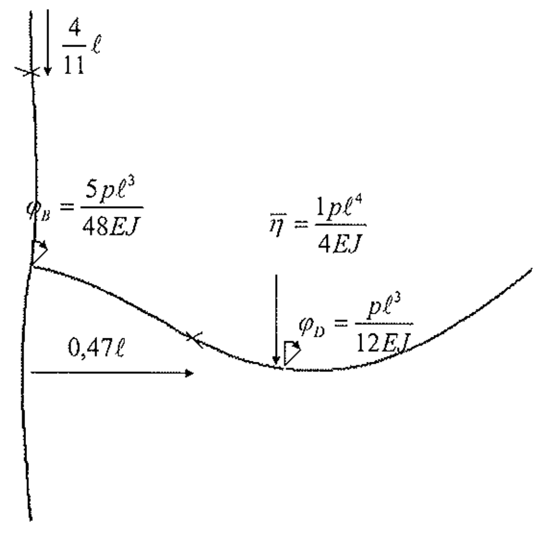
$T\ (\cdot p\ell)$



$N\ (\cdot p\ell)$

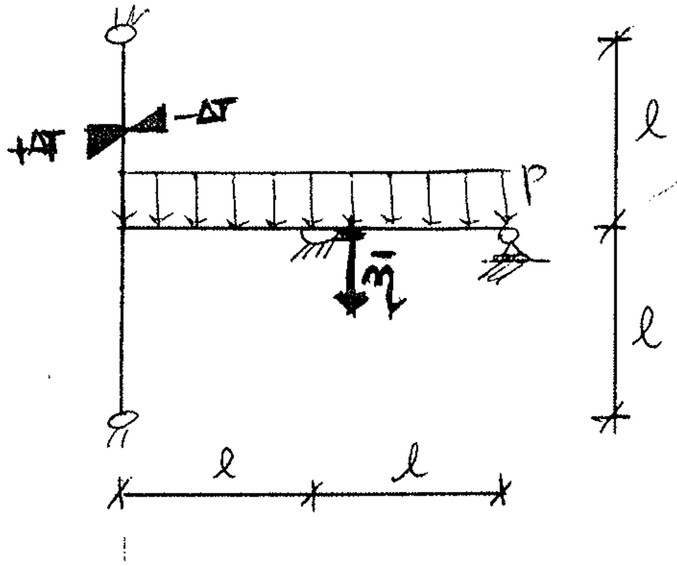


Deformata



La reazione vincolare verticale nel punto D vale $V_D = \dfrac{7}{4}p\ell$ verso il basso

Tema esame del 24/11/08



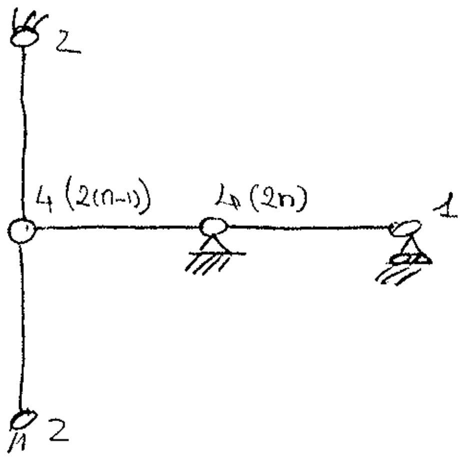
FILA 1

- $\frac{\alpha \Delta T}{h} = \frac{5}{4} \frac{p l^2}{EJ}$
- $\bar{\eta} = \frac{1}{36} \frac{p l^4}{EJ}$

FILA 2

- $\frac{\alpha \Delta T}{h} = \frac{1}{8} \frac{p l^2}{EJ}$
- $\bar{\eta} = \frac{1}{4} \frac{p l^4}{EJ}$

Si tratta di una struttura IPERSTATICA a NODI FISSI

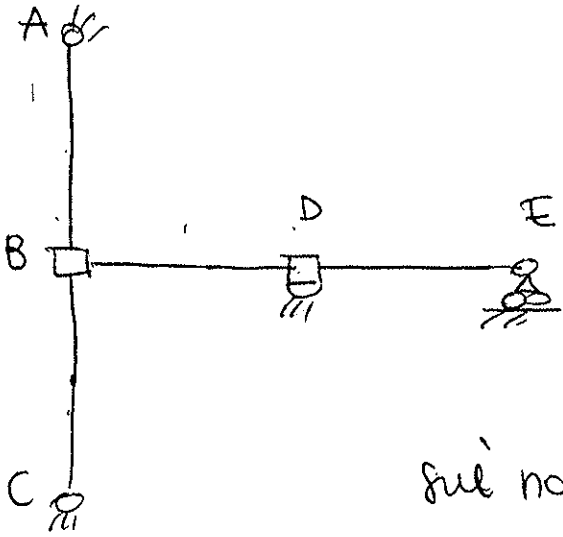


$L = 3 \times 4 = 12$

$V = 2 + 2 + 1 + 4 + 4 = 13$ } IPERSTATICA

Non c'è alcun cinematismo

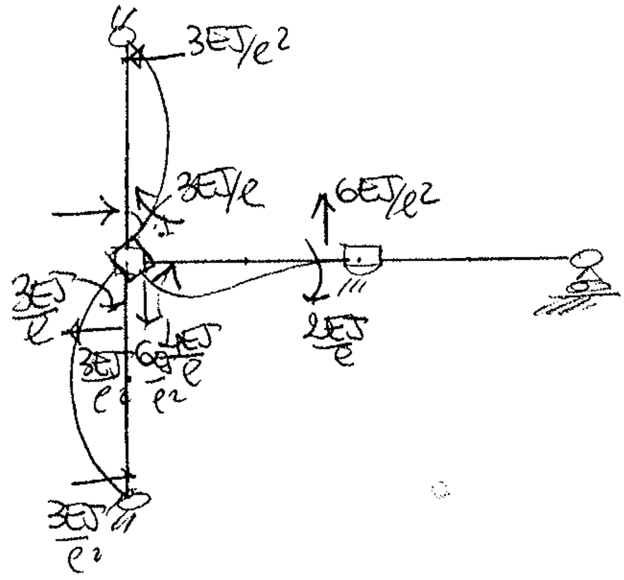
↳ NODI FISSI



$$\begin{cases} m_{BB}\,\varphi_B + m_{BD}\,\varphi_D + m_{B0} = 0 \\ m_{DB}\,\varphi_B + m_{DD}\,\varphi_D + m_{D0} = 0 \end{cases}$$

sui nodi

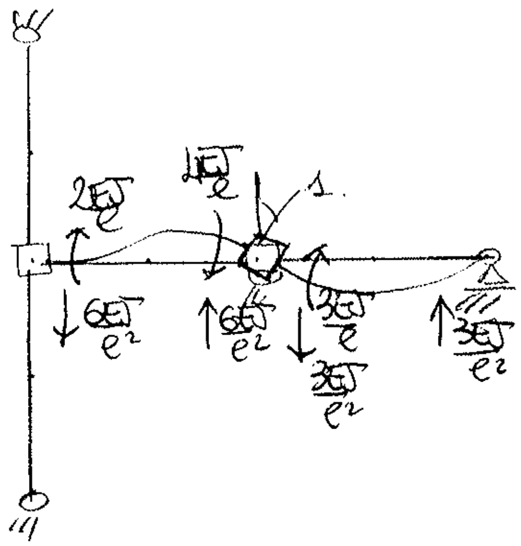
$\varphi_B = 1$



$$m_{BB} = \frac{10EJ}{l}$$

$$m_{DB} = \frac{2EJ}{l}$$

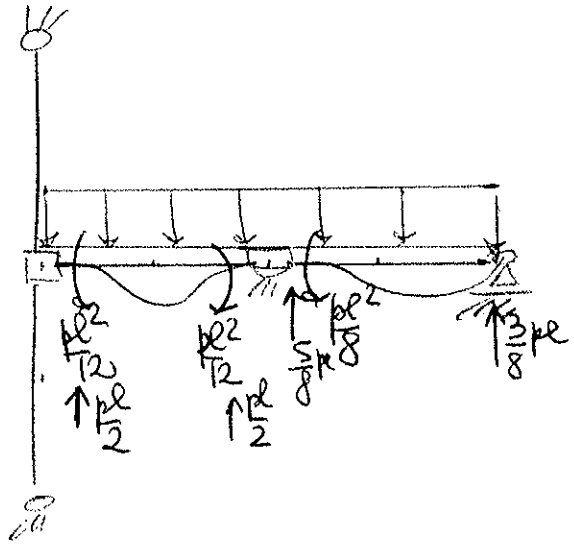
$\varphi_D = 1$



$$m_{DD} = \frac{7EJ}{l}$$

$$m_{BD} = \frac{2EJ}{l}$$

$p \neq 0$

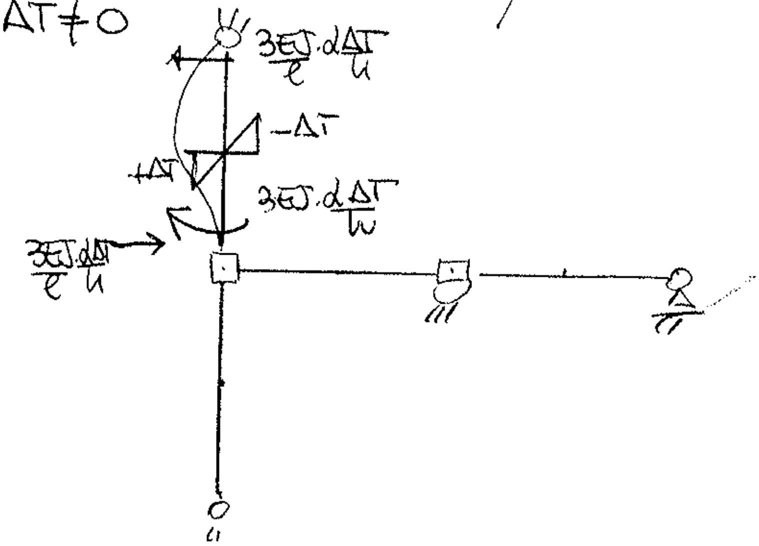


$$m_{B0}^{p\neq 0} = -\frac{pl^2}{12}$$

$$m_{D0}^{p\neq 0} = \frac{pl^2}{12} - \frac{pl^2}{8} = \frac{2-3pl^2}{24} =$$

$$= -\frac{pl^2}{24}$$

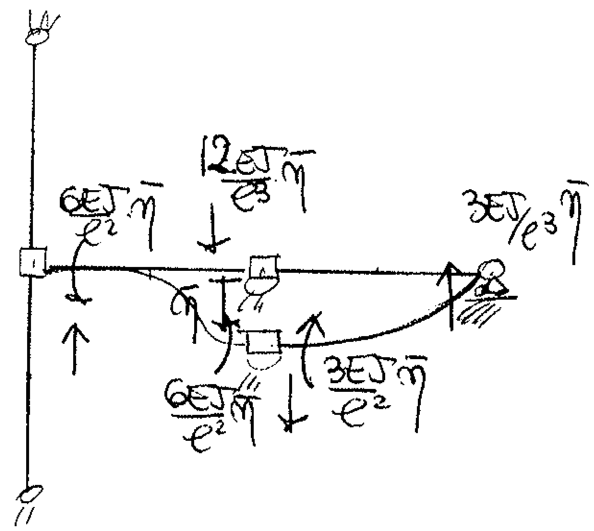
$\Delta T \neq 0$



$$\mathcal{M}_{BO}^{\Delta T \neq 0} = \cdot\, 3EJ\cdot\frac{\alpha\Delta T}{h}$$

$$\mathcal{M}_{DO}^{\Delta T \neq 0} = 0$$

$\bar{\eta} \neq 0$



$$\mathcal{M}_{BO}^{\bar{\eta}\neq 0} = -\frac{6EJ}{\ell^2}\bar{\eta}$$

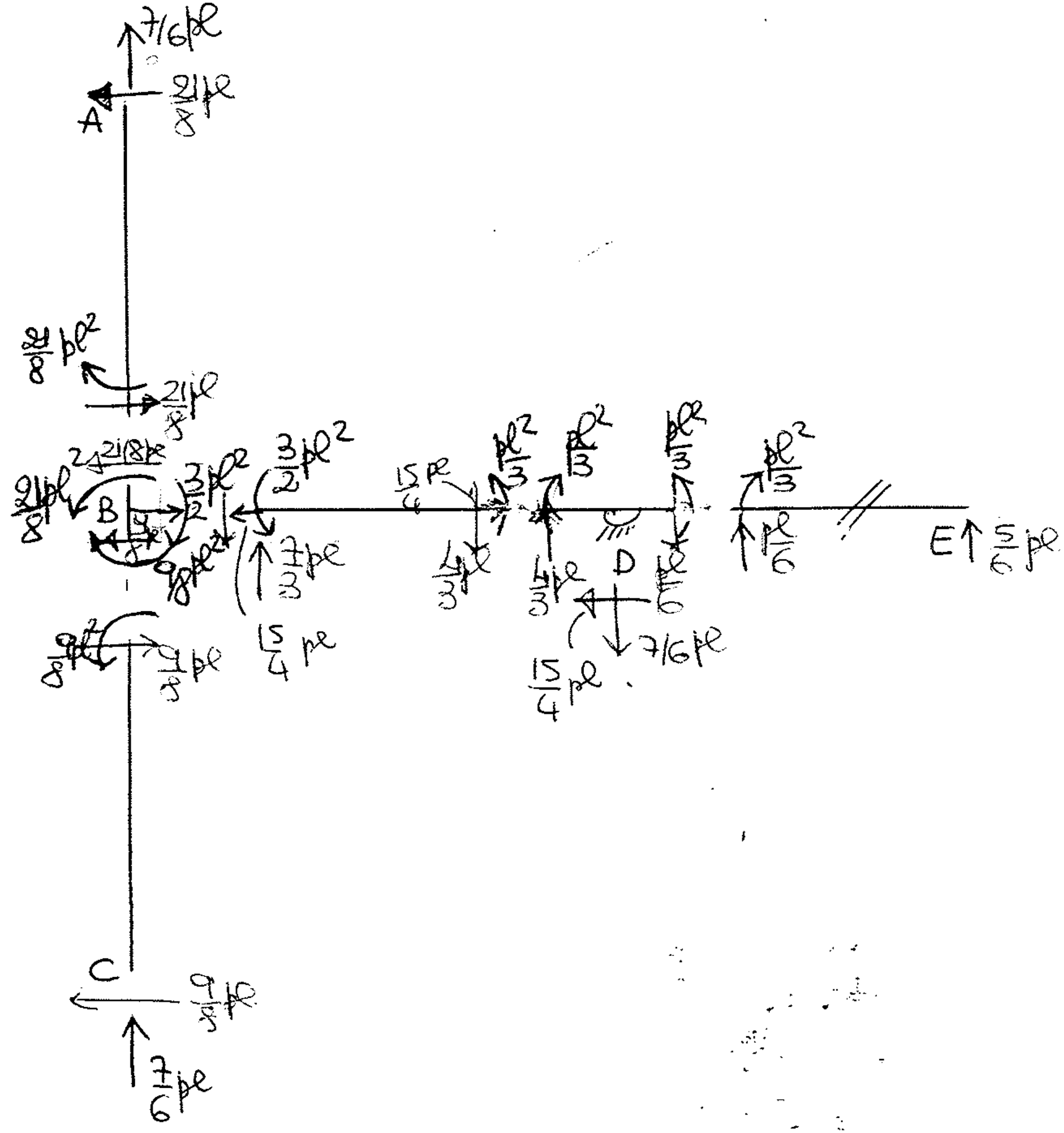
$$\mathcal{M}_{DO}^{\bar{\eta}\neq 0} = \left(\frac{3EJ}{\ell^2} - \frac{6EJ}{\ell^2}\right)\bar{\eta} = -\frac{3EJ}{\ell^2}\bar{\eta}$$

Il sistema risolvente diventa

$$\begin{cases} \frac{10EJ}{\ell}\varphi_B + \frac{2EJ}{\ell}\varphi_D - \frac{p\ell^2}{12} + 3EJ\cdot\frac{\alpha\Delta T}{h} - \frac{6EJ}{\ell^2}\bar{\eta} = 0 \\ \frac{2EJ}{\ell}\varphi_B + \frac{7EJ}{\ell}\varphi_D - \frac{p\ell^2}{24} - \frac{3EJ}{\ell^2}\bar{\eta} = 0 \end{cases}$$

$$\begin{cases} \frac{10EJ}{\ell}\varphi_B + \frac{2EJ}{\ell}\varphi_D - \frac{p\ell^2}{12} + 3EJ\cdot\frac{5}{4}\frac{p\ell^2}{EJ} - \frac{6EJ}{\ell^2}\cdot\frac{1}{36}\frac{p\ell^4}{EJ} = 0 \\ \frac{2EJ}{\ell}\varphi_B + \frac{7EJ}{\ell}\varphi_D - \frac{p\ell^4}{24} - \frac{3EJ}{\ell^2}\cdot\frac{1}{36}\frac{p\ell^4}{EJ} \end{cases}$$

$$\begin{cases} \frac{10EJ}{\ell}\varphi_B + \frac{2EJ}{\ell}\varphi_D + \frac{7}{2}p\ell^2 = 0 \\ \frac{2EJ}{\ell}\varphi_B + \frac{7EJ}{\ell}\varphi_D - \frac{1}{8}p\ell^2 = 0 \end{cases} \qquad \begin{cases} \varphi_B = -\frac{3}{8}\frac{p\ell^3}{EJ} \\ \varphi_D = \frac{p\ell^3}{8EJ} \end{cases}$$


A
B
C
D
E

$$T_A = \frac{3EJ}{l^2}\cdot\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) + \frac{3EJ}{l}\cdot\frac{5}{4}\frac{pl^2}{EJ} = -\frac{9}{8}pl + \frac{15}{4}pl = \frac{-9+30}{8} = \frac{21}{8}pl$$

$$T_B^{sopra} = T_A$$

$$T_C = \frac{3EJ}{l^2}\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) = -\frac{9}{8}pl$$

$$M_B^{sopra} = \frac{3EJ}{l}\cdot\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) + \frac{3EJ}{l}\cdot\frac{5}{4}\frac{pl^2}{EJ} = \frac{21}{8}pl^2$$

$$M_B^{sotto} = \frac{3EJ}{l}\cdot\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) = -\frac{9}{8}pl^2$$

$$M_B^{dx} = \frac{4EJ}{l}\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) + \frac{2EJ}{l}\cdot\frac{pl^3}{8EJ} - \frac{pl^2}{12} - \frac{6EJ}{l^2}\cdot\frac{1}{36}\frac{pl^4}{EJ} =$$

$$= -\frac{3}{2}pl^2 + \frac{pl^2}{4} - \frac{pl^2}{12} - \frac{1}{6}pl^2 = \frac{-18+3-1-2}{12} = -\frac{\cancel{18}^{3}}{\cancel{12}_{2}}pl^2$$

Equilibrio al nodo $\frac{21}{8}pl^2 - \frac{3}{2}pl^2 - \frac{9}{8}pl^2 = 0$

$$T_B^{dx} = \frac{\cancel{6}^{3}EJ}{l^2}\cdot\left(-\frac{3}{\cancel{8}_{4}}\frac{pl^3}{EJ}\right) + \frac{\cancel{6}^{3}EJ}{l^2}\cdot\left(\frac{pl^3}{\cancel{8}_{4}EJ}\right) - \frac{pl}{2} - \frac{\cancel{12}^{\cancel{2}}EJ}{l^3}\cdot\frac{1}{\cancel{36}_{\cancel{6}_{3}}}\frac{pl^4}{EJ} = \left(-\frac{9}{4} + \frac{3}{4} - \frac{1}{3} - \frac{1}{2}\right)$$

$$= \frac{-27+9-4-6}{12} = -\frac{\cancel{28}^{\cancel{14}^{7}}}{\cancel{12}_{\cancel{6}_{3}}}$$

$$M_D^{sx} = \frac{2EJ}{l}\left(-\frac{3}{8}\frac{pl^3}{EJ}\right) + \frac{4EJ}{l}\left(\frac{pl^3}{8EJ}\right) + \frac{pl^2}{12} - \frac{6EJ}{l^2}\left(\frac{1}{36}\frac{pl^4}{EJ}\right) =$$

$$= \left(-\frac{3}{4} + \frac{1}{2} + \frac{1}{12} - \frac{1}{6}\right)pl^2 = \frac{-9+6+1-2}{12}pl^2 = -\frac{pl^2}{3}$$

$$M_B^{dx} = \frac{3EJ}{l}\left(\frac{pl^3}{8EJ}\right) - \frac{pl^2}{8} + \frac{3EJ}{l^2}\cdot\left(+\frac{1}{36}\frac{pl^4}{EJ}\right) = \left(\frac{3}{8} - \frac{1}{8} + \frac{1}{12}\right)pl^2 = \frac{9-3+2}{24}pl^2 = \frac{pl}{3}$$

$$T_D^{sx} = \frac{\cancel{6}^{3}EJ}{l^2}\cdot\left(-\frac{3}{\cancel{8}_{4}}\frac{pl^3}{EJ}\right) + \frac{\cancel{6}^{3}EJ}{l^2}\left(\frac{pl^3}{\cancel{8}_{4}EJ}\right) + \frac{pl}{2} - \frac{12EJ}{l^3}\cdot\frac{1}{\cancel{36}_{3}}\frac{pl^4}{EJ} = \frac{9}{4} + \frac{3}{4} + \frac{1}{2} - \frac{1}{3} = -\frac{16}{12}pl = -\frac{4}{3}$$

$$T_D^{dx} = \frac{3EJ}{l^2}\left(\frac{pl^3}{8EJ}\right) - \frac{5}{8}pl + \frac{3EJ}{l^3}\cdot\frac{1}{36}\frac{pl^4}{EJ} = \frac{3}{8} - \frac{5}{8} + \frac{1}{12} = \frac{9-15+2}{24} = -\frac{pl}{6}$$

$$T_E = \frac{3EJ}{l^2}\left(\frac{pl^3}{8EJ}\right) + \frac{3}{8}pl + \frac{3EJ}{l^3}\frac{1}{36}\frac{pl^4}{EJ} = \left(\frac{3}{8} + \frac{3}{8} + \frac{1}{12}\right)pl = \frac{9+9+2}{24} = \frac{5}{6}pl$$

Tracati i tagli determino le azioni assiali

$$N_{BD} = N_{DE} = \frac{21}{8}pl + \frac{9}{8}pl = \frac{15}{4}pl$$

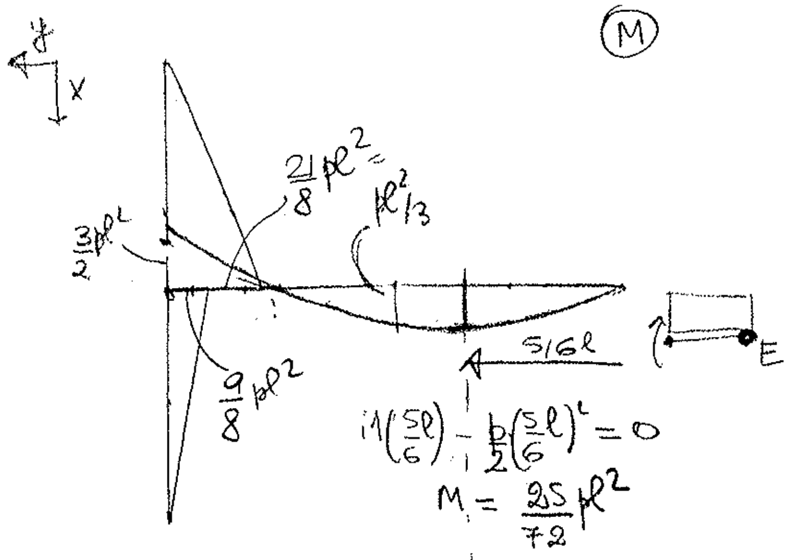
$$N_{AB} = N_{BC} = \frac{7}{3}\,\frac{1}{2}\,pl = \frac{7}{6}pl$$

Verifica equilibrio globale

$$\uparrow + \left(\frac{7}{6} + \frac{7}{6} - \frac{7}{6} + \frac{5}{6} - 2\right) = 0 \quad \text{ok!}$$

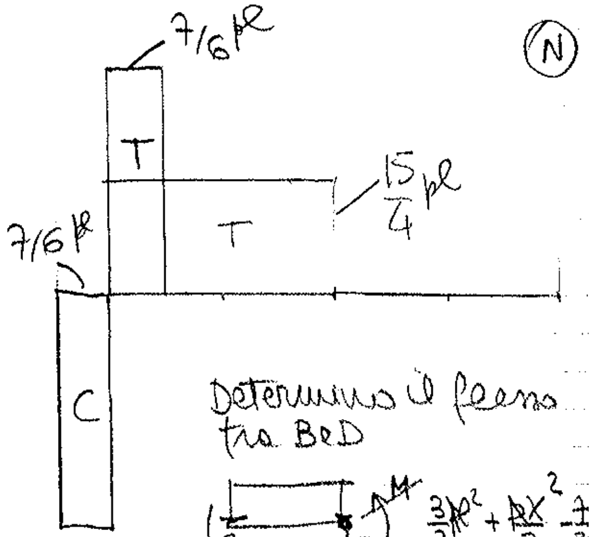
$$\xrightarrow{+} \left(-\frac{21}{8} - \frac{9}{8} + \frac{15}{4}\right) pl = 0 \quad \text{ok!}$$

$$\circlearrowleft + \quad \frac{21}{8}pl^2 - \frac{9}{8}pl^2 - 2pl^2 - \frac{7}{6}pl^2 + \frac{5}{6}pl \cdot 2l = \frac{21-9-16+4}{8}pl^2 = 0 \quad \text{ok!}$$



$$M\left(\frac{5}{6}l\right) - \frac{p}{2}\left(\frac{5}{6}l\right)^2 = 0$$

$$M = \frac{25}{72}pl^2$$



Determino il flesso tra B e D

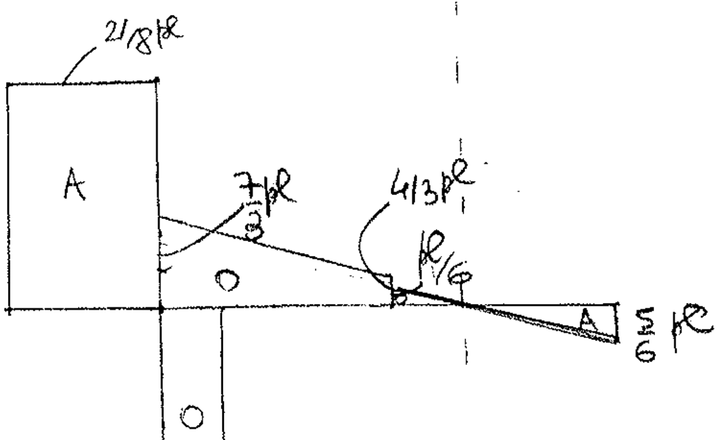
$$\frac{3}{2}pl^2 + \frac{px^2}{2} - \frac{7}{3} \ldots$$

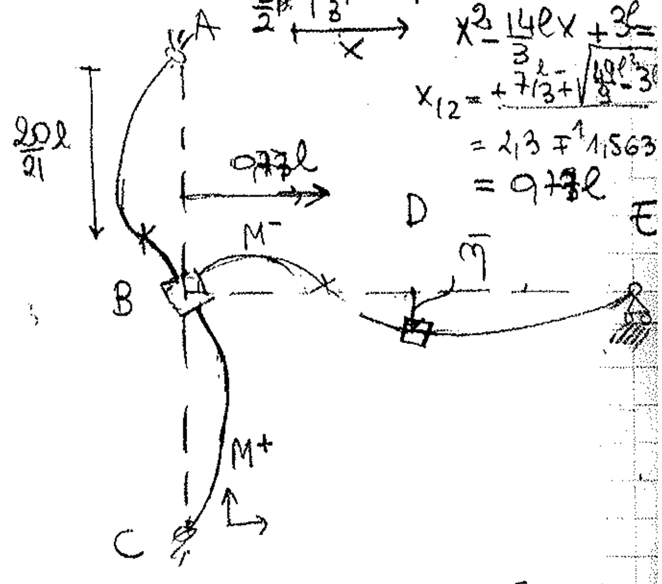
$$x^2 - \frac{14}{3}lx + 3l^2 = \ldots$$

$$x_{12} = +\frac{7}{3} \mp \sqrt{\frac{49}{9}l^2 - 3l^2} \ldots$$

$$= 2{,}3 \mp 1{,}563$$

$$= 0{,}77\,l$$





Det. flesso tra A e B

$$M(x) + \frac{21}{8}pl\,x = 0 \qquad M(x) = -\frac{21}{8}pl\,x$$

$$y''(x) = -\left[-\frac{21}{8}pl\,x\right]\frac{1}{EJ} - \frac{25}{\ldots}\,pl \ldots$$

$$\frac{21}{4}x - \frac{5}{2}l \geq 0 \qquad x \geq \frac{20\,l}{21}$$

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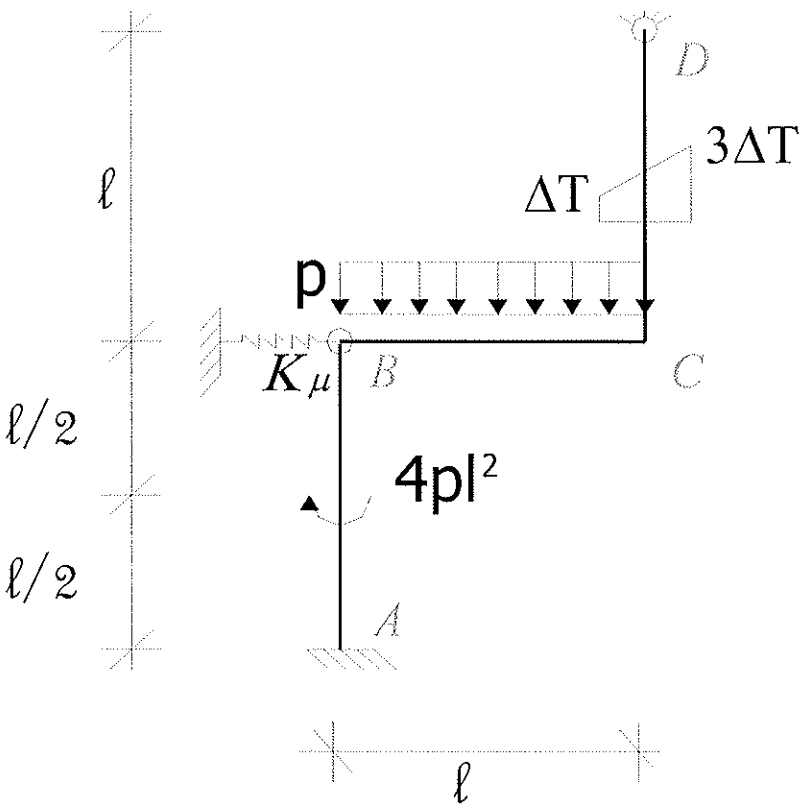
TECNICA DELLE COSTRUZIONI
(Corso di laurea in Ingegneria Civile e Meccanica V.O.)

PROVA SCRITTA
(Fila 1)

18 – 12 – 2008



$$h = \frac{\ell}{12}; \quad \frac{\alpha \Delta T}{h} = \frac{p\ell^2}{4EJ}; \quad k_\mu = \frac{69EJ}{2l^3} \quad EJ = \cos t \quad EA \to \infty$$

Si richiedono:

- Grafico del Momento flettente (con il valore e la posizione dei massimi)
- Grafico del Taglio
- Grafico dell' Azione assiale
- Deformata qualitativa con posizione dei flessi
- Direzione e verso della forza sulla molla in B

SOLUZIONE

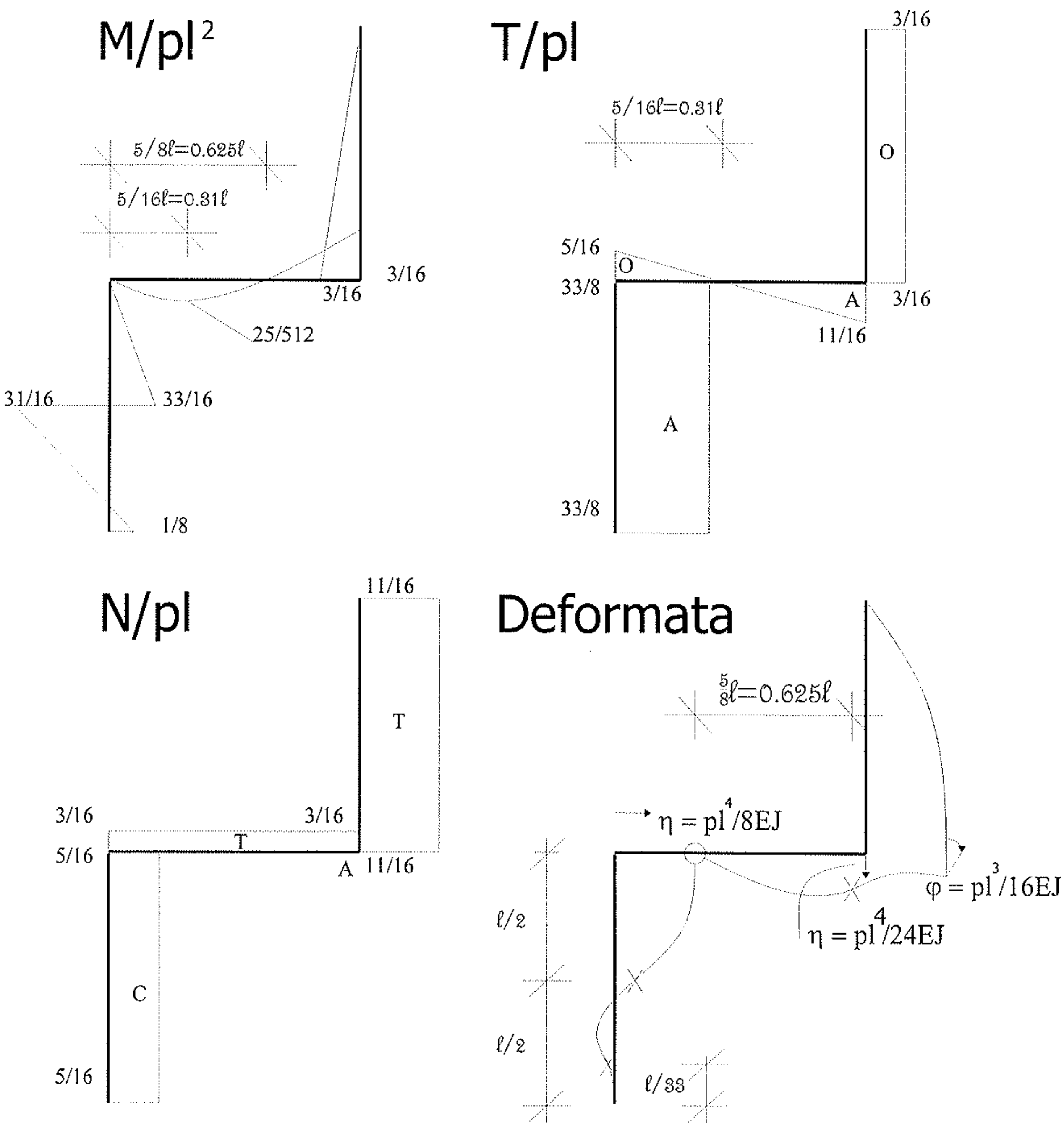
Sistema risolvente

$$\begin{cases} \dfrac{6EJ}{\ell}\varphi_C + \dfrac{3EJ}{\ell^2}\eta_C - \dfrac{3}{4}p\ell^2 = 0 \\ -\dfrac{3EJ}{\ell^2}\varphi_C - \dfrac{81EJ}{2\ell^3}\eta_C + \dfrac{21}{4}p\ell = 0 \end{cases}$$

Soluzioni

$$\begin{cases} \varphi_C = \dfrac{1p\ell^3}{16EJ} \\ \eta_C = \dfrac{1p\ell^4}{8EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata



La forza sulla molla vale **69/16 pl** ed è diretta verso destra.

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FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

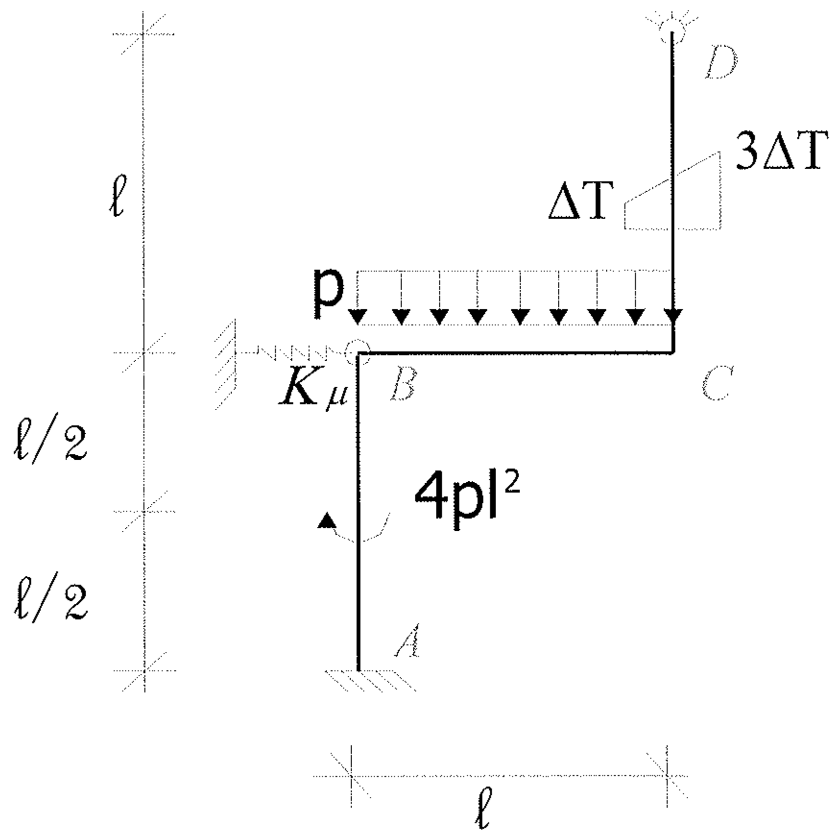
TECNICA DELLE COSTRUZIONI

(Corso di laurea in Ingegneria Civile e Meccanica V.O.)

PROVA SCRITTA

(Fila 2)

18 – 12 – 2008



$$h = \frac{\ell}{8}; \quad \frac{\alpha \Delta T}{h} = \frac{p\ell^2}{6EJ}; \quad k_\mu = \frac{29EJ}{2l^3} \qquad EJ = \cos t \quad EA \to \infty$$

Si richiedono:

- Grafico del Momento flettente (con il valore e la posizione dei massimi)
- Grafico del Taglio
- Grafico dell' Azione assiale
- Deformata qualitativa con posizione dei flessi
- Direzione e verso della forza sulla molla in B

SOLUZIONE

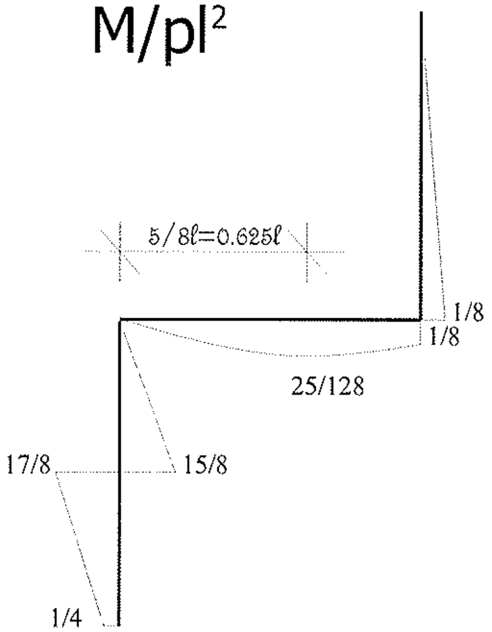
Sistema risolvente

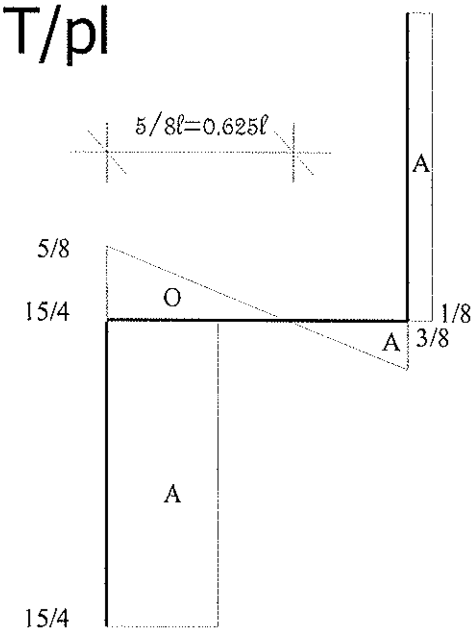
$$\begin{cases} \dfrac{6EJ}{\ell}\varphi_C + \dfrac{3EJ}{\ell^2}\eta_C - \dfrac{1}{2}p\ell^2 = 0 \\ -\dfrac{3EJ}{\ell^2}\varphi_C - \dfrac{81EJ}{2\ell^3}\eta_C + 5p\ell = 0 \end{cases}$$

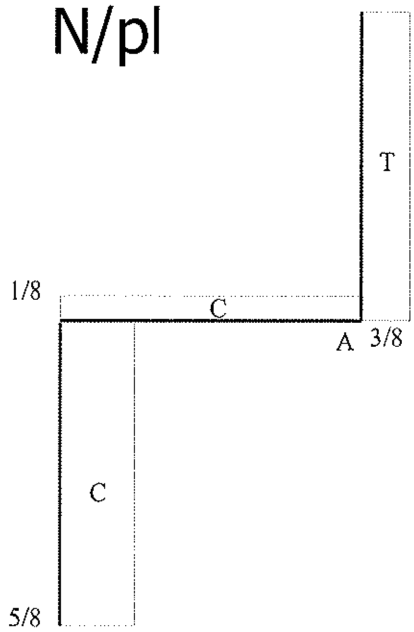
Soluzioni

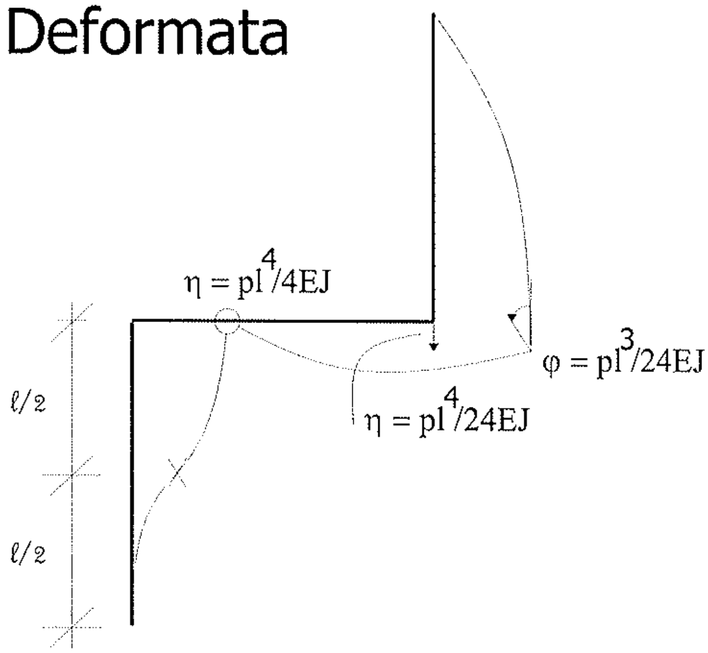
$$\begin{cases} \varphi_C = -\dfrac{p\ell^3}{24EJ} \\ \eta_C = \dfrac{p\ell^4}{4EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata



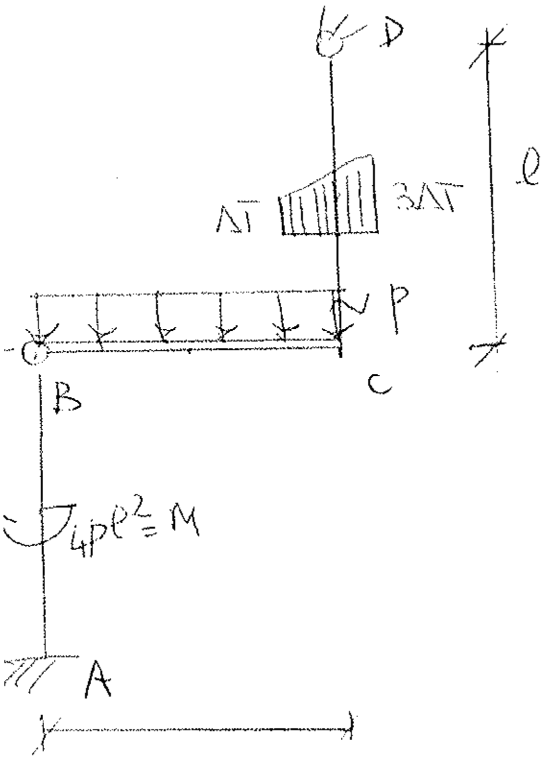






La forza sulla molla vale **29/8 pl** ed è diretta verso destra.

ia esame del 18_12_08



FILA 1

$$\frac{\alpha\Delta T}{h} = \frac{1}{4}\frac{pl^2}{EJ} \qquad h = \frac{1}{12}l$$

$$\frac{\alpha\Delta T}{l} = \frac{1}{48}\frac{pl^2}{EJ}$$

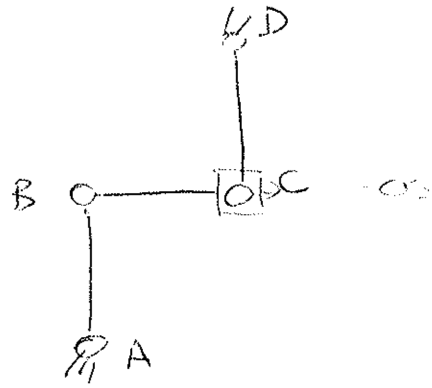
$$k_J = \frac{69}{2}\frac{EJ}{l^3}$$

FILA 2

$$\frac{\alpha\Delta T}{h} = \frac{1}{6}\frac{pl^2}{EJ} \qquad h = \frac{1}{8}l$$

$$\frac{\alpha\Delta T}{l} = \frac{1}{48}\frac{pl^2}{EJ}$$

$$k_J = \frac{24}{2}\frac{EJ}{l^3}$$

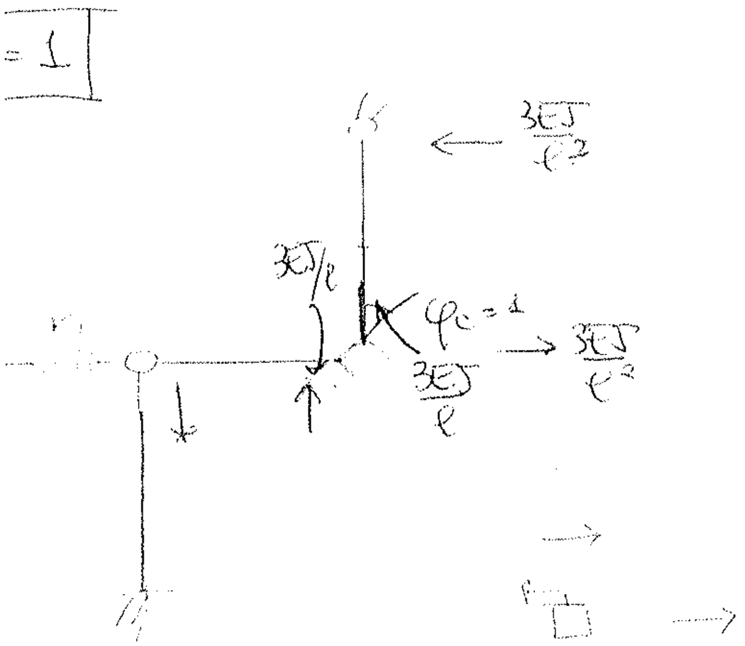


Si tratta di una struttura IPERSTATICA a NODI SPOSTABILI

[+] ↑ →o—+—o←

$$m_{cc}\varphi_c + m_{c\eta}\eta + m_{c0} = 0$$

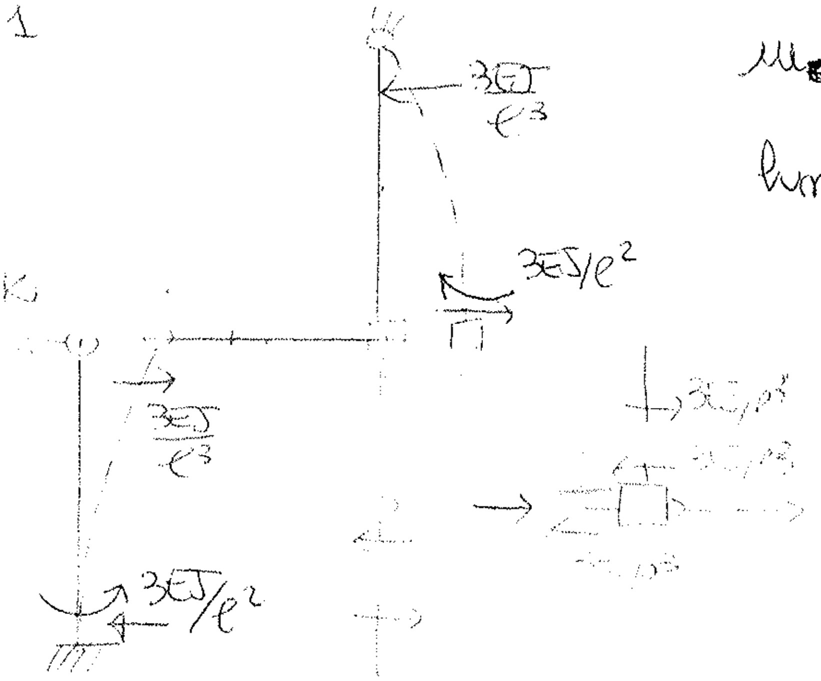
$$h_{c\eta}\varphi_c + h_{\eta\eta}\eta + h_{\eta 0} = 0$$



$$m_{cc} = \frac{6EJ}{l}$$

$$h_{1c\eta} = -\frac{3EJ}{l^2}$$

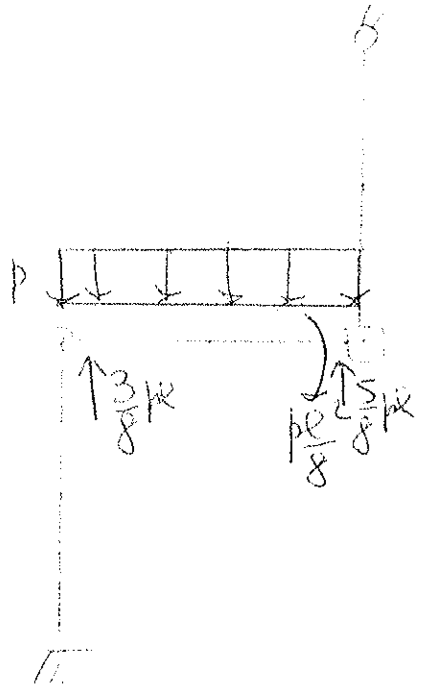
1



$m_{\varphi\eta} = 3EJ/\ell^2$

$k_{\eta\eta} = -\left[\frac{3EJ}{\ell^3}\cdot 2 + k\right]$

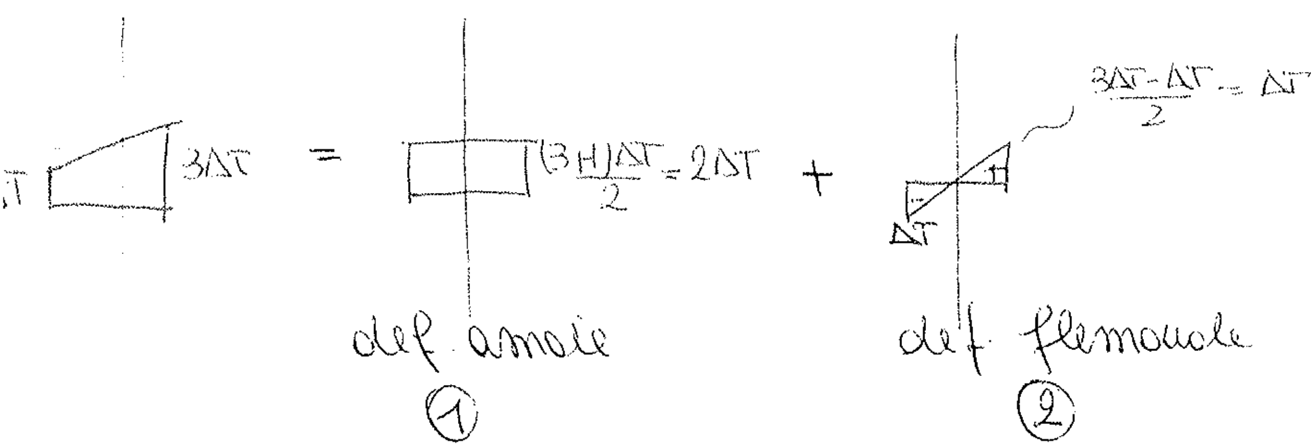
p ≠ 0



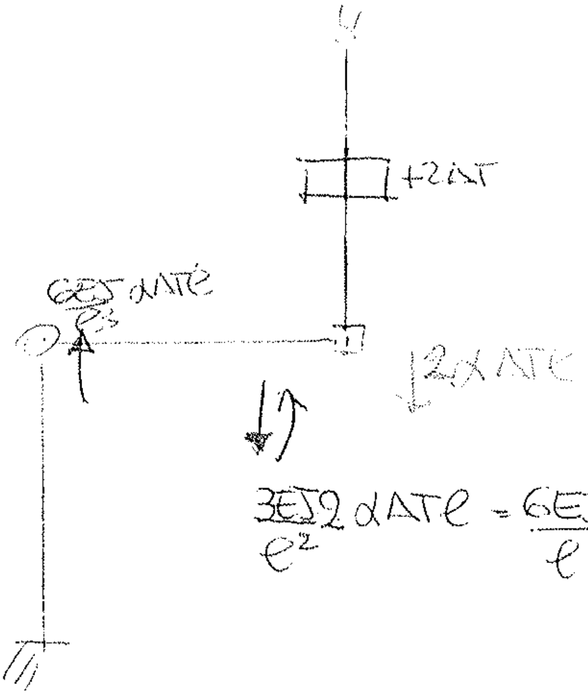
$m_{\varphi 0}^{p\neq 0} = \frac{p\ell^2}{8}$

$k_{\eta 0}^{p\neq 0} = 0$

T ≠ 0 CI SONO 2 CONTRIBUTI



(1)

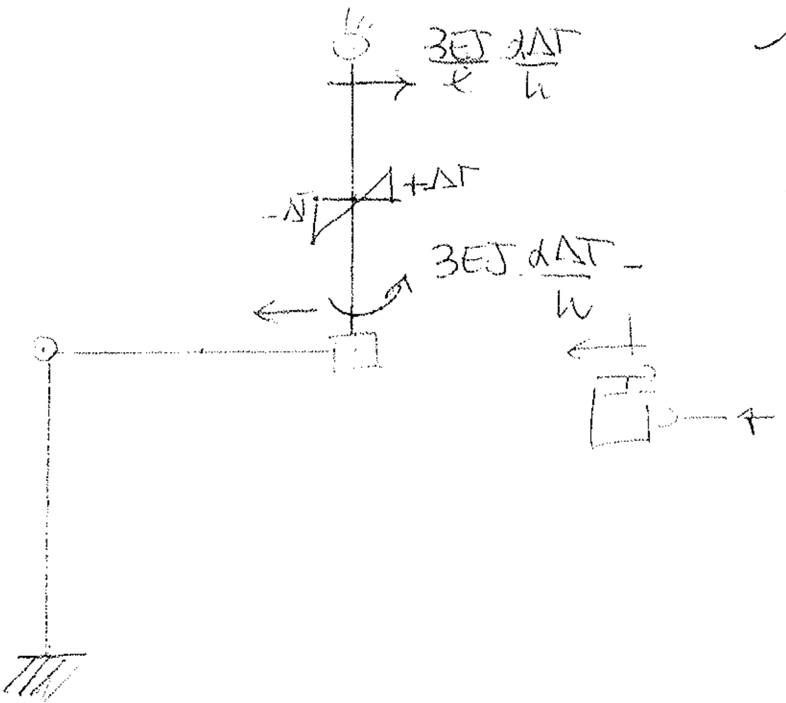


$$\frac{3EJ\,2\alpha\Delta T l}{l^2} = \frac{6EJ}{l^2}\alpha\Delta T l$$

$$\mathcal{M}_{CO}^{\Delta T_1} = -\frac{6EJ}{l^2}\alpha\Delta T l$$

$$\ell_{\eta_0}^{\Delta T_1} = 0$$

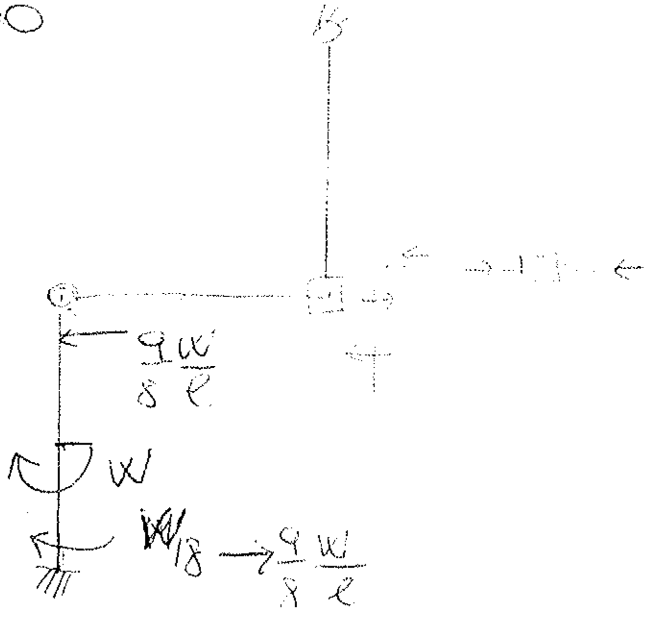
(2)



$$\mathcal{M}_{CO}^{\Delta T_2} = -3EJ\,\frac{\alpha\Delta T}{h}$$

$$\ell_{\eta_0}^{\Delta T_2} = \frac{3EJ}{l}\cdot\frac{\alpha\Delta T}{h}$$

$W \neq 0$



$$\mathcal{M}_{CO}^{W} \neq 0$$

$$\ell_{\eta_0}^{W} = \frac{9}{8}\frac{W}{l}$$

(*) Si imposta il sistema per la FILA 1 (analogamente si procede per la FILA 2)

$$\begin{cases} \frac{6EJ}{\ell}\varphi_C + \frac{3EJ}{\ell^2}\eta + \frac{p\ell^2}{8} - \frac{6EJ}{\ell^2}\alpha\Delta T\ell - 3EJ\cdot\frac{\alpha\Delta T}{h} = 0 \\ -\frac{3EJ}{\ell^2}\varphi_C - \left(\frac{3EJ}{\ell^3}\cdot 2 + k\right)\eta + \frac{3EJ}{\ell}\cdot\frac{\alpha\Delta T}{h} + \frac{9}{8}\frac{W}{\ell} = 0 \end{cases}$$

(*)

$$\begin{cases} \frac{6EJ}{\ell^2}\varphi_C + \frac{3EJ}{\ell^2}\eta + \frac{p\ell^2}{8} - \frac{6EJ}{\ell^2}\frac{1}{48}\frac{p\ell^4}{EJ} - 3EJ\cdot\frac{p\ell^2}{4EJ} = 0 \\ -\frac{3EJ}{\ell^2}\varphi_C - \left(\frac{6EJ}{\ell^3} + \frac{69}{2}\frac{EJ}{\ell^3}\right)\eta + \frac{3EJ}{\ell}\cdot\frac{p\ell^2}{4EJ} + \frac{9}{8}\cdot\frac{4p\ell^2}{\ell} = 0 \end{cases}$$

$$\begin{cases} \frac{6EJ}{\ell}\varphi_C + \frac{3EJ}{\ell^2}\eta - \frac{3}{4}p\ell^2 = 0 \\ -\frac{3EJ}{\ell^2}\varphi_C - \frac{81}{2}\frac{EJ}{\ell^3}\eta + \frac{21}{4}p\ell = 0 \end{cases} \longrightarrow \begin{cases} \varphi_C = \frac{1}{16}\frac{p\ell^3}{EJ} \\ \eta = \frac{1}{8}\frac{p\ell^4}{EJ} \end{cases}$$ FIL

N.B. Fila 2

$$\begin{cases} \frac{6EJ}{\ell}\varphi_C + \frac{3EJ}{\ell^2}\eta - \frac{1}{2}p\ell^2 = 0 \\ -\frac{3EJ}{\ell^2}\varphi_C - \frac{41}{2}\frac{EJ}{\ell^3}\eta + 5p\ell^2 = 0 \end{cases} \quad \begin{cases} \varphi_C = -\frac{p\ell}{24} \\ \eta = \frac{p\ell}{4EJ} \end{cases}$$

Apro la struttura e valuto i momenti, i tagli e le azioni assiali (per la FILA 1, analogamente fila 2)

$$M_A = \frac{3EJ}{\ell^2}\left(\frac{p\ell^4}{8EJ}\right) - \frac{4p\ell^2}{8} = -\frac{p\ell^2}{8}$$

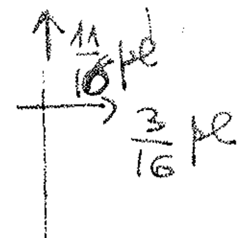
$$T_A = -\frac{3EJ}{\ell^3}\left(\frac{p\ell^4}{8EJ}\right) + \frac{9}{8}\frac{4p\ell^2}{\ell} = \frac{33}{8}p\ell \rightarrow N_{BC} = \frac{69}{16} - \frac{33}{8} = \frac{3}{16}p\ell$$

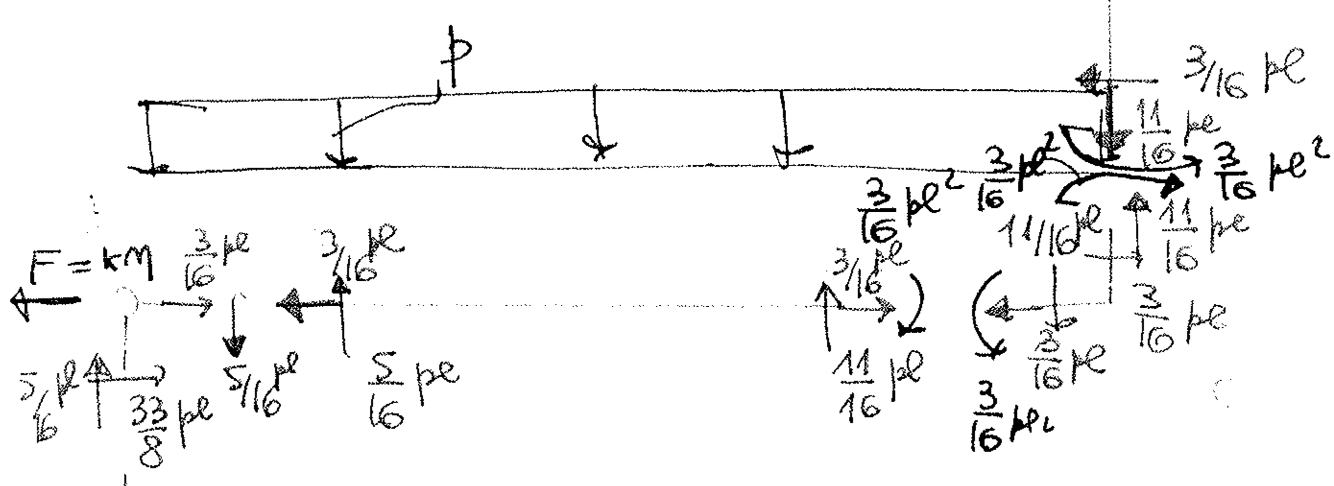
$$T_B = -\frac{3EJ}{\ell^2}\left(\frac{p\ell^3}{16EJ}\right) + \frac{3}{8}p\ell + \frac{6EJ}{\ell^3}\left(\frac{p\ell^4}{48EJ}\right) = \frac{5}{16}p\ell$$

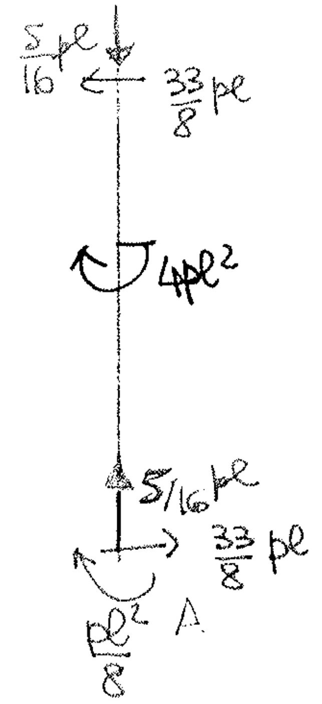
$$T_C = \frac{3EJ}{\ell^2}\left(\frac{p\ell^3}{16EJ}\right) + \frac{5}{8}p\ell - \frac{6}{48}p\ell = \frac{11}{16}p\ell$$ ... v. 2 pagine succ.

$$F = k\,\eta = \frac{69}{2}\frac{EJ}{l^3}\cdot\frac{1}{8}\frac{pl^4}{EJ} = \frac{69}{16}pl$$

diretta verso dx







VERIFICA degli EQUILIBRI

$\uparrow +$ $\quad \frac{5}{16}pl + \frac{11}{16}pl - pl = 0$ $\quad$ ok!

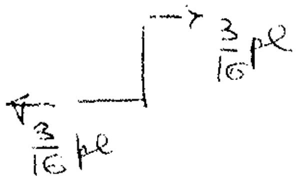
$\xrightarrow{+}$ $\quad \frac{33}{8}pl - \frac{69}{16}pl + \frac{3}{16}pl = 0$ $\quad$ ok!

$\circlearrowleft +_A$ $\quad -4pl^2 + \frac{69}{16}pl\cdot l - pl\cdot\frac{l}{2} - \frac{3}{16}pl\cdot 2l - \frac{pl^2}{8} + \frac{11}{16}pl^2 = 0$

$$\frac{-64+69-8-6-2+11}{16} = 0 \quad \text{Ok!}$$

$$T_C^{sopra} = -\frac{3EJ}{\ell^2}\left(\frac{1}{16}\frac{p\ell^3}{EJ}\right) - \frac{3EJ}{\ell^3}\left(\frac{p\ell^4}{8EJ}\right) + \frac{3EJ}{\ell}\cdot\left(\frac{1}{4}\frac{p\ell^2}{EJ}\right) = \frac{3}{16}p\ell$$

ok! Equilibrio al nodo C

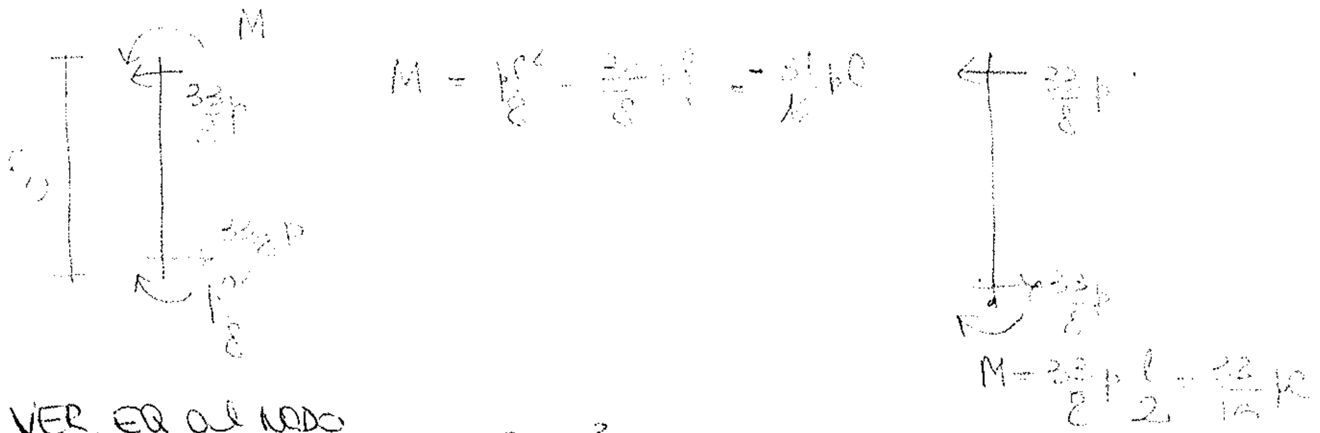


$T_D = T_C^{sopra}$

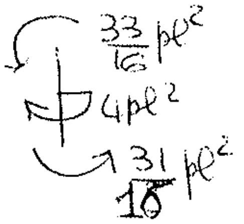
$$M_C^{sx} = \frac{3EJ}{\ell}\cdot\frac{1}{16}\frac{p\ell^3}{EJ} + \frac{p\ell^2}{8} - \frac{6EJ}{\ell^2}\cdot\frac{p\ell^4}{48EJ} = \frac{3}{16}p\ell^2$$

$$M_C^{dx} = \frac{3EJ}{\ell}\cdot\frac{1}{16}\frac{p\ell^3}{EJ} + \frac{3EJ}{\ell^2}\cdot\frac{p\ell^4}{8EJ} - \frac{3EJ}{\ell}\cdot\frac{p\ell^2}{4EJ} = -\frac{3}{16}p\ell^2$$

Asta AB (andam. dei momenti)

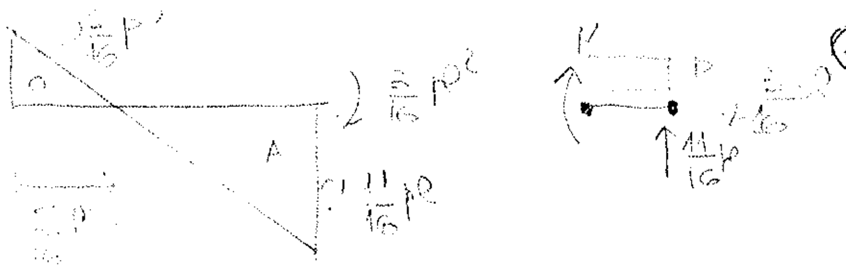


VER. EQ al nodo di mezzeria AB



$$\frac{33}{16} + \frac{31}{16} \overset{ok!}{=} 4$$

ASTA BC



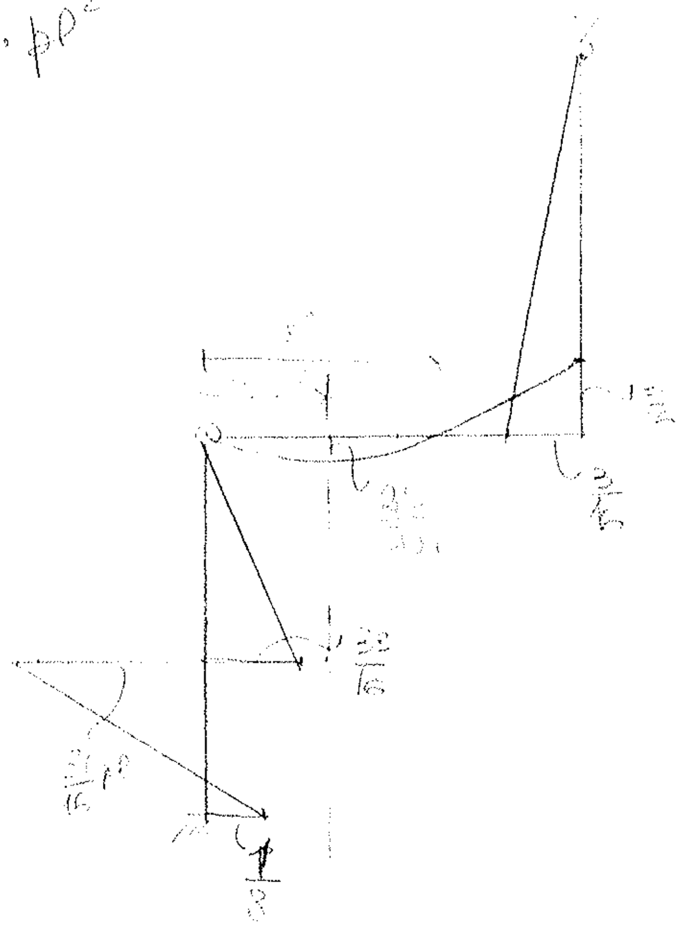
(*) $M + \frac{3}{16}p\ell^2 + p\frac{11\ell}{16}\cdot\frac{11\ell}{32} - \frac{11}{16}\ldots$

$$M = \frac{121}{256}p\ell^2 - \frac{121}{512}p\ell^2 - \frac{3}{16}p\ell^2$$

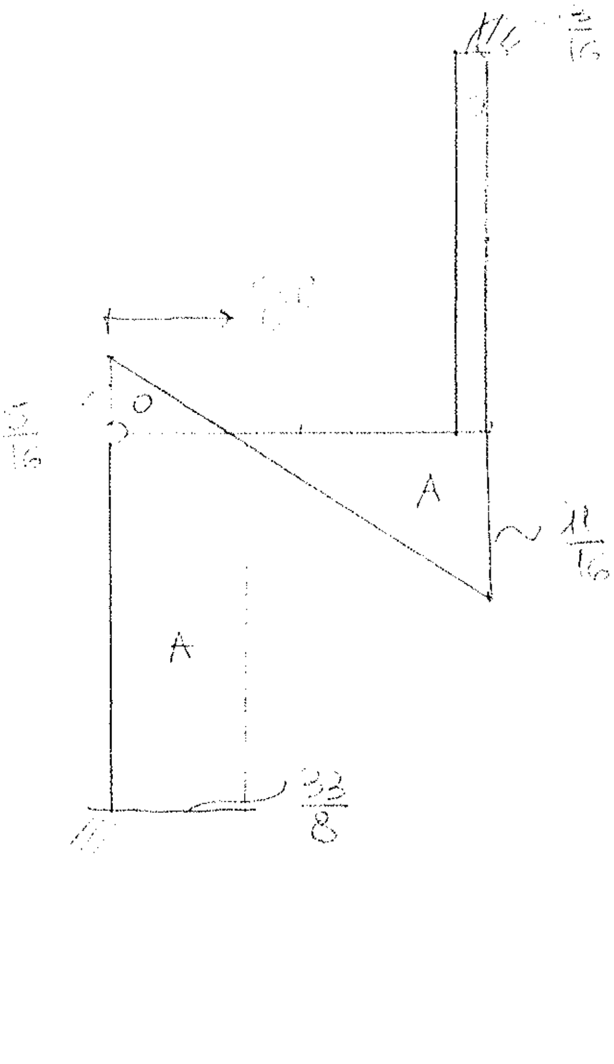
$$= \frac{242 - 121 - 96}{512} = \frac{25}{512}p\ell$$

(*) Calcolo del momento max

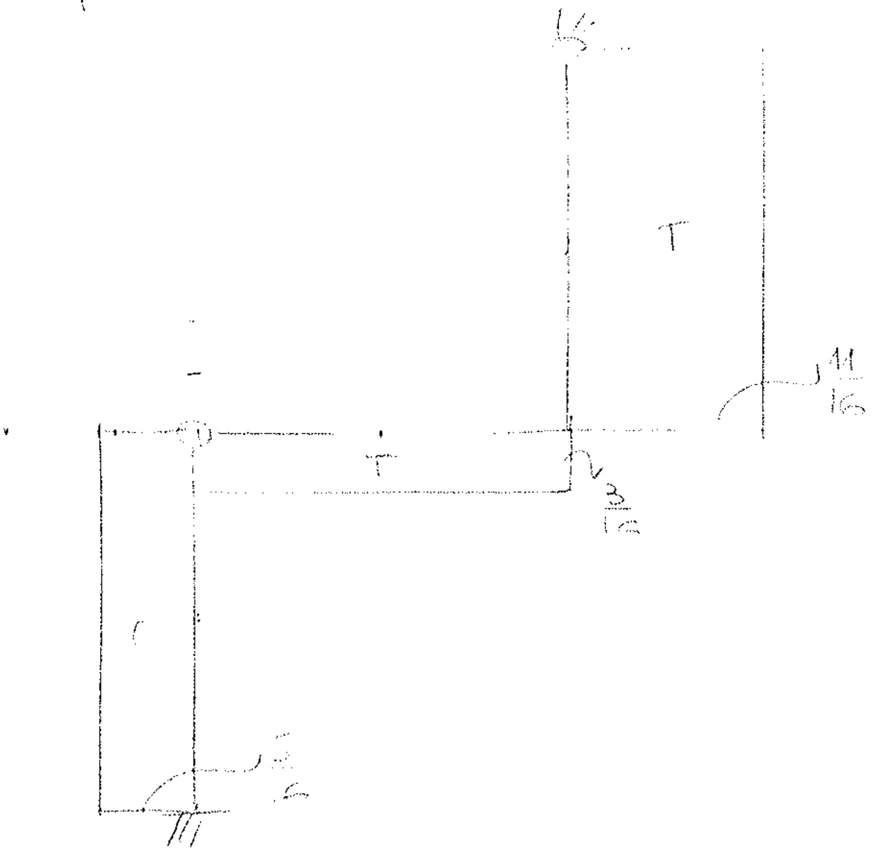
M · pl^2



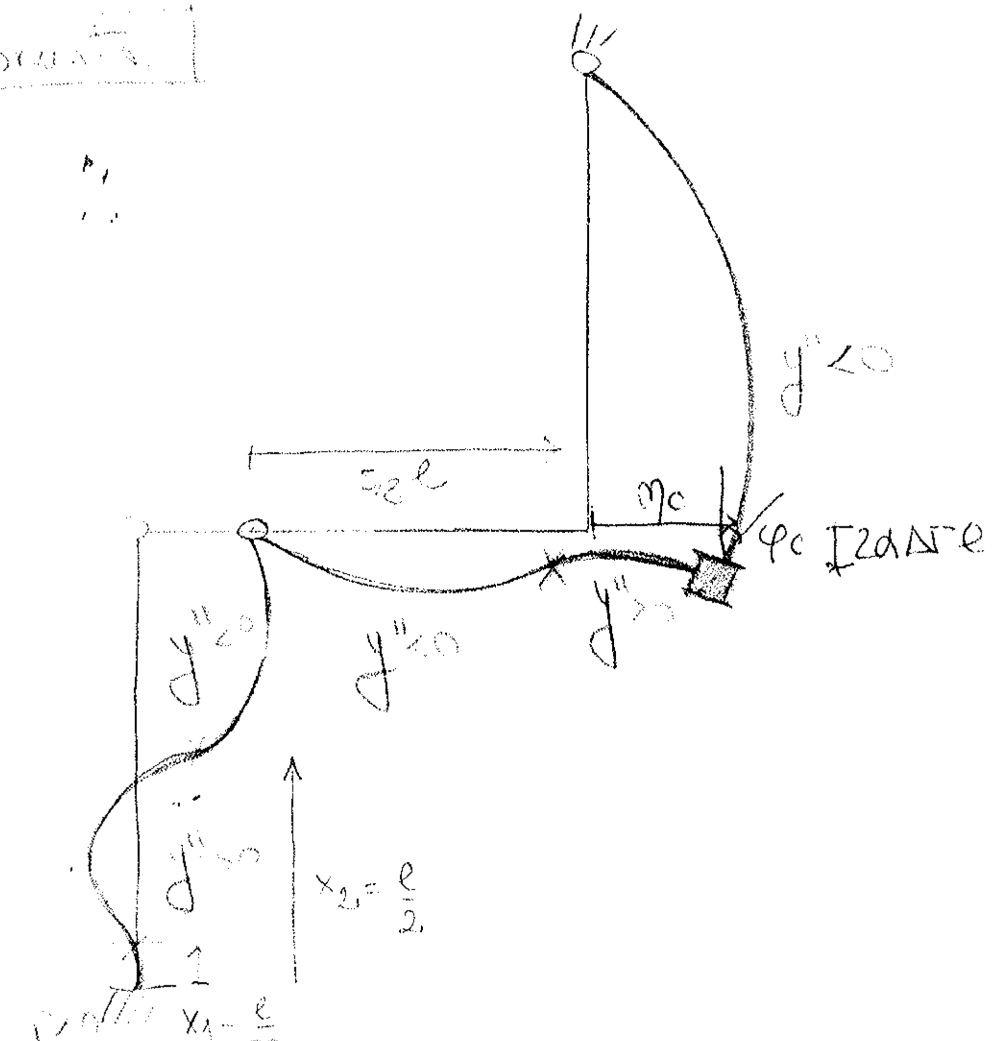
T pl



[M] · p^0

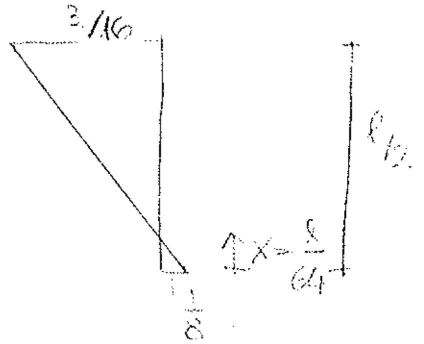


[Deformata]



Calcolo dei flessi della deformata

AB

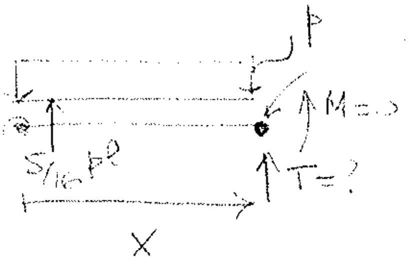


$$\frac{3}{16} : \left(\frac{\ell}{2} - x\right) = \frac{1}{8} : x$$

$$\frac{1}{8}\left(\frac{\ell}{2} - x\right) = \frac{3}{16}x \qquad \left(\frac{3}{16} + \frac{1}{8}\right) = \frac{\ell}{16}$$

$$x_1 = \frac{\ell}{...}$$

BC

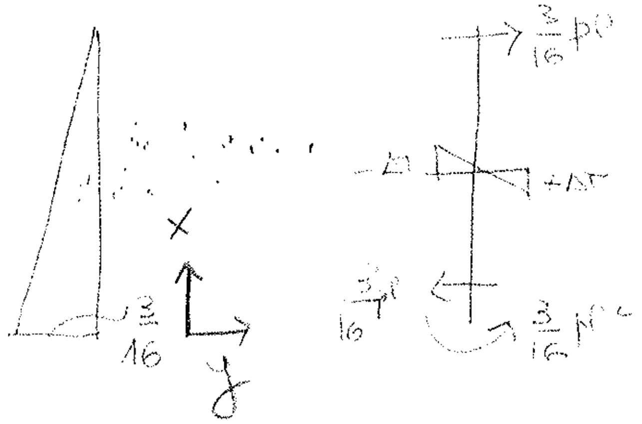


— ROT.

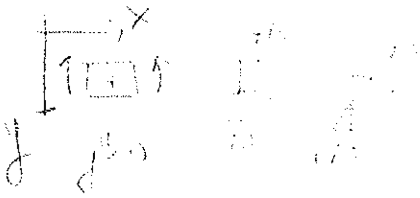
$$\frac{5}{16}p\ell x - p\frac{x^2}{2} = 0$$

$$x\left(\frac{5}{16}\ell - \frac{x}{2}\right) = 0 \qquad x = 0 \qquad x = \frac{5}{16}\ell \cdot 2 = \frac{5}{8}\ell$$

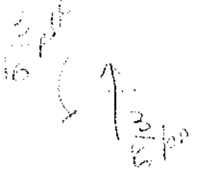
CD



$$y''(x) = -\frac{M(x)}{EJ} \mp \frac{2\alpha\Delta T}{h}$$



Nel nostro caso $y''(x) = -\frac{1}{EJ}\frac{3}{16}p\ell(x-\ell) - 2\cdot\frac{1}{4}\frac{p\ell^2}{EJ}$



$$M(x) = -\frac{3}{16}p\ell^2 + \frac{3}{16}p\ell x = \frac{3}{16}p\ell(-\ell + x)$$

$$y''(x) \overset{?}{<} 0 \qquad -\frac{1}{EJ}\frac{3}{16}p\ell(x-\ell) - \frac{p\ell^2}{2EJ} \leq 0$$

$$\frac{3}{16}x \geq \frac{3}{16}\ell - \frac{\ell}{2} \qquad x \geq \frac{16}{3}\left(\frac{3-8\ell}{16}\right)$$

$$x \geq -\frac{5}{3}\ell$$

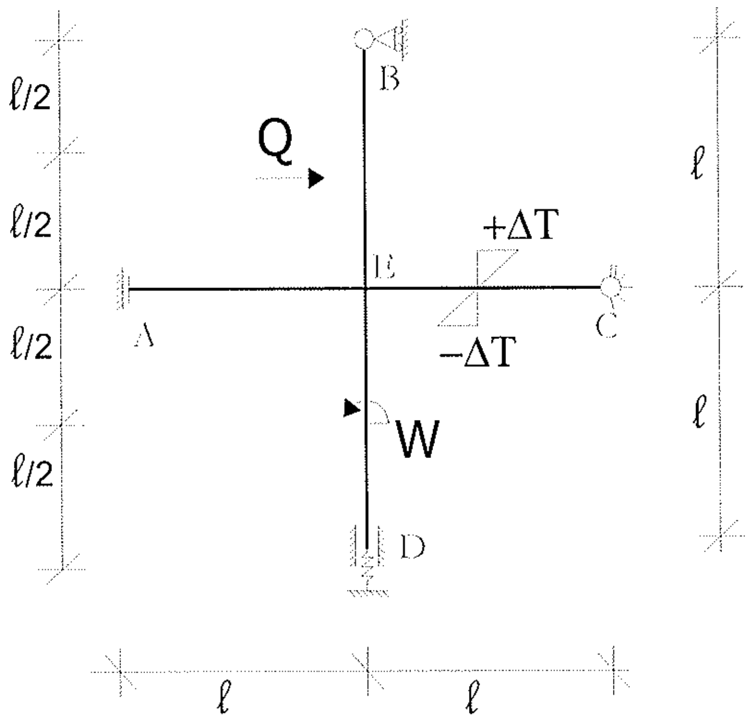
$$\rightarrow \forall x \geq -\frac{5}{3}\ell \qquad y''(x) < 0$$

Esame di
FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

PROVA SCRITTA

20 – 03 – 2009



$$k_\eta = \frac{6EJ}{\ell^3}$$

$$\frac{\alpha \Delta T}{h} = \frac{p\ell^2}{EJ}$$

$$W = 4p\ell^2$$

$$Q = 2p\ell$$

Si richiedono i grafici di:

- Momento flettente (con il valore e la posizione dei massimi)
- Taglio
- Azione assiale
- Deformata qualitativa con posizione dei flessi

SOLUZIONE

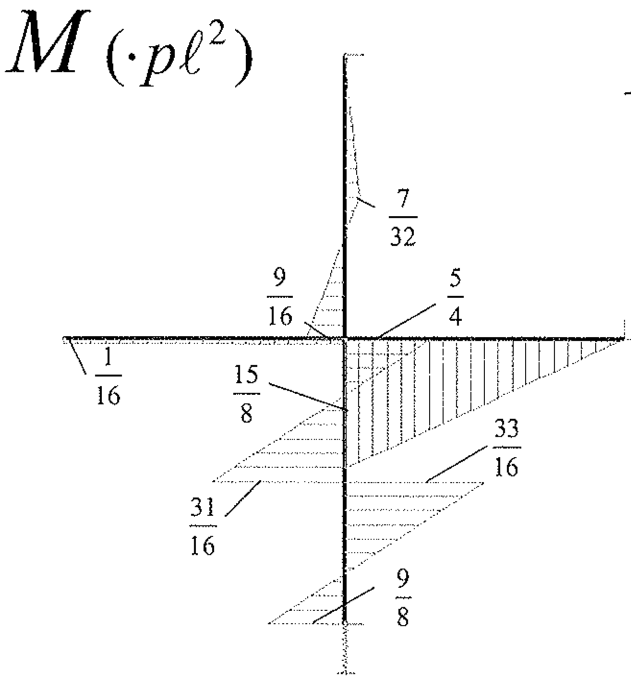
Sistema risolvente

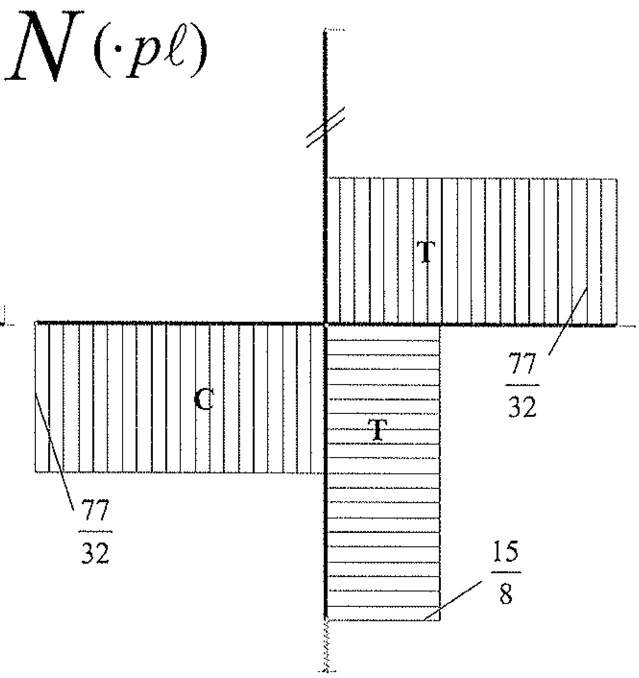
$$\begin{cases} \dfrac{11EJ}{\ell}\varphi_E + \dfrac{3EJ}{\ell^2}\eta_D + \dfrac{13}{8}p\ell^2 = 0 \\ \dfrac{3EJ}{\ell^2}\varphi_E + \dfrac{9EJ}{\ell^3}\eta_D + 3p\ell = 0 \end{cases}$$

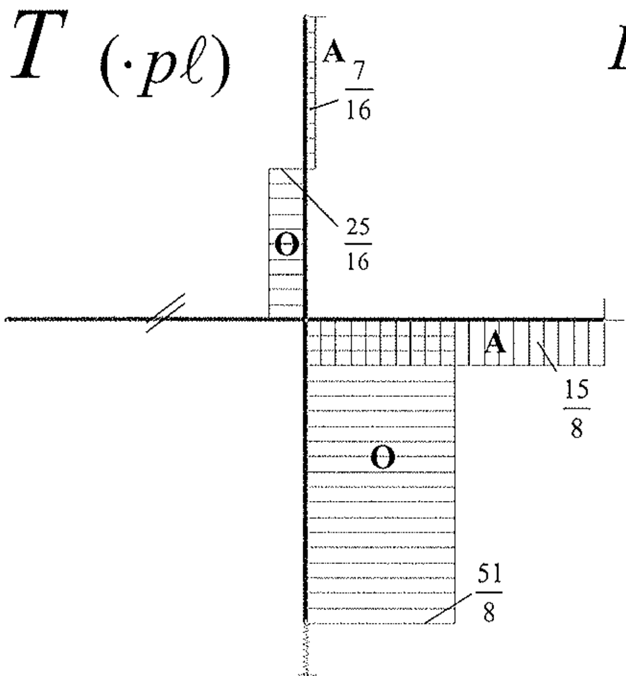
Soluzioni

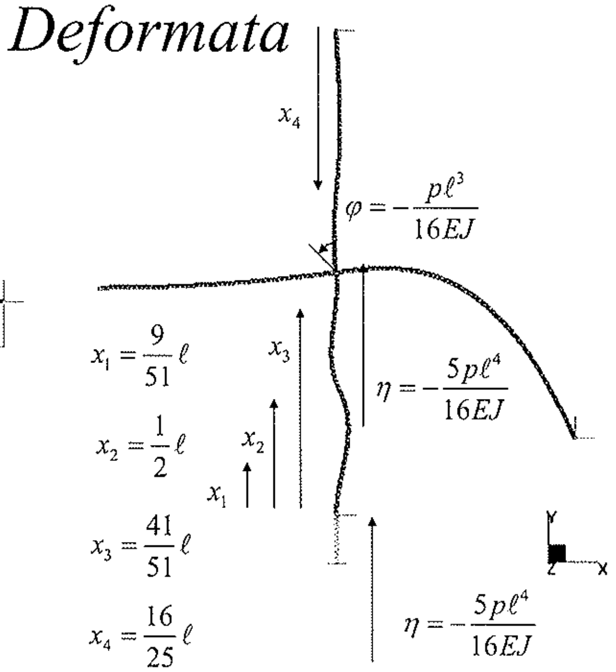
$$\begin{cases} \varphi_E = -\dfrac{p\ell^3}{16EJ} \\ \eta_D = -\dfrac{5p\ell^4}{16EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata



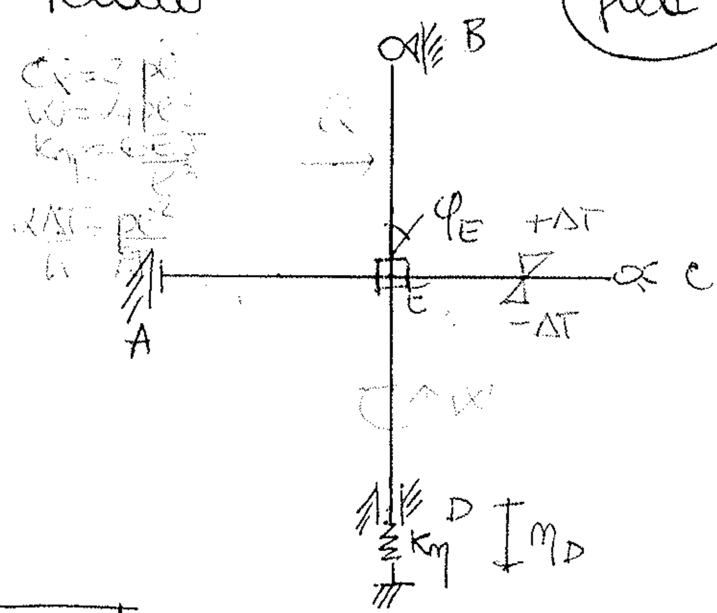






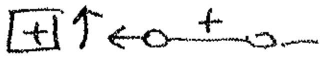
Telaio

fila 1

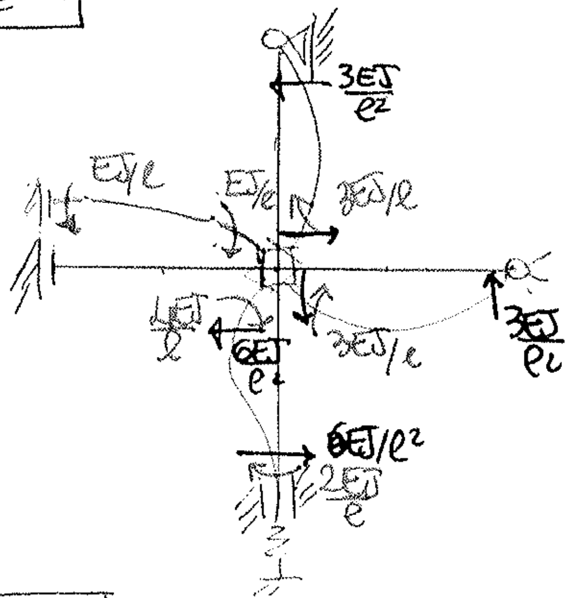


SISTEMA RISOLVENTE

$$\begin{cases} m_{E\varphi}\varphi_E + m_{E\eta}\eta_D + m_{E0} = 0 \\ h_{D\varphi}\varphi_E + h_{D\eta}\eta_D + h_{D0} = 0 \end{cases}$$



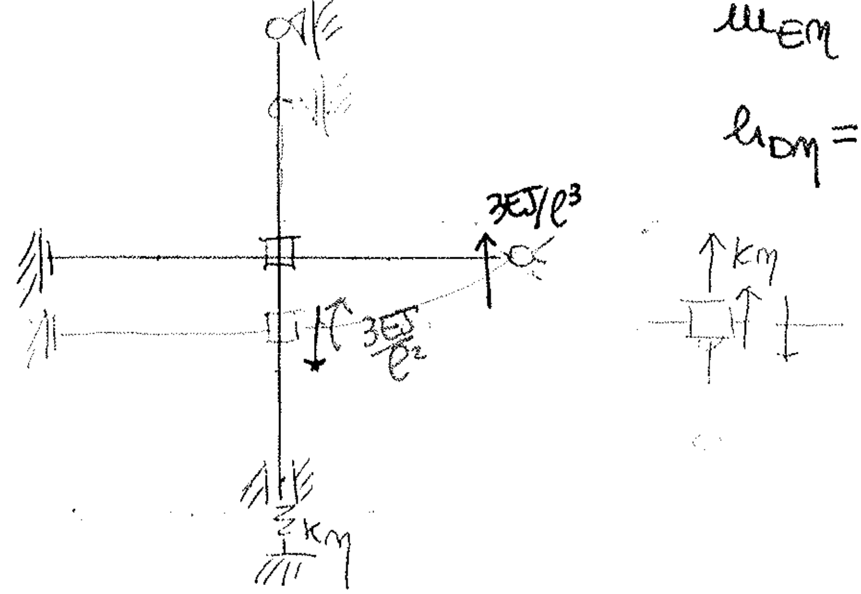
$\varphi_E = 1$



$$m_{E\varphi} = 11EJ/l$$

$$h_{D\varphi} = \frac{3EJ}{l^2}$$

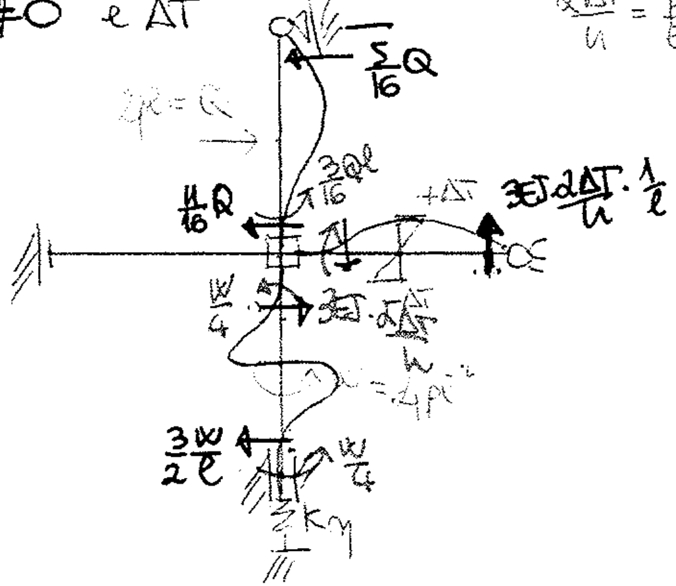
$\eta_D = 1$



$$m_{E\eta} = 3EJ/l^2$$

$$h_{D\eta} = \frac{3EJ}{l^3} + k_\eta = \frac{3EJ}{l^3} + \frac{6EJ}{l^3} = \frac{9EJ}{l^3}$$

$p \neq 0$ e ΔT $\qquad \frac{\alpha \Delta T}{h} = \frac{p\ell^2}{EJ}$



$$u_{EO} = -\left[\frac{3}{16} Q\ell + \frac{W}{4} - 3EJ \cdot \frac{\alpha \Delta T}{h}\right] =$$

$$= -\left[\frac{3}{8} p\ell^2 + p\ell^2 - 3p\ell^2\right] = +\frac{13}{8} p\ell^2$$

$$h_{Do} = 3p\ell = 3EJ \cdot \frac{\alpha \Delta T}{h} \cdot \frac{1}{\ell}$$

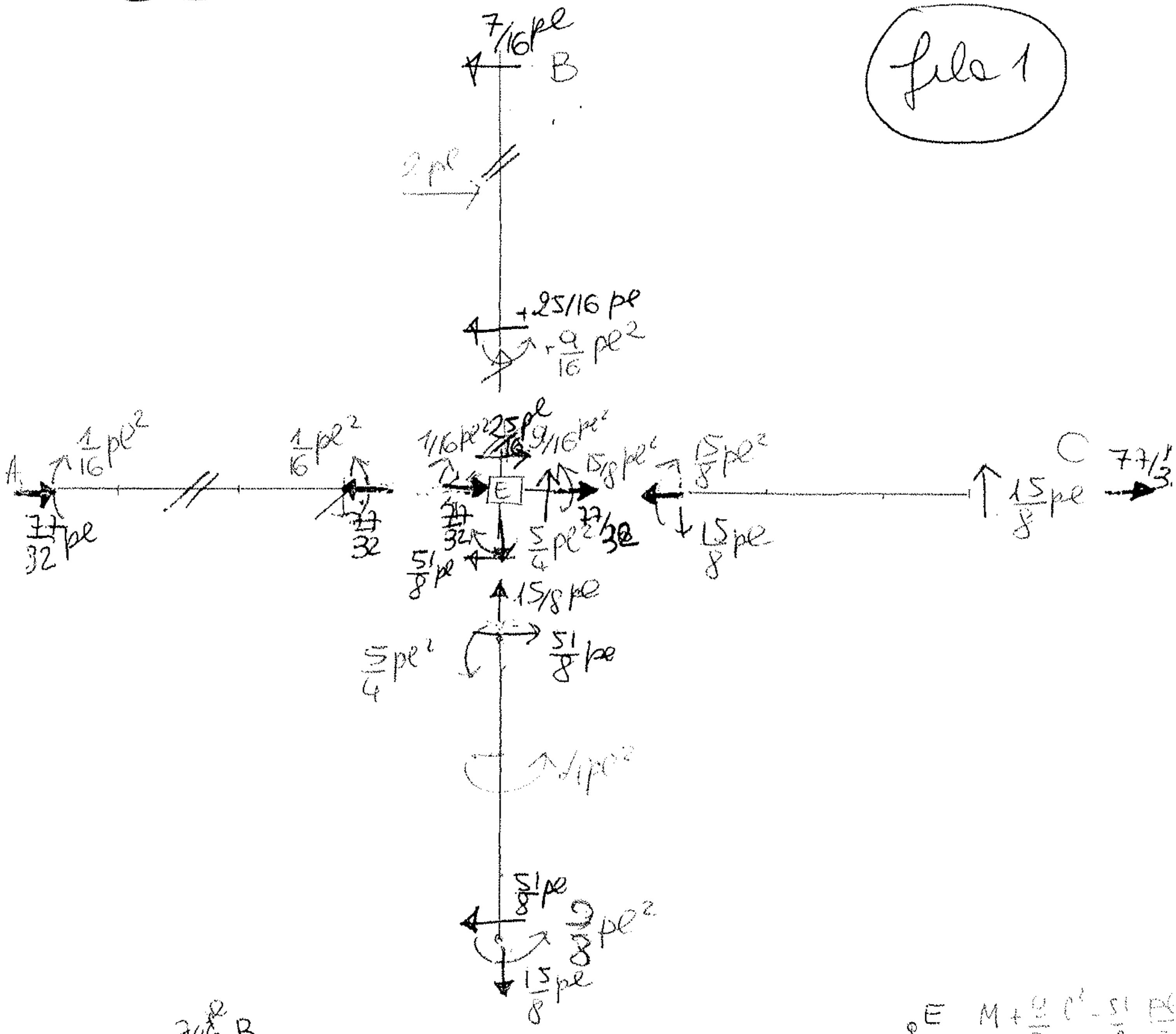
sistema risolvente

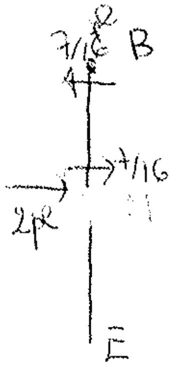
$$\begin{cases} \frac{11EJ}{\ell} \varphi_E + \frac{3EJ}{\ell^2} m_D + \frac{13}{8} p\ell^2 = 0 \\ \frac{3EJ}{\ell^2} \varphi_E + \frac{9EJ}{\ell^3} m_D + 3p\ell = 0 \end{cases}$$

$$\varphi_E = -1/16 \, p\ell^3/EJ$$

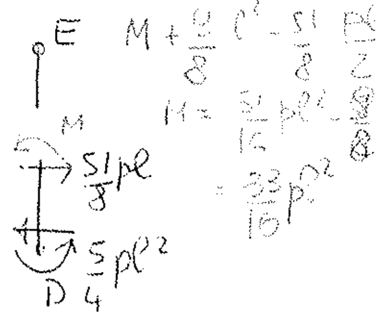
$$m_D = -5/16 \, \frac{p\ell^4}{EJ}$$

fila 1



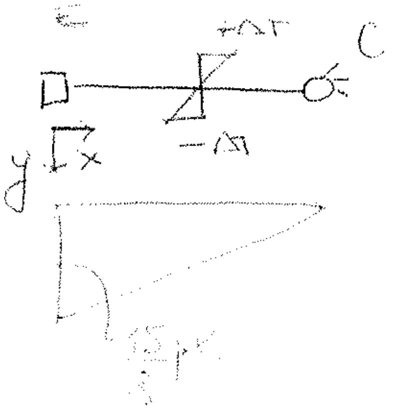


$$M + \frac{7}{16} \frac{pl^2}{2} - M = 0 \qquad M = \frac{7}{32} pl^2$$

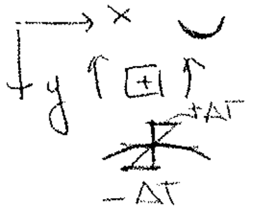


DEFORMATA & FLESSI

1)



$$M(x) + \frac{15}{8}pl\,x - \frac{15}{8}pl^2 = 0 \;\rightarrow\; M(x) = \frac{15}{8}pl(l - x)$$



$$y'' = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{H}$$

$$y'' = -\frac{15}{8}pl(l-x)\frac{1}{EJ} + \frac{2pl^2}{EJ} \geq 0$$

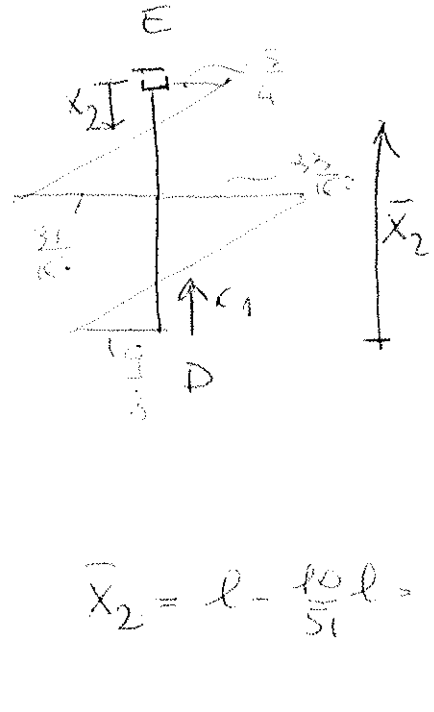
$$-15pl(l-x) + 16pl^2 \geq 0$$

$$-15pl^2 + 15pl\,x + 16pl^2 \geq 0 \qquad 15pl\,x + pl^2 \geq 0$$

$$x \geq -\frac{l}{15}$$

quindi $y'' \geq 0 \quad \forall x$

2)



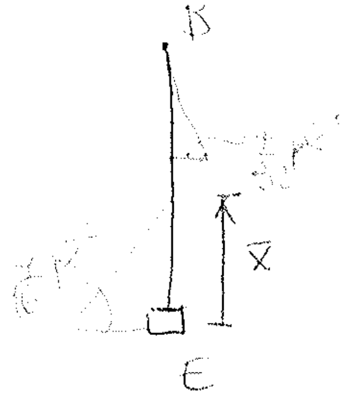
$$\left(\frac{33}{16} + \frac{18}{16}\right) : \frac{l}{2} = \frac{18}{16} \cdot x$$

$$x_1 = \frac{\frac{l}{2}\cdot\frac{18}{16}}{\frac{51}{16}} = \frac{9}{16}\cdot\frac{16}{51}l = \frac{9}{51}l$$

$$x_2 = \frac{l}{2}\cdot\frac{5}{4} \Big/ \left(\frac{31}{16} + \frac{20}{16}\right) = \frac{5l}{8}\cdot\frac{16}{51} = \frac{10}{51}l$$

$$\bar{x}_2 = l - \frac{10}{51}l = \frac{41}{51}l$$

3)



$$\left(\frac{7}{32}+\frac{18}{32}\right)\cdot\frac{l}{2} = \frac{9}{16}\cdot x$$

$$x = \frac{9}{16\cdot 2}\Big/\frac{25}{32}\,l = \frac{9}{16}\,\frac{32}{25}\,\frac{1l}{2} = \frac{9}{25}\,l$$

Esame di
FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

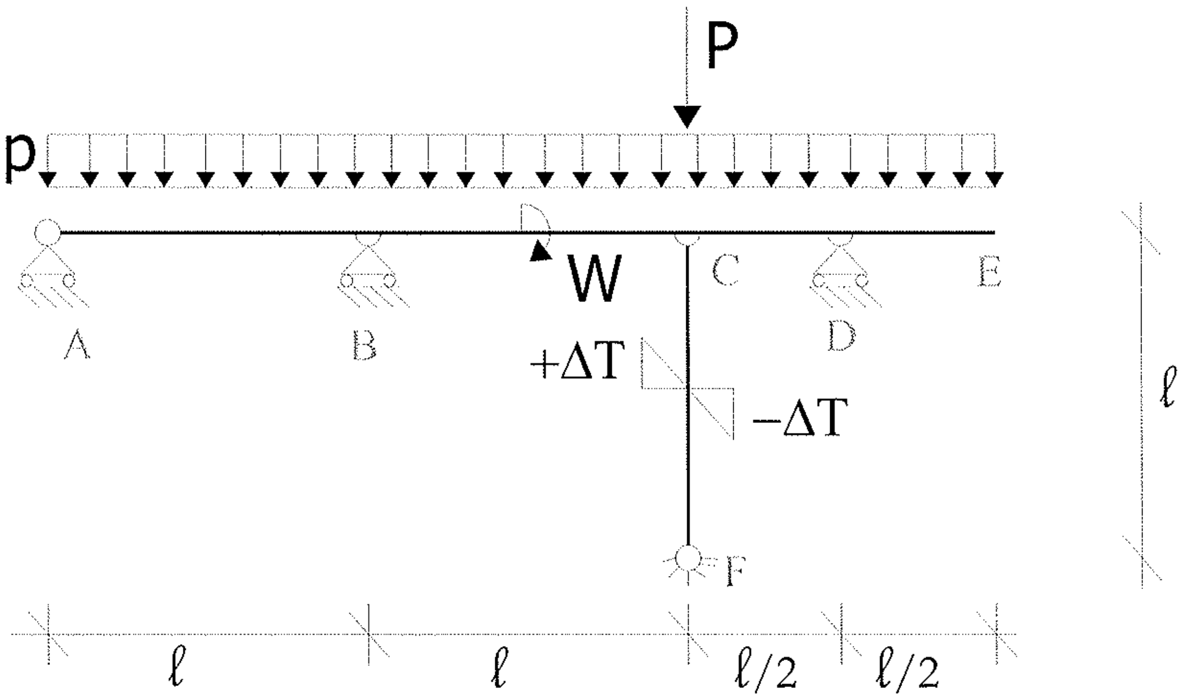
e

TECNICA DELLE COSTRUZIONI

(Corsi di laurea: Edile – Architettonico, Civile e per l'Ambiente e il Territorio V.O.)

PROVA SCRITTA

22 – 06 – 2009



$$\frac{\alpha\Delta T}{h} = \frac{5p\ell^2}{4EJ} \qquad W = 3p\ell^2 \qquad \mathrm{P} = \frac{7p\ell}{32}$$

Si richiedono i grafici di:

- Momento flettente (con il valore e la posizione dei massimi)
- Taglio
- Azione assiale
- Deformata qualitativa con posizione dei flessi

SOLUZIONE

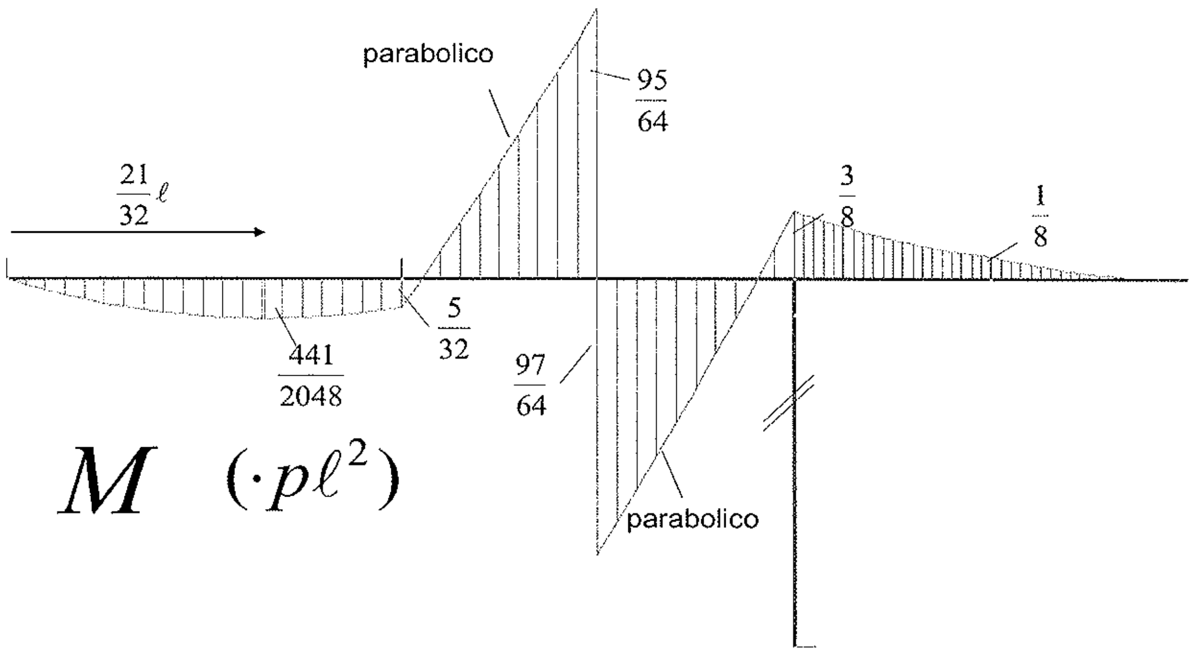
Sistema risolvente

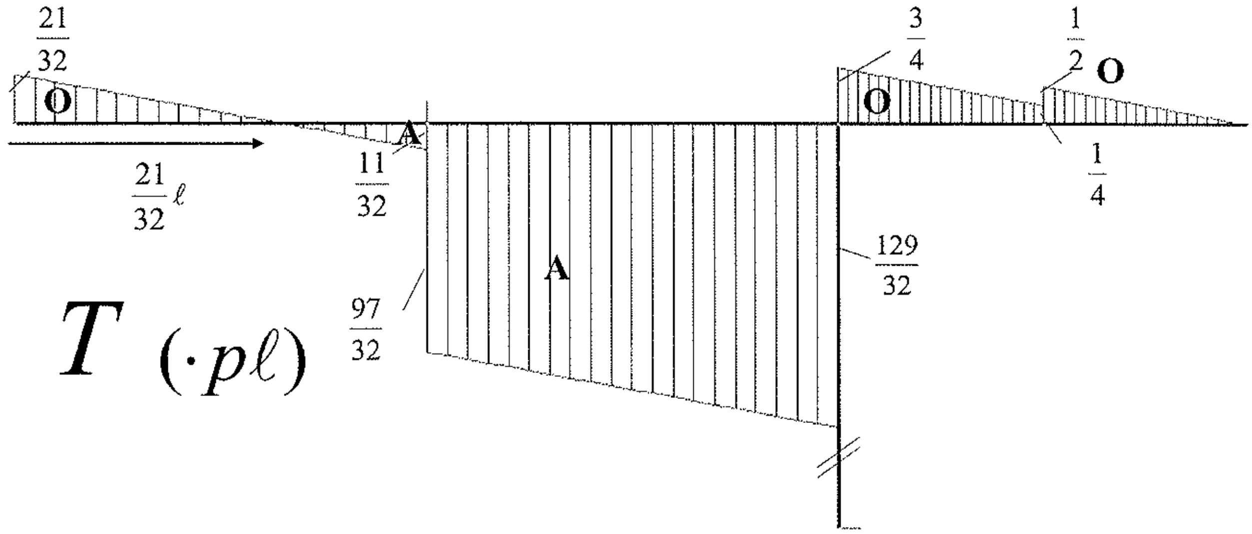
$$\begin{cases} \dfrac{2\ell}{3EJ}X_B + \dfrac{\ell}{6EJ}X_C - \dfrac{p\ell^3}{24EJ} = 0 \\ \dfrac{\ell}{6EJ}X_B + \dfrac{\ell}{2EJ}X_C + \dfrac{31p\ell^3}{192EJ} = 0 \end{cases}$$

Soluzioni

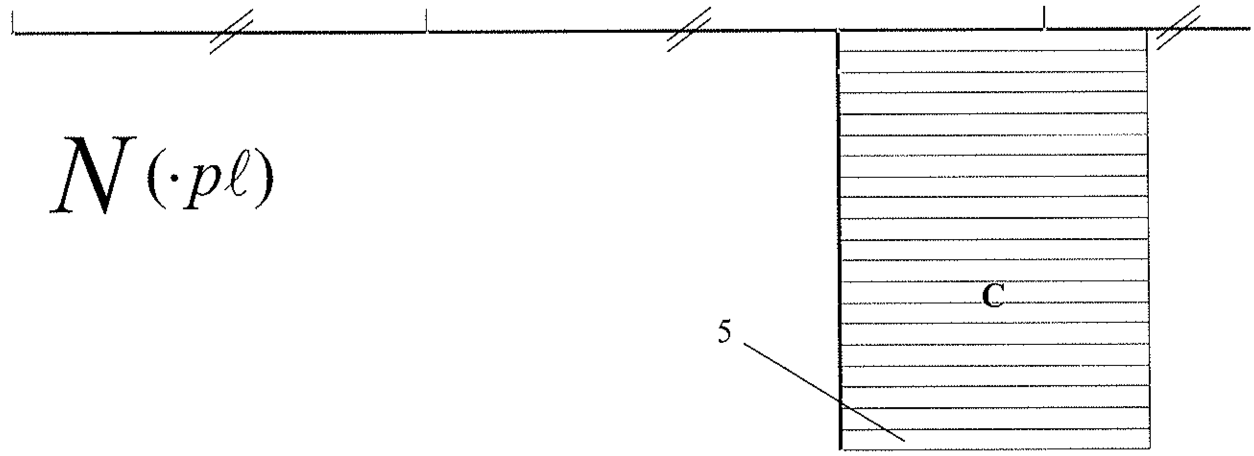
$$\begin{cases} X_B = +\dfrac{5}{32}p\ell^2 \\ X_C = -\dfrac{3}{8}p\ell^2 \end{cases}$$

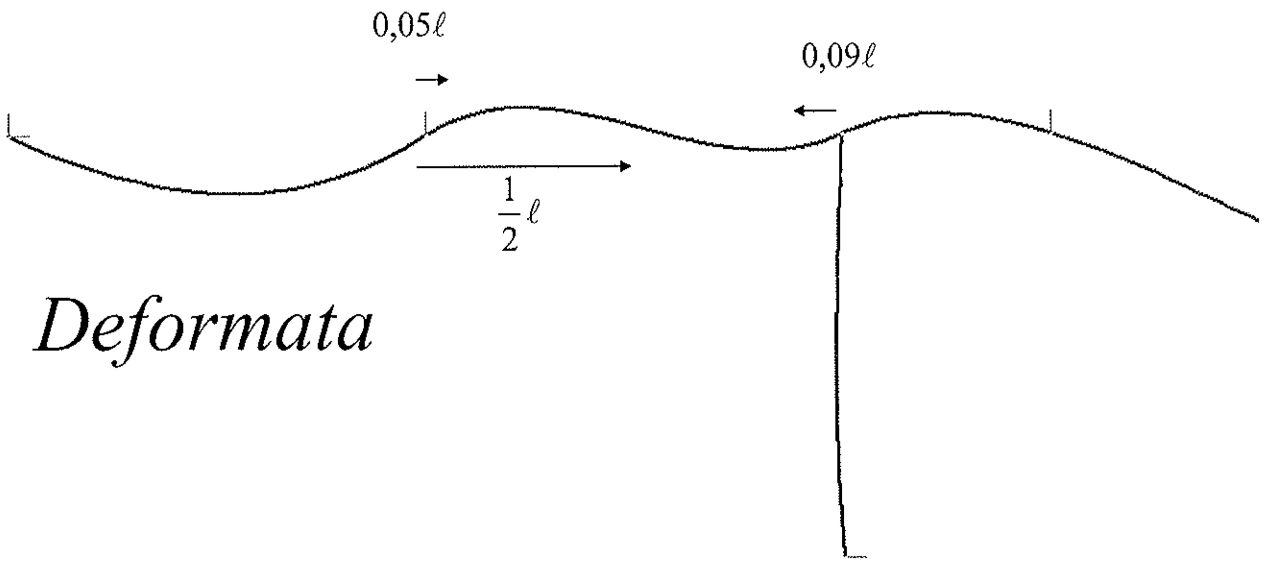
Diagrammi delle azioni interne e Deformata





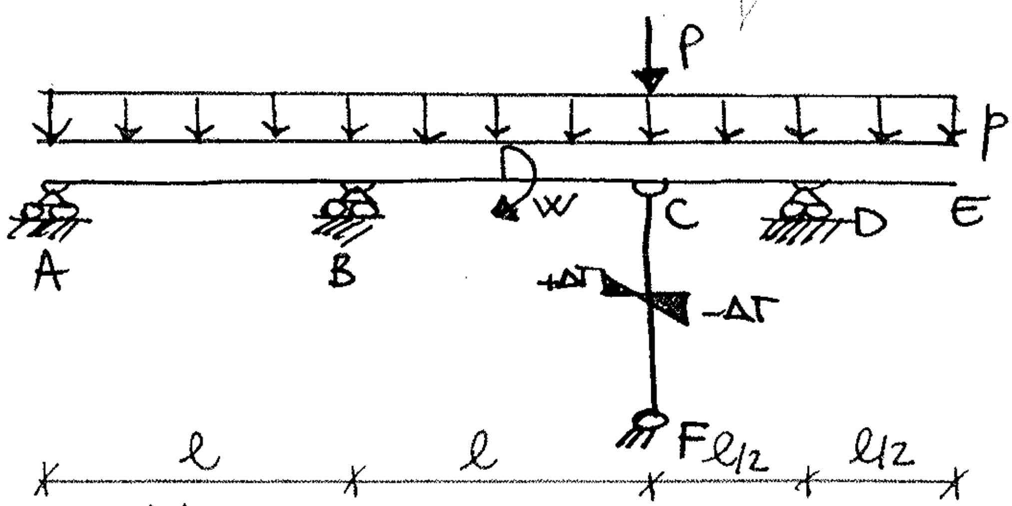
$N\,(\cdot p\ell)$





Deformata

Tema esame del 22 giugno 2009



$P = \frac{7pl}{32}$

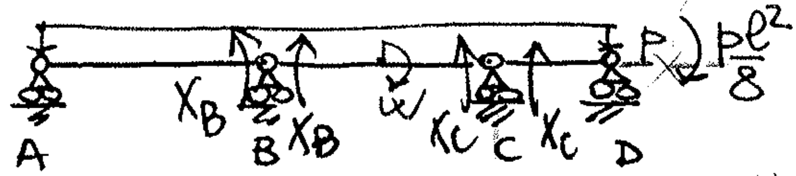
$W = 3pl^2$

$\frac{\alpha \Delta T}{h} = \frac{5}{4}\frac{pl^2}{EJ}$

$$\begin{cases} X_B \varphi_{BB} + X_C \varphi_{BC} + \varphi_{B0} = 0 \\ X_B \varphi_{CB} + X_C \varphi_{CC} + \varphi_{C0} = 0 \end{cases}$$

si tratta di una trave continua* → → MTD FORZE

* mi accorgo dell'appendice isostatica DE e CF quindi riduco la struttura a:

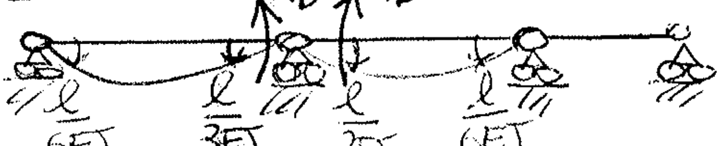


mensola

$M_C = \frac{pl}{2} \cdot \frac{l}{4} = \frac{pl^2}{8}$

↖ si tratta di un'appendice isostatica → la def. termica non produce alcuna azione interna

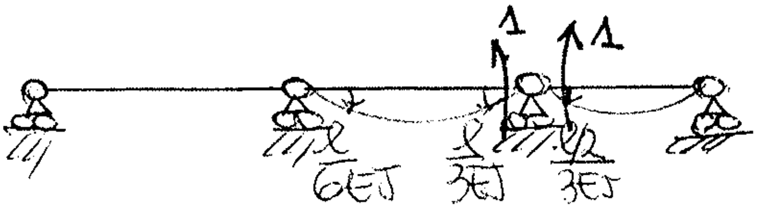
$X_B = 1$



$\varphi_{BB} = \frac{l}{3EJ} - \left(-\frac{l}{3EJ}\right) = \frac{2}{3}\frac{l}{EJ}$

$\varphi_{CB} = 0 - \left(-\frac{l}{6EJ}\right) = \frac{l}{6EJ}$

$X_C = 1$

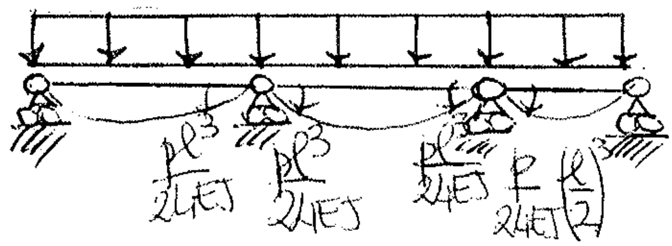


$\varphi_{BC} = \frac{l}{6EJ}$

$\varphi_{CC} = \frac{l}{6EJ} - \left(-\frac{l}{3EJ}\right) = \frac{1+2}{6} = \frac{l}{2EJ}$

≠0

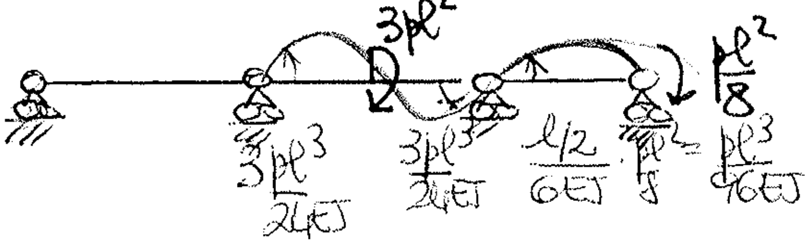
1)



$$\varphi^1_{B0} = \frac{pl^3}{24EJ} - \left(-\frac{pl^3}{24EJ}\right) = \frac{pl^3}{12EJ}$$

$$\varphi^1_{C0} = \frac{pl^3}{24\cdot 8EJ} - \left(-\frac{pl^3}{24EJ}\right) = \frac{1+8}{24\cdot 8}\frac{pl^3}{EJ} = \frac{9}{192}\frac{pl^3}{EJ}$$

2)



$$\varphi^2_{B0} = -\frac{3}{24}\frac{pl^3}{EJ} = -\frac{pl^3}{8EJ}$$

$$\varphi^2_{C0} = -\frac{pl^3}{96EJ} - \left(-\frac{pl^3}{8EJ}\right) = \frac{-1+12}{96}\frac{pl^3}{EJ} = \frac{11}{96}\frac{pl^3}{EJ}$$

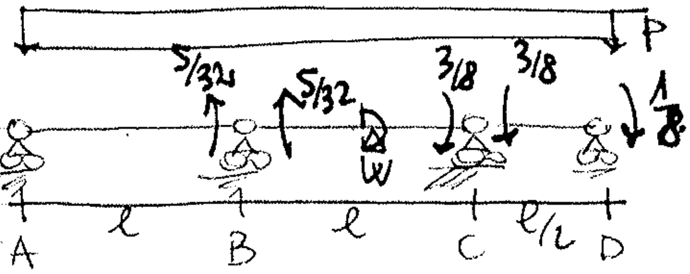
$$\varphi_{B0} = \varphi^1_{B0} + \varphi^2_{B0} = \frac{1}{12} - \frac{1}{8} = \frac{2-3}{24} = -\frac{pl^3}{24EJ}$$

$$\varphi_{C0} = \varphi^1_{C0} + \varphi^2_{C0} = \frac{9}{192} + \frac{11}{96} = \frac{9+22}{192}\frac{pl^3}{EJ} = \frac{31}{192}\frac{pl^3}{EJ}$$

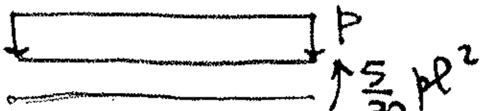
$$\frac{2}{3}\frac{l}{EJ}X_B + \frac{l}{6EJ}X_C - \frac{pl^3}{24EJ} = 0$$

$$\frac{l}{6EJ}X_B + \frac{l}{2EJ}X_C + \frac{31}{192}\frac{pl^3}{EJ} = 0$$

$$\begin{cases} X_B = \frac{5}{32}pl^2 \\ X_C = -\frac{3}{8}pl^2 \end{cases}$$



AB

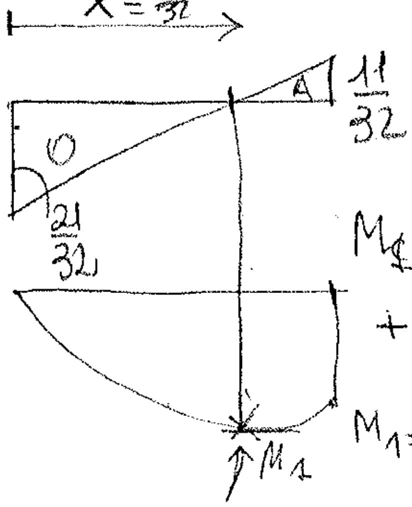


A: $\uparrow \frac{5}{32}pl$, $\uparrow \frac{pl}{2}$

B: $\downarrow \frac{5}{32}pl$, $\uparrow \frac{pl}{2}$

$=$

$\uparrow \frac{1}{2} + \frac{5}{32} = \frac{21}{32} pl \qquad \uparrow \frac{1}{2} - \frac{5}{32} = \frac{11}{32} pl$

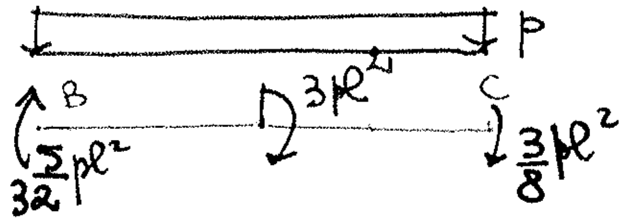


$M_1 - \frac{21}{32}pl \cdot \frac{21}{32}l + \frac{21}{32}pl \cdot \frac{1}{2}\frac{21}{32}l = 0$

$M_1 = \left(\frac{21}{32}\right)^2 pl^2 - \frac{1}{2}\left(\frac{21}{32}\right)^2 pl^2$

$= \frac{1}{2}\left(\frac{21}{32}\right)^2 pl^2$

BC



B: $\downarrow \frac{5}{32}pl$, $\downarrow \frac{3}{8}pl$, $\uparrow \frac{1}{2}pl$, $\downarrow 3pl$

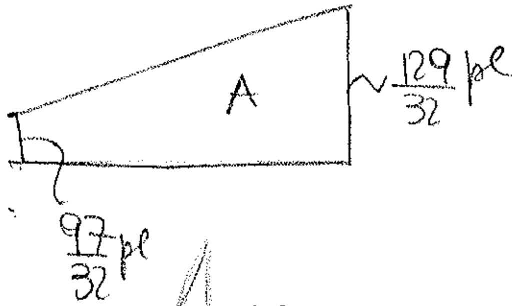
C: $\uparrow \frac{5}{32}pl$, $\uparrow \frac{3}{8}pl$, $\uparrow \frac{1}{2}pl$, $\uparrow 3pl$

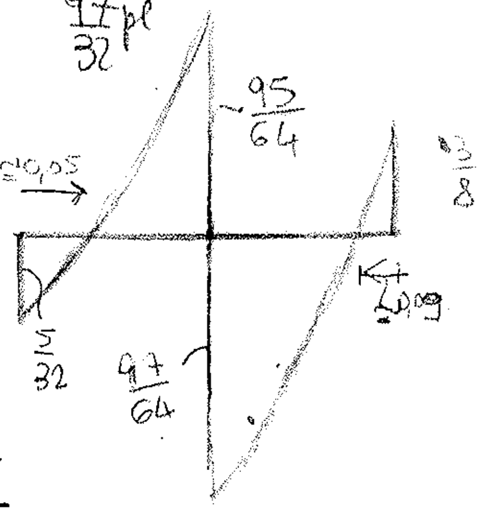
$=$

$\downarrow 3pl - \frac{1}{2} + \frac{3}{8} + \frac{5}{32} = \qquad \uparrow 3 + \frac{1}{2} + \frac{3}{8} + \frac{5}{32} =$

$= \frac{96 - 16 + 12 + 5}{32} \qquad = \frac{96 + 16 + 12 + 5}{32}$

$= 97/32\, pl \qquad = \frac{129}{32} pl$





flem:

$x_1 = ?$ $\quad \frac{5}{32}pl^2 - \frac{97}{32}pl\,x - \frac{px^2}{2} = 0$

$16px^2 + 97pl\,x - 5pl^2 = 0$

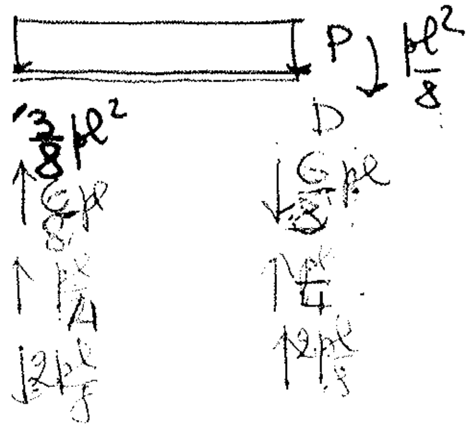
$x_{12} = \frac{-97 \mp \sqrt{97^2 + 320}}{32}$ < 0,05l, NA

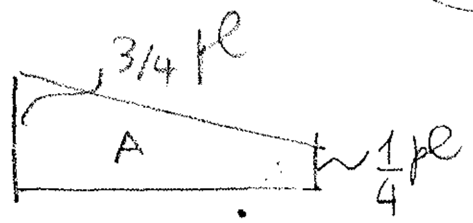
$x_2 = :$ $\quad \frac{3}{8}pl^2 - \frac{129}{32}pl\,x + \frac{px^2}{2} = 0$

$16px^2 - 129pl\,x + 12pl = 0$

x — 0,09l

(CD)

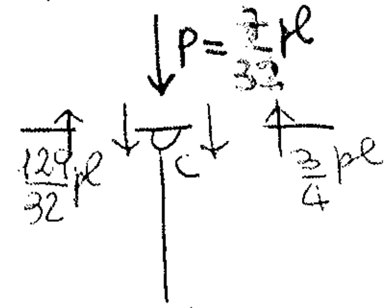




$= \uparrow + \frac{6}{8} - \frac{1}{4} = \frac{3}{4}pl$

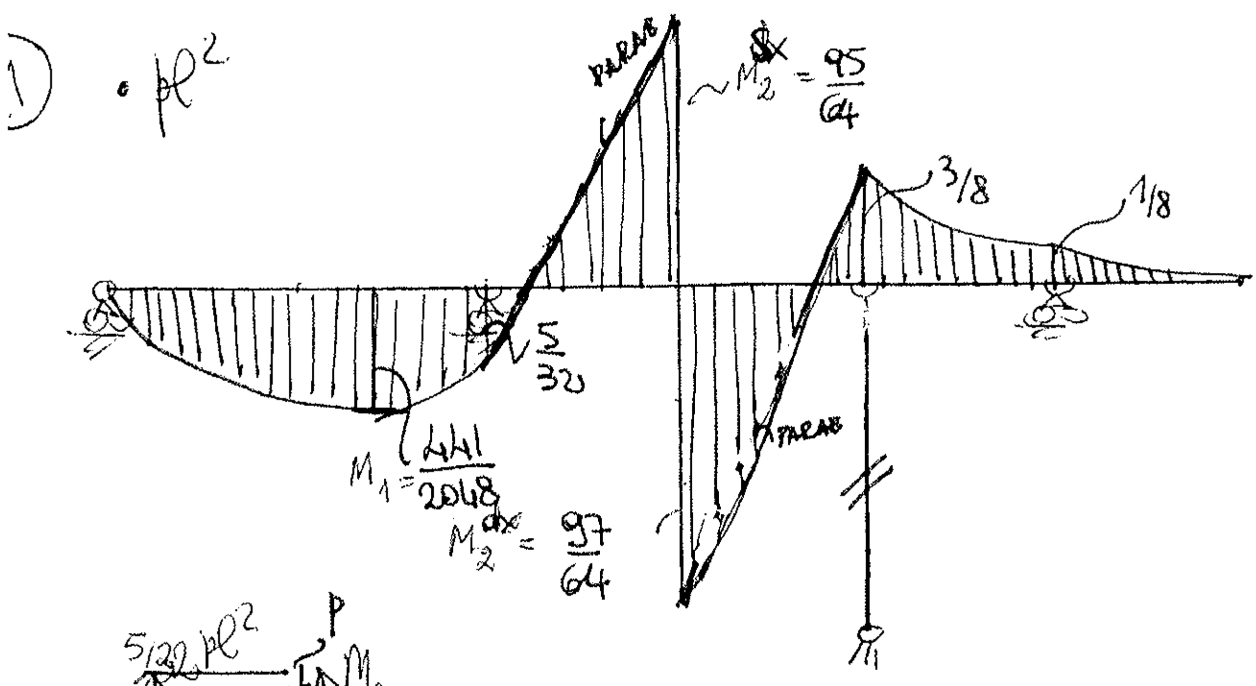
$= \uparrow \frac{1}{4} + \frac{1}{4} - \frac{6}{8} = \frac{4-6}{8} = -\frac{1}{4}pl$

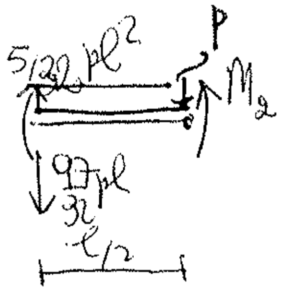
...zione assiale in CF



$\uparrow\left(\frac{129}{32} + \frac{3}{4} + \frac{7}{32}\right)pl = 5pl$

1) $\cdot pl^2$



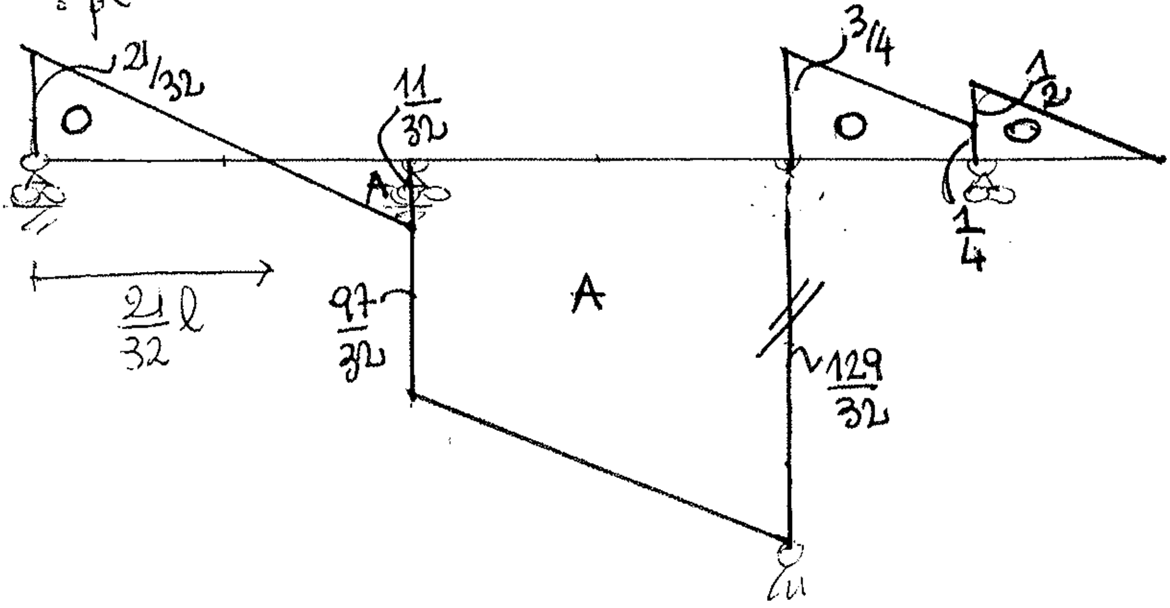


$M_2^{sx} - \frac{5}{32}pl^2 + \frac{97}{32}\frac{1}{2}pl^2 + \frac{pl^2}{8} = 0$

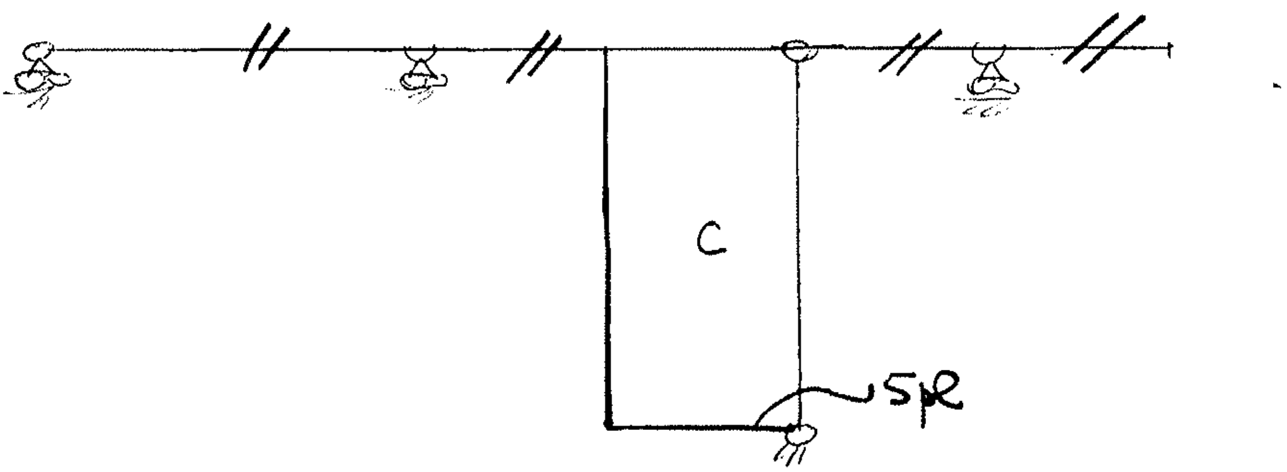
$M_2^{sx} = \frac{+10-97-8}{64} = -\frac{95}{64}pl^2$

$\frac{95}{64}pl^2 + M_2^{dx} - 3pl^2 = 0 \quad \rightarrow \quad M_2^{dx} = 97/64\,pl^2$

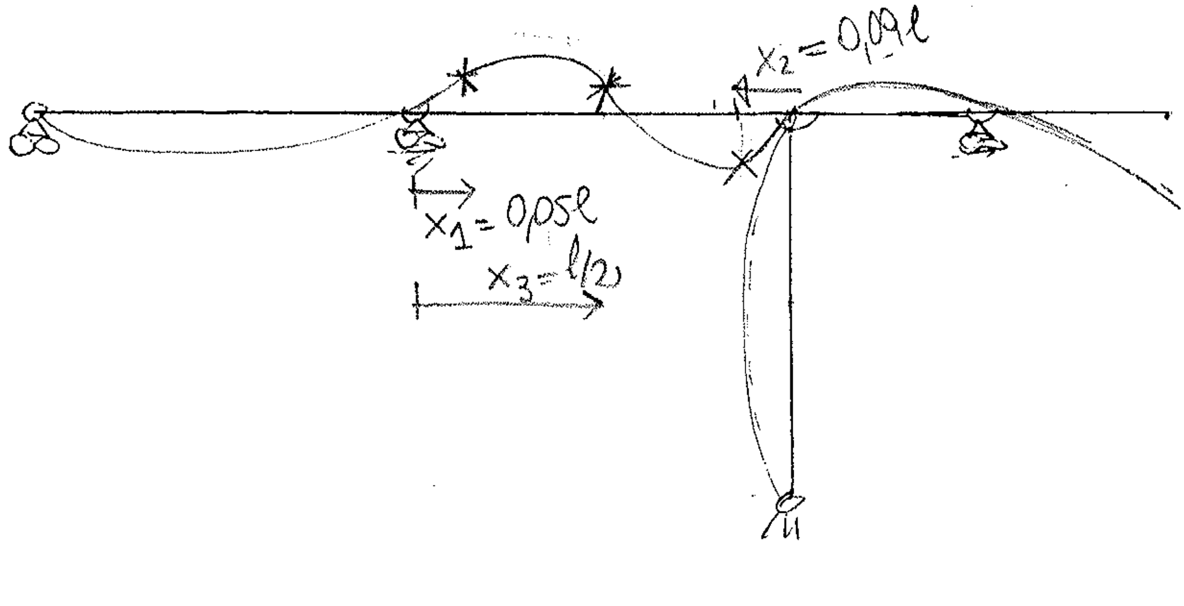
(T) · pl

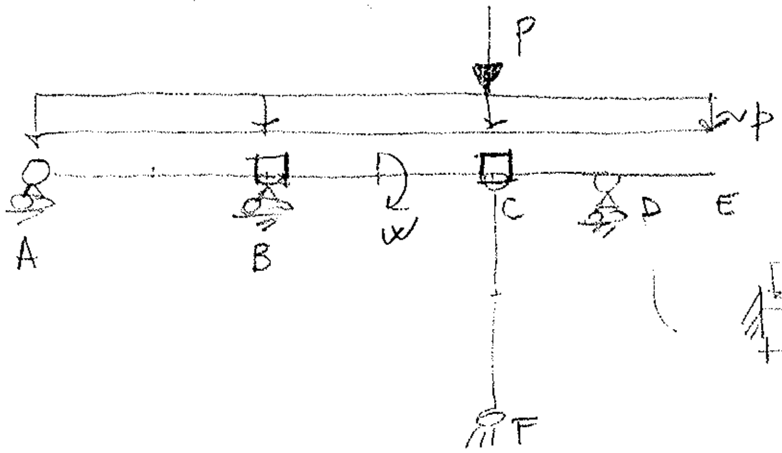


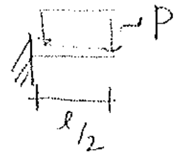
(N) · pl



deformata



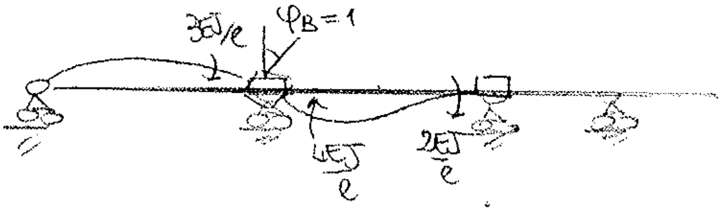




$M = \frac{p\ell}{2}\,\frac{1}{2}\,\frac{\ell}{2} = \frac{p\ell^2}{8}$

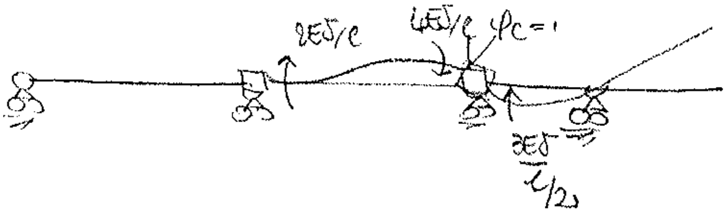
$M_{BB}\varphi_B + M_{BC}\varphi_C + M_{B0} = 0$

$M_{CB}\varphi_B + M_{CC}\varphi_C + M_{C0} = 0$



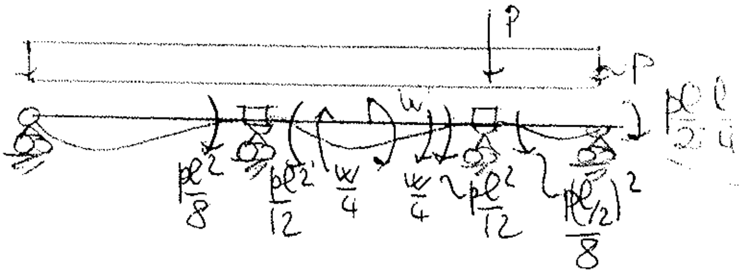
$M_{BB} = \frac{7EJ}{\ell}$

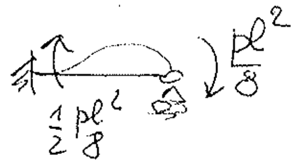
$M_{CB} = \frac{2EJ}{\ell}$



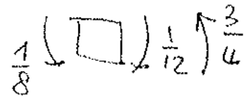
$M_{CC} = \frac{10EJ}{\ell}$

$M_{BC} = \frac{2EJ}{\ell}$



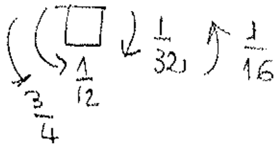


$M_{B0} = \frac{p\ell^2}{8} - \frac{p\ell^2}{12} + \frac{W}{4} = \frac{1}{8} - \frac{1}{12} + \frac{3}{4} = \frac{3-2+18}{24} = \frac{19}{24}p\ell^2$



$M_{C0} = \frac{W}{4} + \frac{p\ell^2}{12} - \frac{p\ell^2}{32} + \frac{p\ell^2}{16} = \frac{3}{4} + \frac{1}{12} - \frac{1}{32} + \frac{\ell}{16} = \frac{72+8-3+6}{96} = \frac{83}{96}p\ell^2$

$2^2 \cdot 3 \quad 2^5$



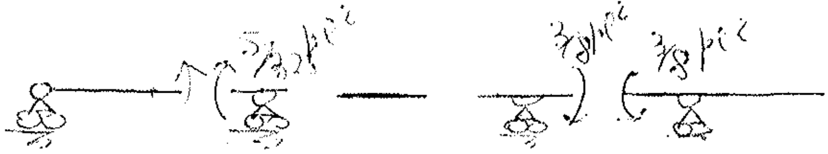
Risolvo il sistema

$$\begin{cases} \frac{7EJ}{l}\varphi_B + \frac{2EJ}{l}\varphi_C + \frac{19}{24}pl^2 = 0 \\ \frac{2EJ}{l}\varphi_B + \frac{10EJ}{l}\varphi_C + \frac{83}{96}pl^2 = 0 \end{cases} \qquad \begin{cases} \varphi_B = -\frac{3}{32}\frac{pl^3}{EJ} \\ \varphi_C = -\frac{13}{192}\frac{pl^3}{EJ} \end{cases}$$

quindi

$$M_B^{sx} = \frac{3EJ}{l}\left(-\frac{3}{32}\frac{pl^3}{EJ}\right) + \frac{pl^2}{8} = -\frac{5}{32}pl^2$$ ok! (= prima)

$$M_C^{dx} = \frac{6EJ}{l}\left(-\frac{13}{192}\frac{pl^3}{EJ}\right) - \frac{pl^2}{32} + \frac{pl^2}{16} = \frac{-14+2}{32} = -\frac{12}{32}pl^2 = -\frac{3}{8}pl^2$$ ok (= prima)



poi si procede come svolto prima

L'esercizio, in definitiva, non è altro che una trave continua su quattro appoggi.

→ È MEGLIO UTILIZZARE IL METODO delle FORZE (+ immediato xché mi permette di calcolare subito i momenti ai nodi evitando, così un passaggio)

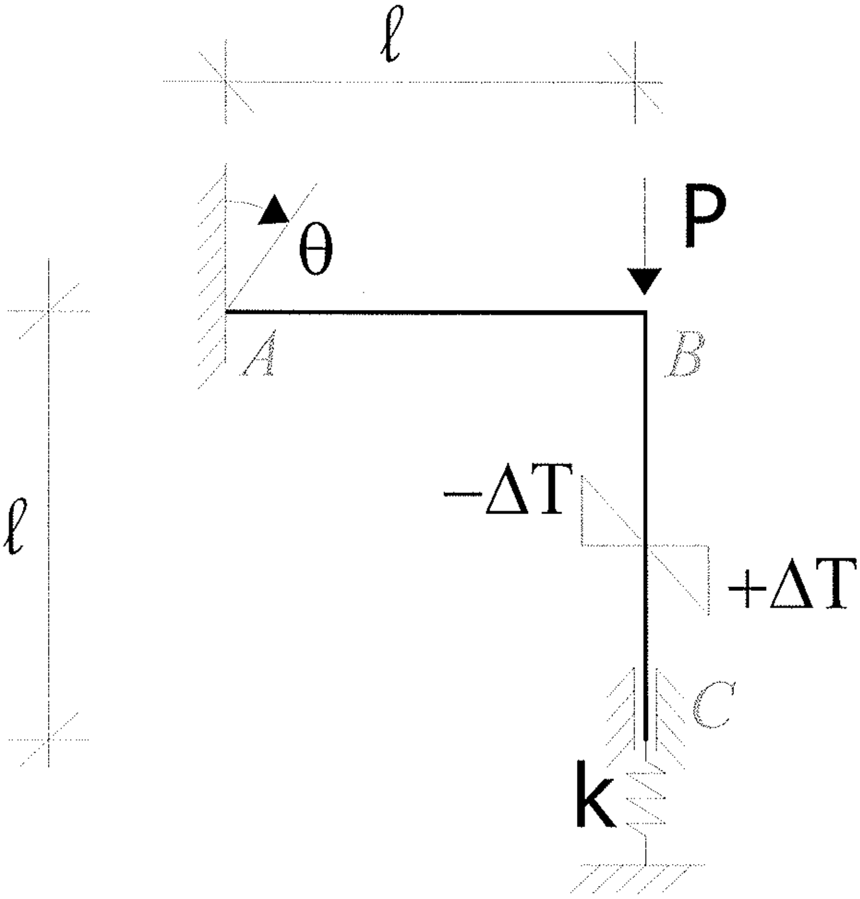
Esame di
FONDAMENTI di PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

APPELLO STRAORDINARIO

PROVA SCRITTA

8 – 07 – 2009



$$\frac{\alpha \Delta T}{t} = \frac{2P\ell}{3EJ}$$

$$\theta = \frac{P\ell^2}{3EJ}$$

$$k = \frac{6EJ}{\ell^2}$$

Si richiedono i grafici di:

- Momento flettente (con il valore e la posizione dei massimi)
- Taglio
- Azione assiale
- Deformata qualitativa con posizione dei flessi

SOLUZIONE

Sistema risolvente

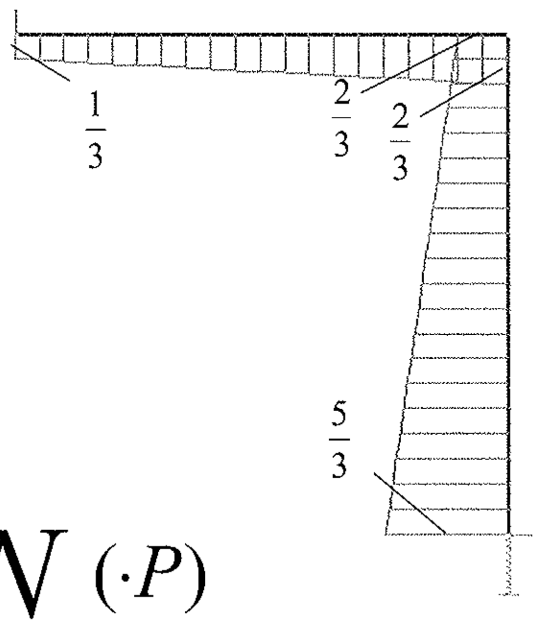
$$\begin{cases} \dfrac{8EJ}{\ell}\varphi_B - \dfrac{6EJ}{\ell^2}\eta_C + 2P\ell = 0 \\ -\dfrac{6EJ}{\ell^2}\varphi_B + \dfrac{18EJ}{\ell^3}\eta_C - 3P = 0 \end{cases}$$

Soluzioni

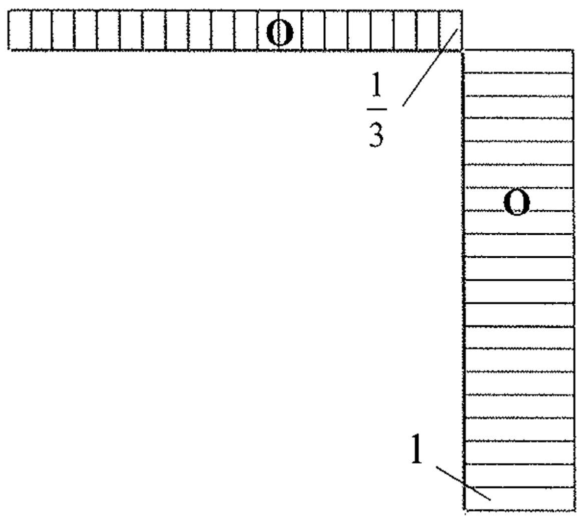
$$\begin{cases} \varphi_B = -\dfrac{P\ell^2}{6EJ} \\ \eta_C = \dfrac{P\ell^3}{9EJ} \end{cases}$$

Diagrammi delle azioni interne e Deformata

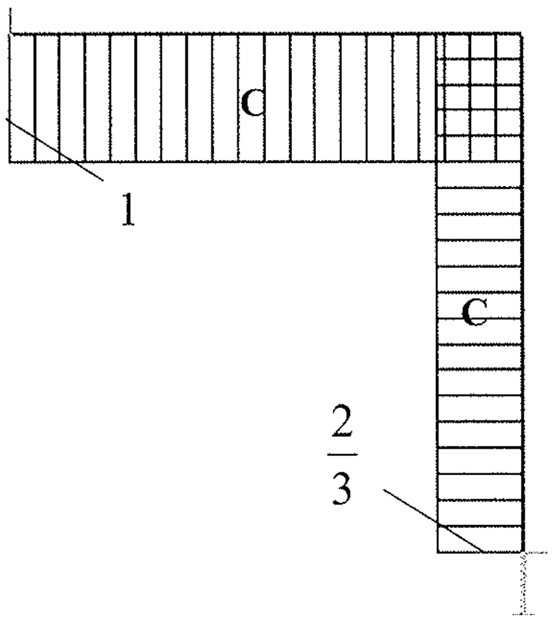
$M\ (\cdot P\ell)$



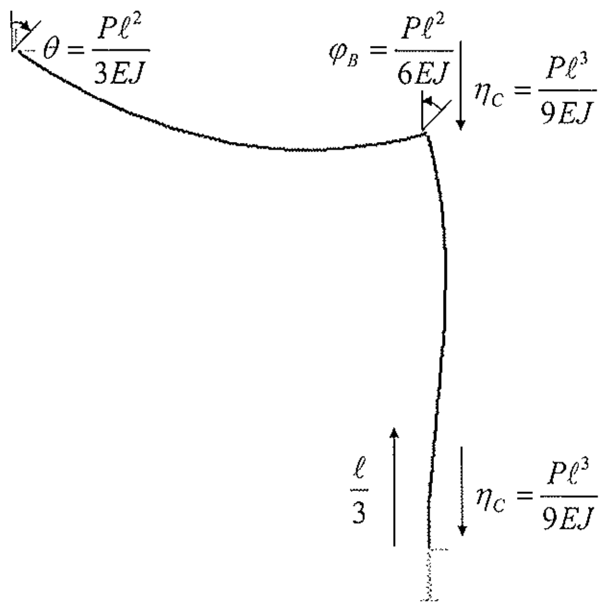
$T\ (\cdot P)$



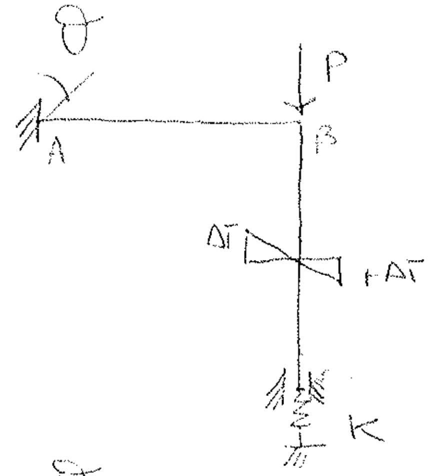
$N\ (\cdot P)$



Deformata

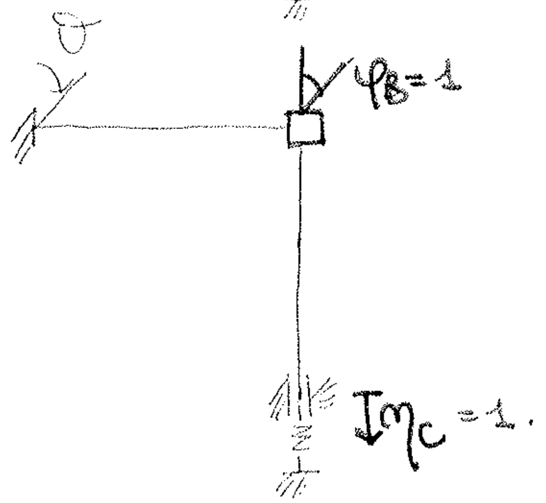


[illegible]



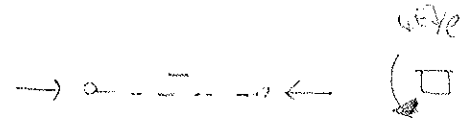
$$\begin{cases} m_{BB}\varphi_B + m_{BC}\eta_C + m_{B0} = 0 \\ h_{CB}\varphi_B + h_{CC}\eta_C + h_{C0} = 0 \end{cases}$$

$$\frac{\alpha \Delta T}{h} = \frac{2Pl}{3EJ}$$

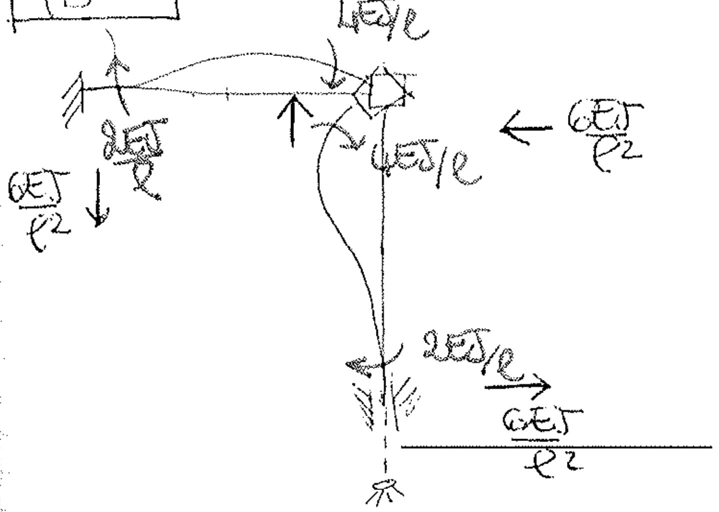


$$\theta = \frac{Pl^2}{3EJ}$$

$$K = \frac{6EJ}{l^3}$$

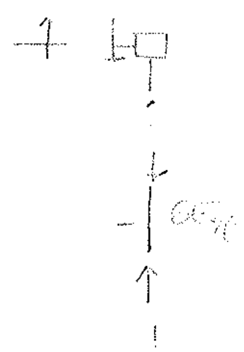


$\varphi_B = 1$

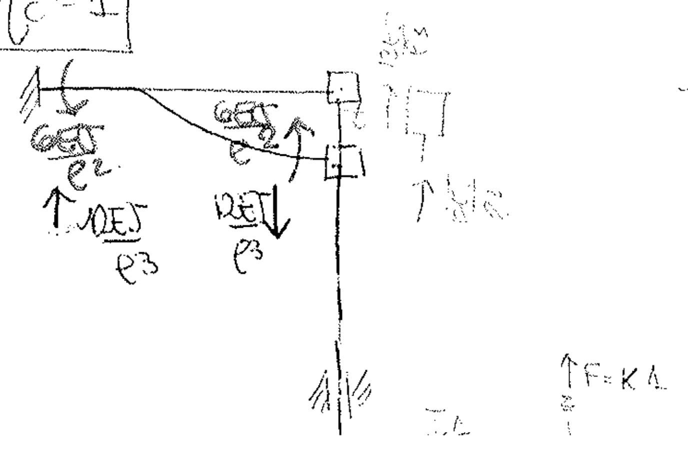


$$m_{BB} = \frac{8EJ}{l}$$

$$h_{CB} = -\frac{6EJ}{l^2}$$

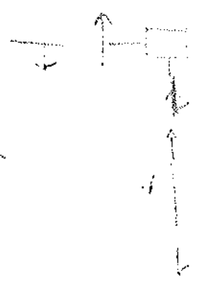


$\eta_C = 1$

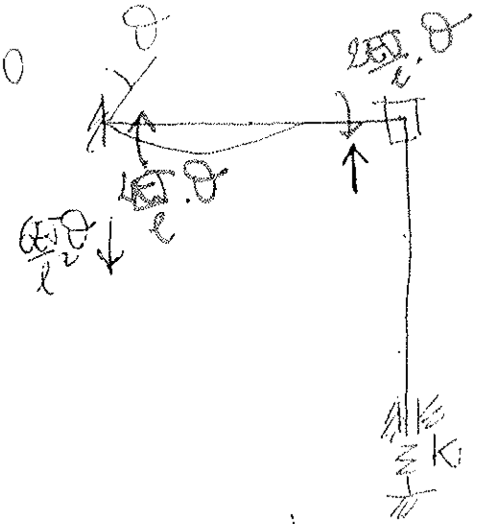


$$m_{BC} = -6EJ/l^2$$

$$h_{CC} = +12EJ/l^3 + K$$

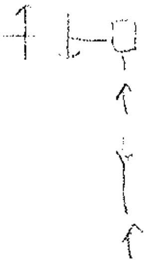


$\theta \neq 0$

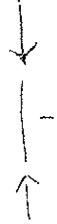


$$m_{B0} = \frac{2EJ}{\ell}\cdot\theta$$

$$r_{C0} = -\frac{6EJ}{\ell^2}\cdot\theta$$



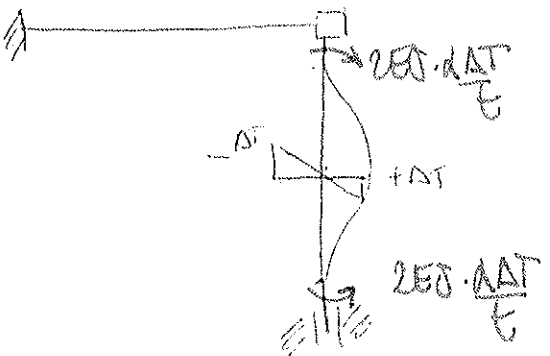
$P \neq 0$



$$m_{B0} = 0$$

$$r_{C0} = -P$$

$\Delta T \neq 0$



$$m_{B0} = +2EJ\cdot\frac{\alpha\Delta T}{t}$$

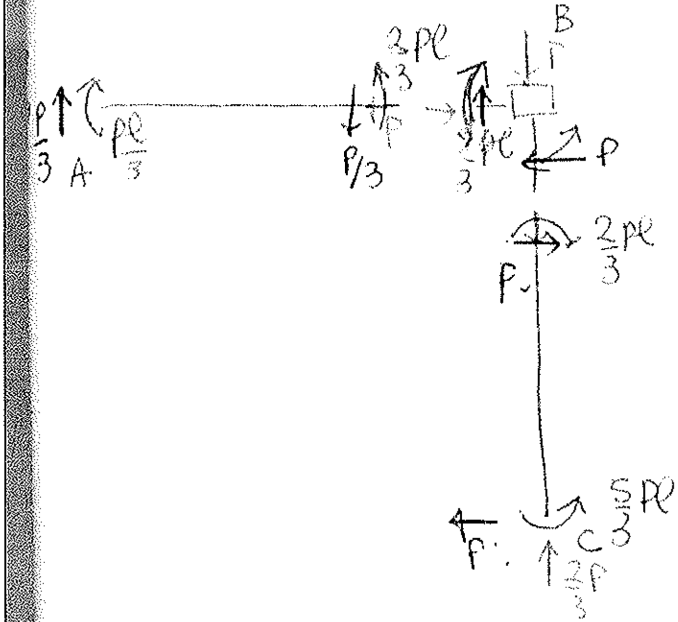
$$r_{C0} = 0$$

$$\begin{cases} \frac{8EJ}{\ell}\varphi_B - \frac{6EJ}{\ell^2}\eta_C + \frac{2EJ}{\ell}\cdot\theta + 2EJ\cdot\frac{\alpha\Delta T}{t} = 0 \\ -\frac{6EJ}{\ell^2}\varphi_B + \left(\frac{12EJ}{\ell^3} + \tilde{k}\right)\eta_C - \frac{6EJ}{\ell^2}\theta - P = 0 \end{cases} \qquad \tilde{k} = 6EJ/\ell^3$$

$$\begin{cases} \frac{8EJ}{\ell}\cdot\varphi_B - \frac{6EJ}{\ell^2}\eta_C + \frac{2EJ}{\ell}\cdot\frac{P\ell^2}{3EJ} + 2EJ\cdot\frac{2P\ell}{3EJ} = 0 \\ -\frac{6EJ}{\ell^2}\varphi_B + \frac{18EJ}{\ell^3}\eta_C - \frac{6EJ}{\ell^2}\cdot\frac{P\ell^2}{3EJ} - P = 0 \end{cases}$$

$$\begin{cases} \frac{8EJ}{\ell}\varphi_B - \frac{6EJ}{\ell^2}\eta_C + 2P\ell = 0 \\ -\frac{6EJ}{\ell^2}\varphi_B + \frac{18EJ}{\ell^3}\eta_C - 3P = 0 \end{cases}$$

$$\boxed{\varphi_B = -\frac{1}{6}\frac{P\ell^2}{EJ} \qquad \eta_C = \frac{1}{9}\frac{P\ell^3}{EJ}}$$



(*) Determinazione flesso

$$y''(x) = -\frac{M(x)}{EJ} \mp \frac{2\alpha\Delta T}{t}$$

$\frac{5Pl}{3}$, P, $M(x)$: $\quad M(x) + \frac{5}{3}Pl - Px = 0 \;\rightarrow\; M(x) = P\left(x - \frac{5}{3}l\right)$

$$y''(x) = -P\left(x - \frac{5}{3}l\right)\frac{1}{EJ} - 2\cdot\frac{2Pl}{3EJ} \leq 0$$

$$-3Px + 5lP - 4Pl \leq 0$$

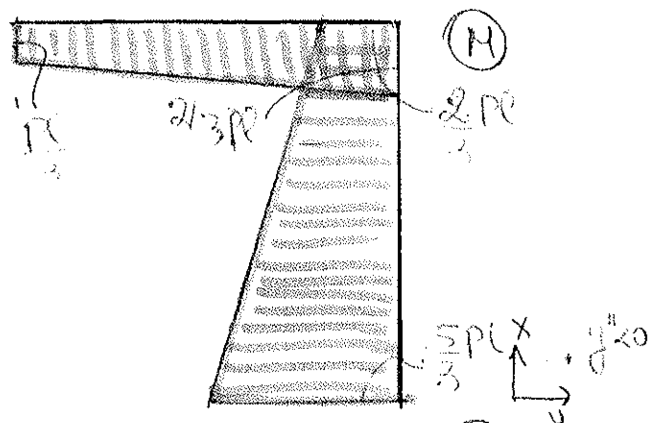
$$-3x \leq -l \qquad x \geq \frac{l}{3}$$

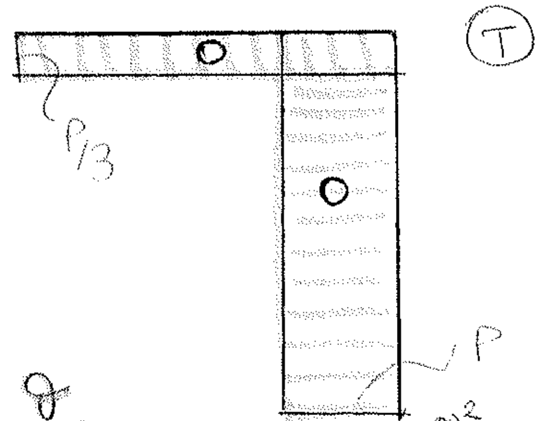
$$M_B^{sx} = \frac{2EJ}{l}\left(-\frac{1}{3}\frac{Pl^2}{EJ}\right) - \frac{2EJ}{l^2}\left(\frac{1}{3}\frac{Pl^3}{EJ}\right) + \frac{2}{3}Pl = -\frac{2}{3}Pl$$

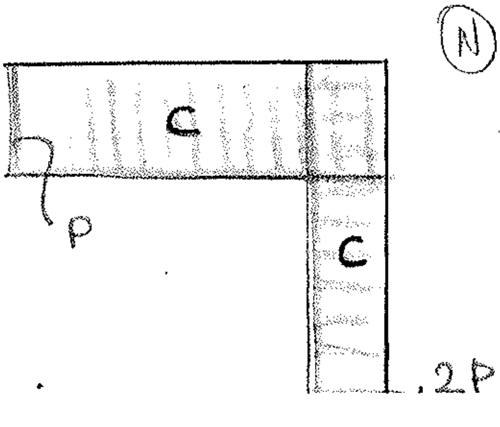
$$M_A = \frac{2EJ}{l}\left(-\frac{1}{3}\frac{Pl^2}{EJ}\right) - \frac{2EJ}{l^2}\left(\frac{1}{3}\frac{Pl^3}{EJ}\right) + \frac{4}{3}Pl = +\frac{Pl}{3}$$

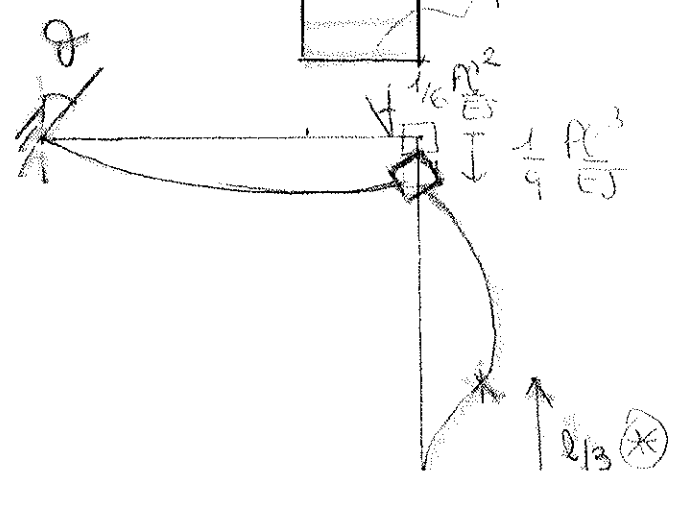
$$M_C = \frac{2EJ}{l}\left(-\frac{1}{3}\frac{Pl^2}{EJ}\right) - 2EJ\,\frac{2Pl}{3EJ} = -\frac{5}{3}Pl$$

$$T_A = \frac{6EJ}{l^2}\left(-\frac{1}{6}\frac{Pl^2}{EJ}\right) + \frac{4EJ}{l^3}\left(\frac{1}{9}\frac{Pl^3}{EJ}\right) - \frac{2EJ}{l^2}\left(\frac{Pl^2}{3EJ}\right) = \frac{P}{3}$$









$$K = \frac{3EJ}{L^3}$$

$$Q = \frac{16}{5} pL$$

$$\frac{\alpha \Delta T}{h} = \frac{pL^2}{6EJ}$$

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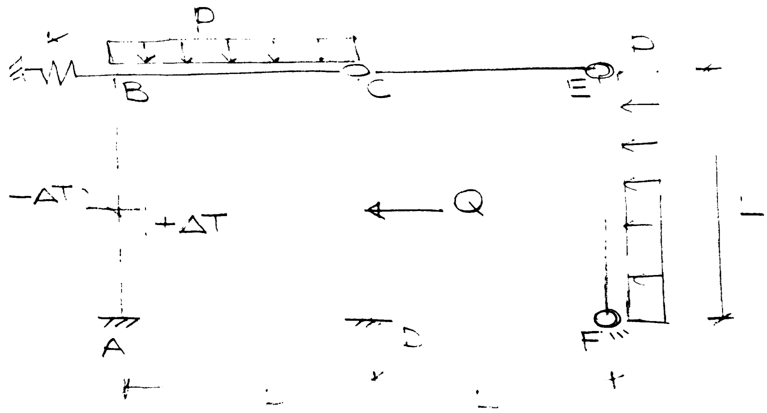
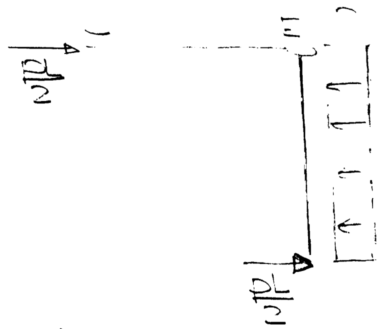
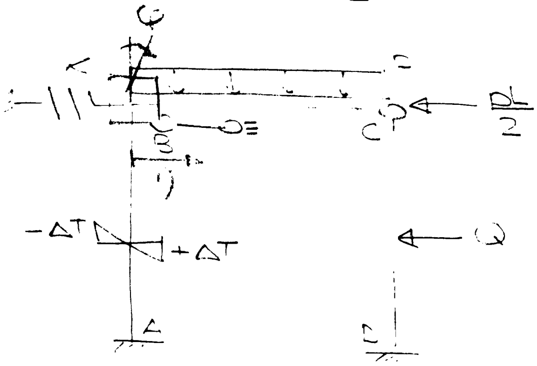


SOLUZIONE APPENDICE ISOSTATICA CEF

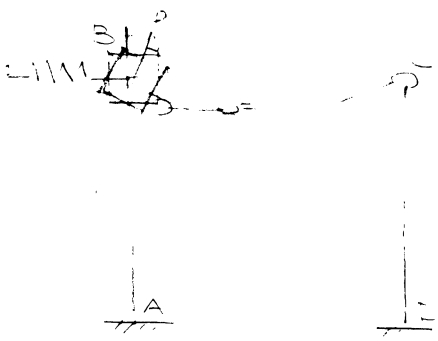




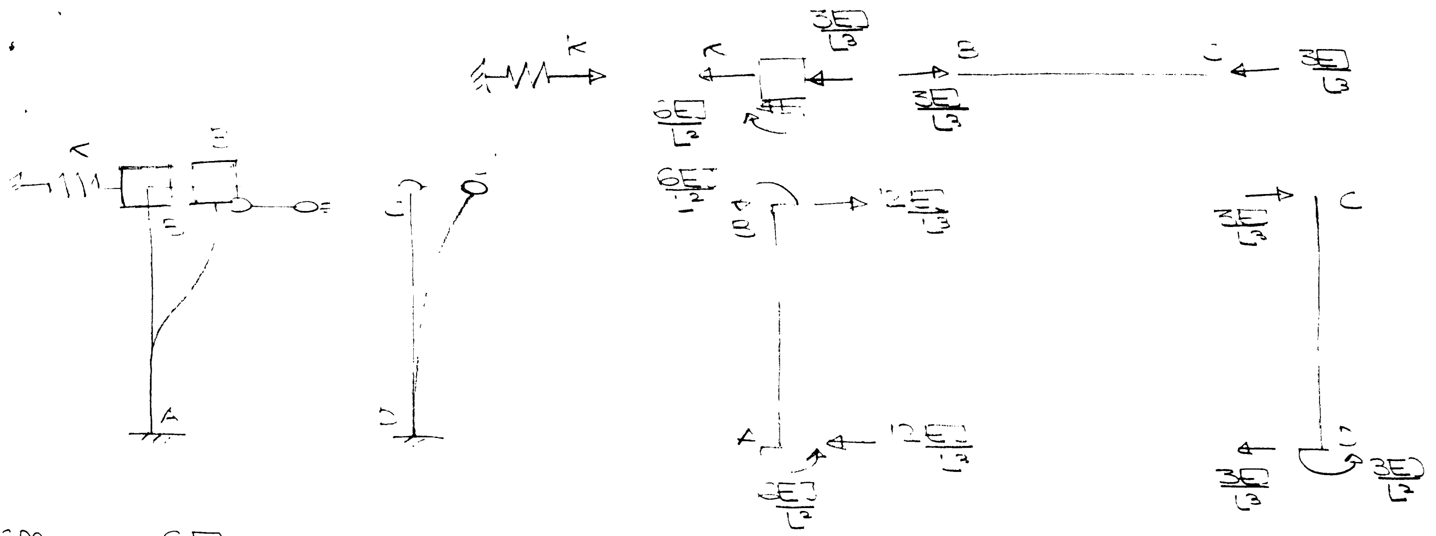
$$m_{BB}\varphi_B + m_{BC}\eta + m_{B0} = 0$$

$$n_{BB}\varphi_B + n_{BC}\eta + n_{B0} = 0$$

$\varphi = 1$

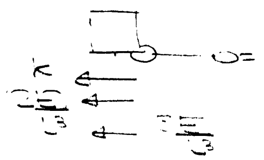


$$\begin{cases} m_{BB} = \frac{4EJ}{L} + \frac{3EJ}{L} = 7\frac{EJ}{L} \\ n_{BB} = 6\frac{EJ}{L^2} \end{cases}$$

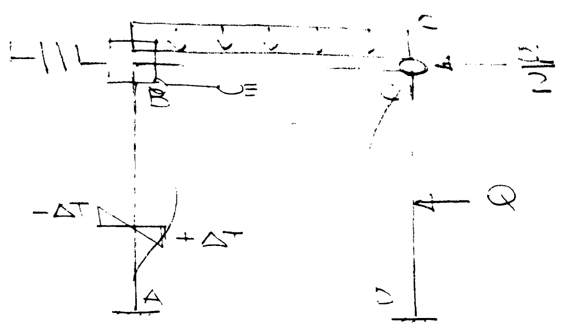


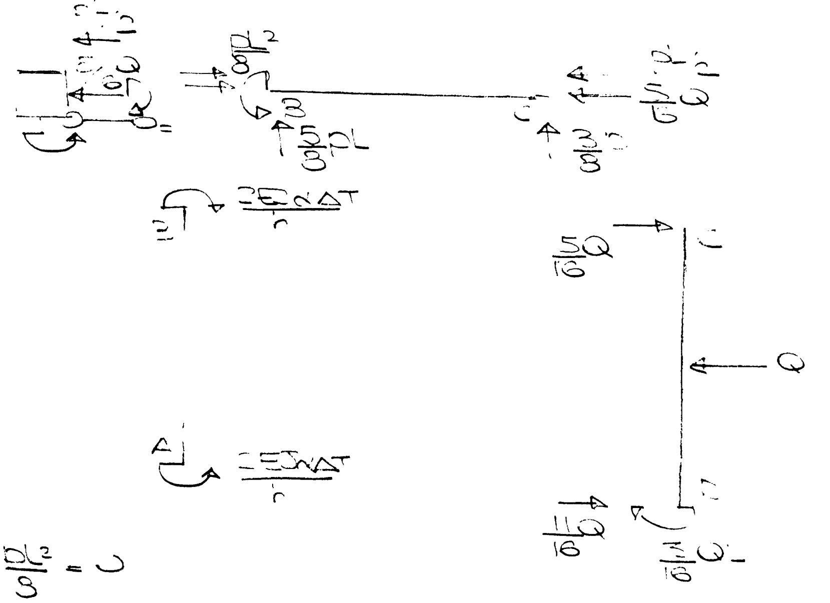
$m_{BD} = -6\frac{EJ}{L^2}$

$h_{BD} = -15\frac{EJ}{L^3} - K = (-15-3)\frac{EJ}{L^3} = -18\frac{EJ}{L^3}$



$P \neq 0 \quad Q \neq 0 \quad \Delta T \neq 0$





$m_{BO} = \frac{3EJ\alpha\Delta T}{h} - \frac{pL^2}{8} = 2\frac{pL^2}{16} - \frac{pL^2}{8} = 0$

$h_{BO} = \frac{5}{16}Q + \frac{p}{2} = -\frac{5}{16}\frac{16}{5}pL - \frac{pL}{2} = -\frac{3}{2}pL$

$7\frac{EJ}{L}\varphi - 6\frac{EJ}{L^2}\eta = 0$

$6\frac{EJ}{L^2}\varphi - 18\frac{EJ}{L^3}\eta - \frac{3}{2}pL = 0$

$\varphi = \frac{6}{7}\frac{\eta}{L}$

$6\frac{EJ}{L^2}\varphi - 18\frac{EJ}{L^3}\eta = -\frac{3}{2}pL$

$\left(\frac{36}{7}\eta - 18\eta\right)\frac{EJ}{L^3} = +\frac{3}{2}pL$

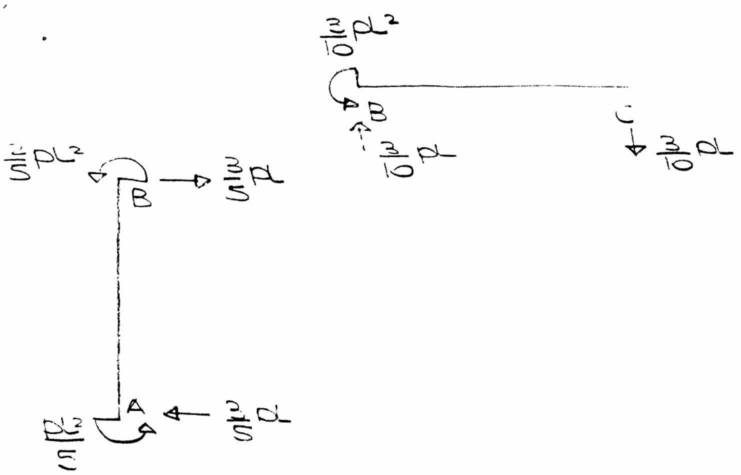
$-\frac{90}{7}\frac{EJ}{L^3}\eta = \frac{3}{2}pL$

$\boxed{\eta = -\frac{7}{60}\frac{pL^4}{EJ}}$

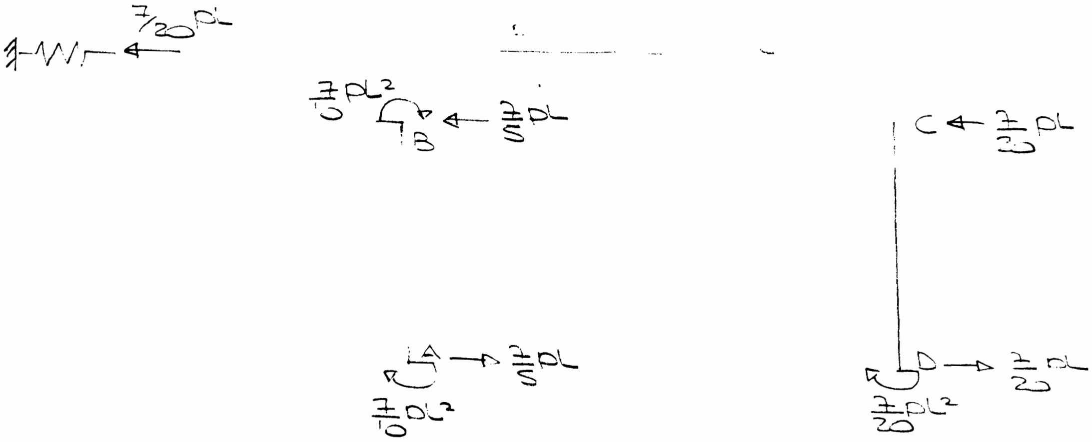
$\boxed{\varphi} = -\frac{7}{60}\frac{pL^4}{EJ}\cdot\frac{6}{7}\frac{1}{L} = \boxed{-\frac{1}{10}\frac{pL^3}{EJ}}$

$- \frac{pL^3}{10EJ}$

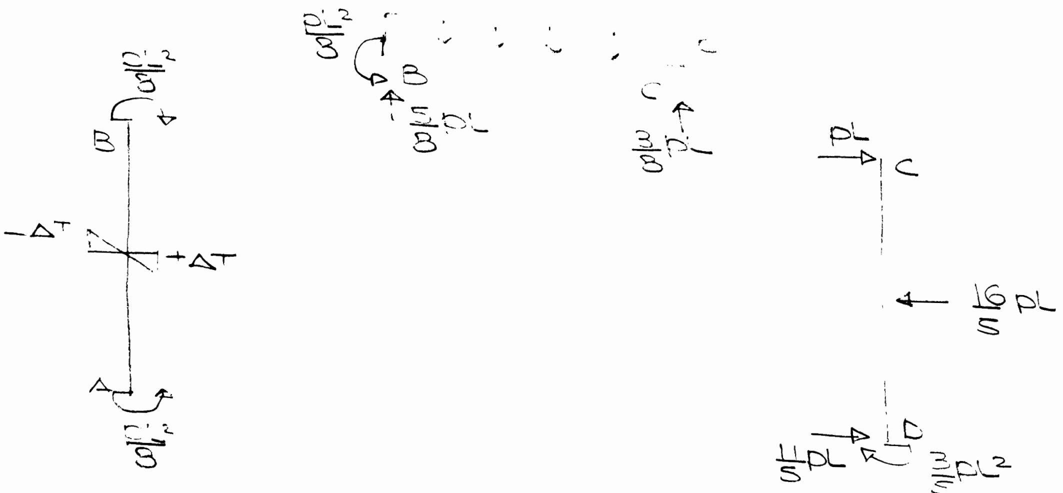
$\frac{4EJ}{L} \cdot \frac{1}{10}\frac{pL^3}{EJ} = \frac{2}{5} pL^2$



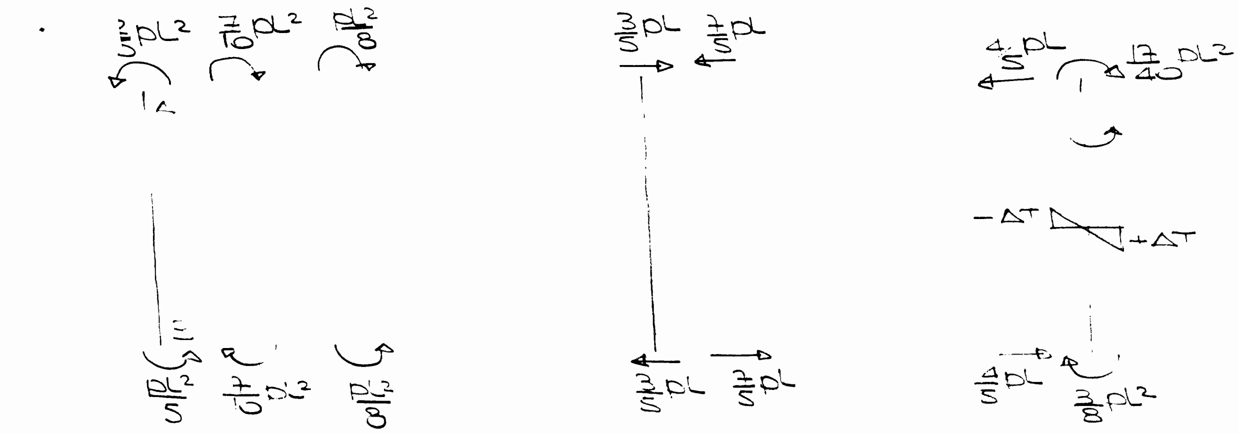
$\eta = -\frac{7}{60}\frac{pL^4}{EJ}$



$\Delta T \neq 0 \quad \ldots \neq 0 \quad \ldots \neq 0$



AB



$\circlearrowright + \quad \frac{7}{10} + \frac{1}{8} - \frac{1}{5} = \frac{28+5-16}{40} = \frac{17}{40}$

$\circlearrowleft + \quad \frac{1}{5} + \frac{1}{8} - \frac{7}{10} = \frac{8+5-28}{40} = -\frac{15}{40} = -\frac{3}{8}$

$\leftarrow + \quad \frac{7}{5} - \frac{3}{5} = \frac{4}{5}$

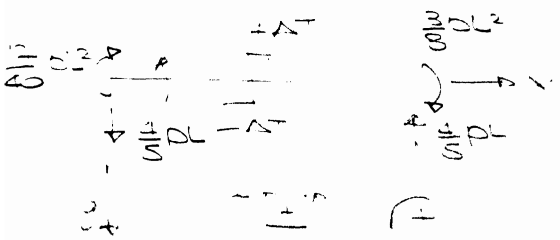
calcolo dove $M=0$

$M \quad -\frac{17}{40}pL^2 + \frac{4}{5}pLx = 0$

$M = \frac{17}{40}pL^2 - \frac{4}{5}pLx = 0$

$\frac{17}{40_8}L = \frac{4}{5}x \qquad x = \frac{17}{32}L$

studio curvatura



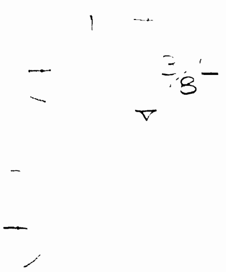
$M = \frac{17}{40}pL^2 - \frac{4}{5}pLx$

$y'' = -\frac{M}{EJ} + \frac{2\alpha\Delta T}{h} = -\frac{17}{40}\frac{pL^2}{EJ} + \frac{4}{5}\frac{pLx}{EJ} + \frac{pL^2}{8EJ} \geq 0$

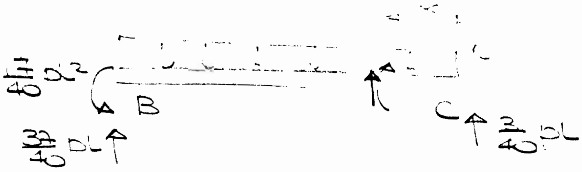
$-\frac{17}{40} + \frac{1}{8} = \frac{-17+5}{40} = -\frac{12}{40} = -\frac{3}{10}$

$-\frac{3}{10}L + \frac{4}{5}x \geq 0$

$\frac{4}{5}x \geq \frac{3}{10}L \qquad x \geq \frac{3}{8}L$



BC



$T + \frac{3}{40} pL - px = 0$

$T = 0 \quad \frac{3}{40} pL = px \quad x = \frac{3}{40} L$

In $x = \frac{3}{40} L$ momento è massimo

$M - \frac{3}{40} pLx + \frac{px^2}{2} = 0$

$M = \frac{3}{40} pLx - \frac{px^2}{2}$

$M = 0 \quad \frac{3}{40} pLx = \frac{px^2}{2} \quad x = \frac{3}{20} L$

$M(x = \frac{3}{40} L) = \frac{3}{40} \cdot \frac{3}{40} - \frac{1}{2} \left(\frac{3}{40}\right)^2 = \frac{9}{1600} - \frac{9}{3200} = \frac{9}{3200} pL^2$

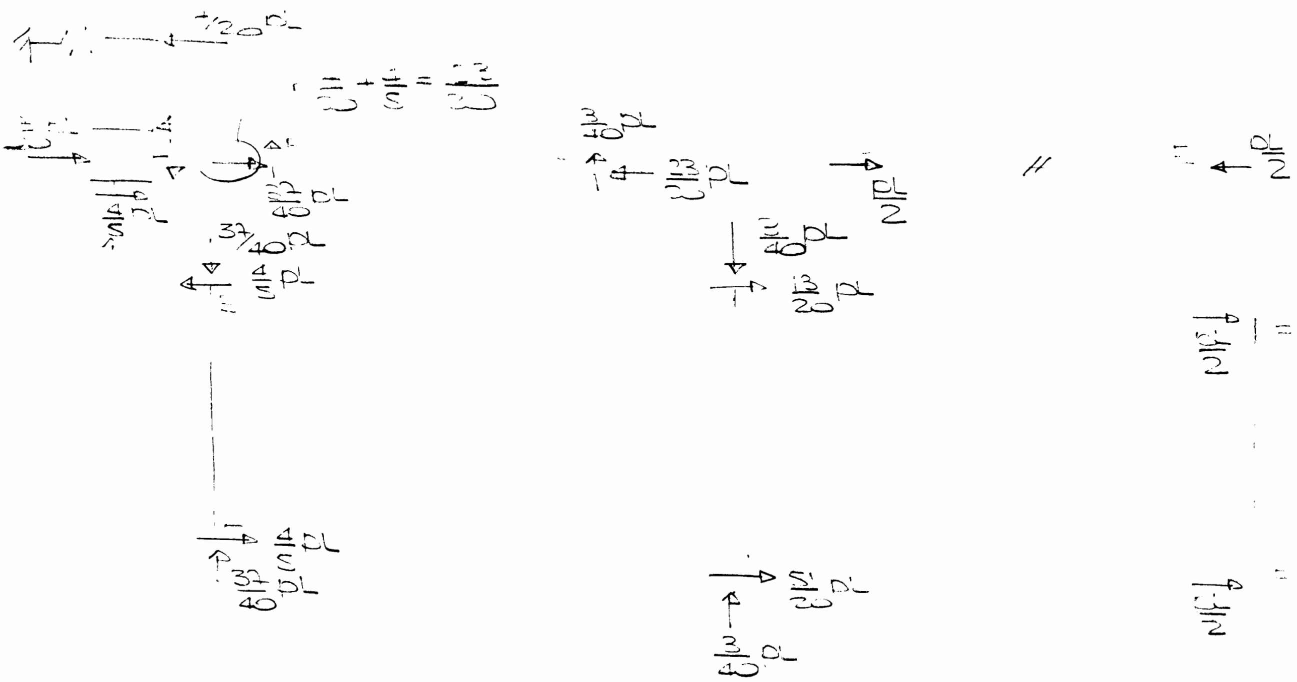
Tratto CD

$M + \frac{9}{32} pL^2 - \frac{5}{32} pLx = 0$

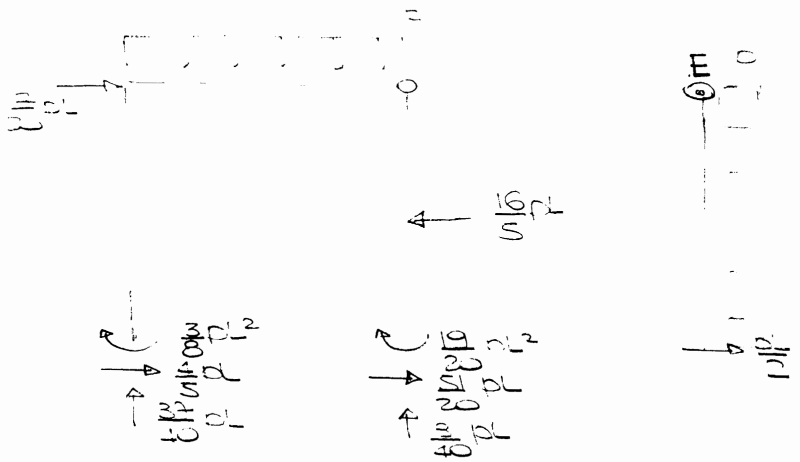
$M = \frac{5}{32} pLx - \frac{9}{32} pL^2$

$M = 0 \quad \frac{5}{32} x = \frac{9}{32} L$

$x = \frac{9}{5} L$



VERIFICA

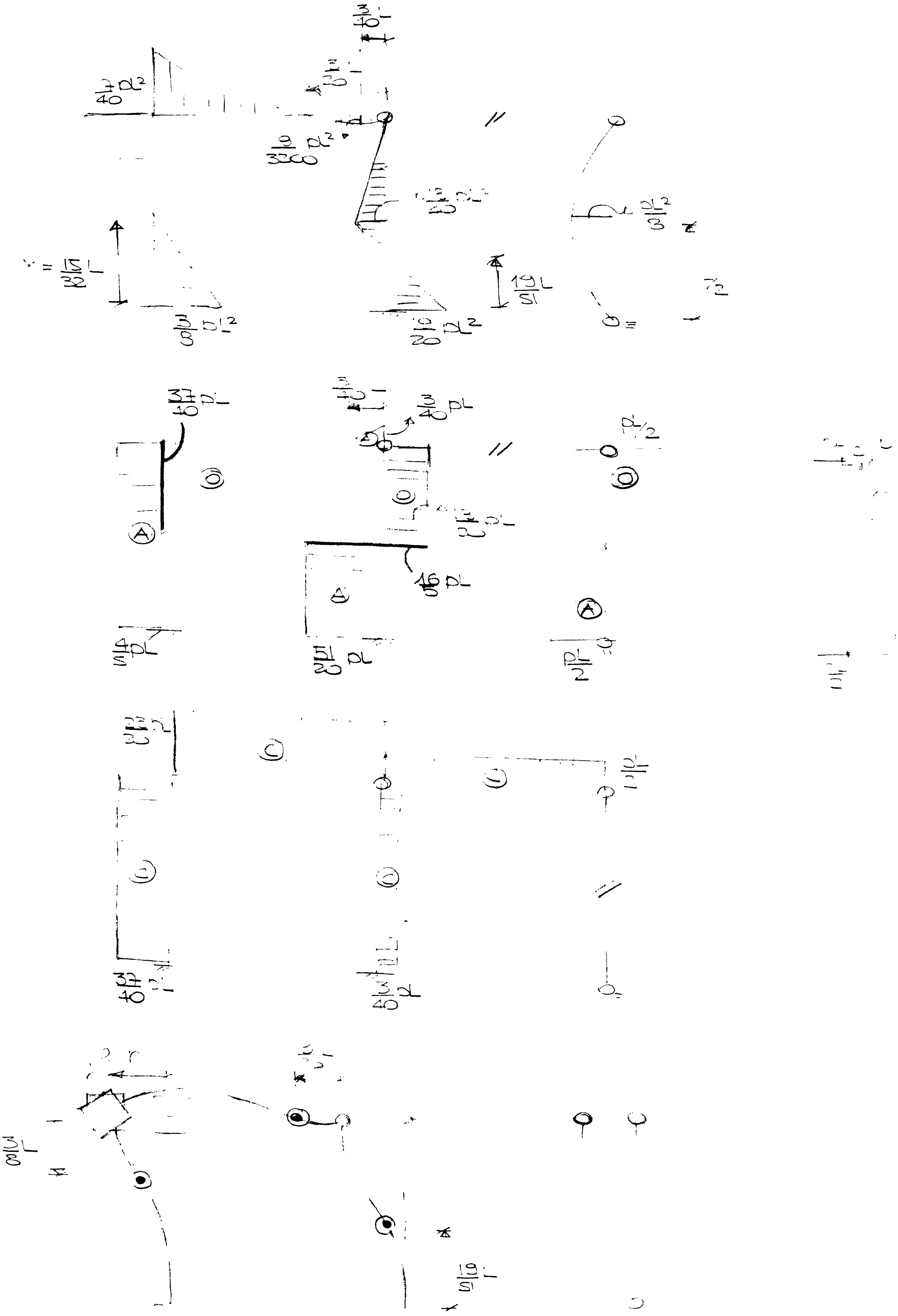


$\Sigma F_x = 0 \quad \frac{7}{20} - \frac{[illegible]}{5} + \frac{[illegible]}{20} - \frac{16}{5} + \frac{1}{2} - 1 = \frac{29}{10} - \frac{12}{5} - \frac{1}{2} = \frac{29 - 24 - [illegible]}{10} = 0$

$\Sigma F_y = 0 \quad 1 - \frac{37}{40} - \frac{3}{40} = \frac{40 - 37 - 3}{40} = 0$

$\Sigma M_E = 0 \quad [illegible] + \frac{[illegible]}{5} - \frac{[illegible]}{5} + [illegible] - [illegible] + [illegible] - [illegible] - [illegible] \; 8 \; [illegible] - [illegible] - [illegible] =$

$\frac{[illegible]}{3} + \frac{[illegible]}{5} - [illegible] - [illegible] + \frac{3}{40} = \frac{[illegible] + 32 + [illegible] - 80 + 3}{[illegible]} = 0$



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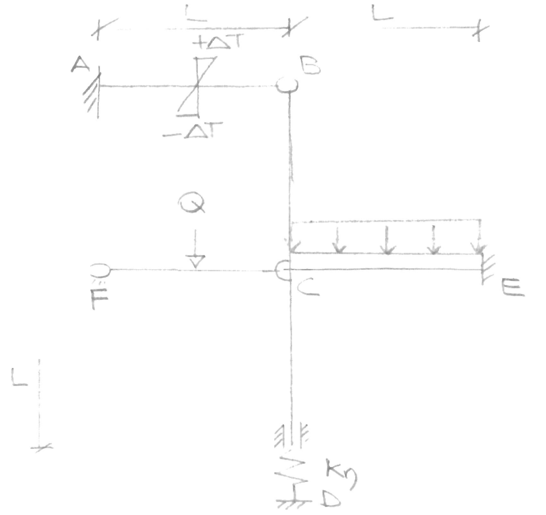
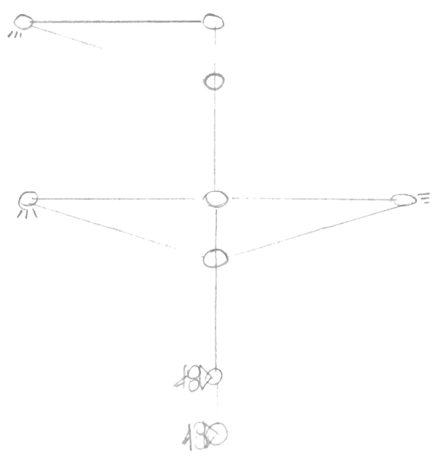
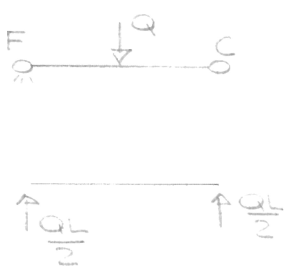


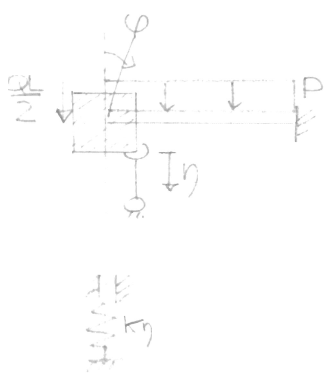
$Q = pL$

$k_\eta = 3\frac{EJ}{L^3}$

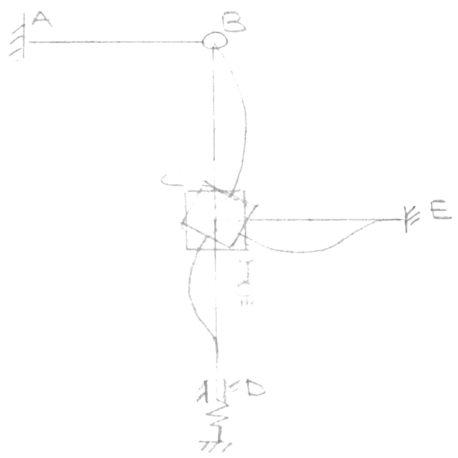
$\frac{\alpha \Delta T}{h} = \frac{1}{2}\frac{pL^2}{EJ}$

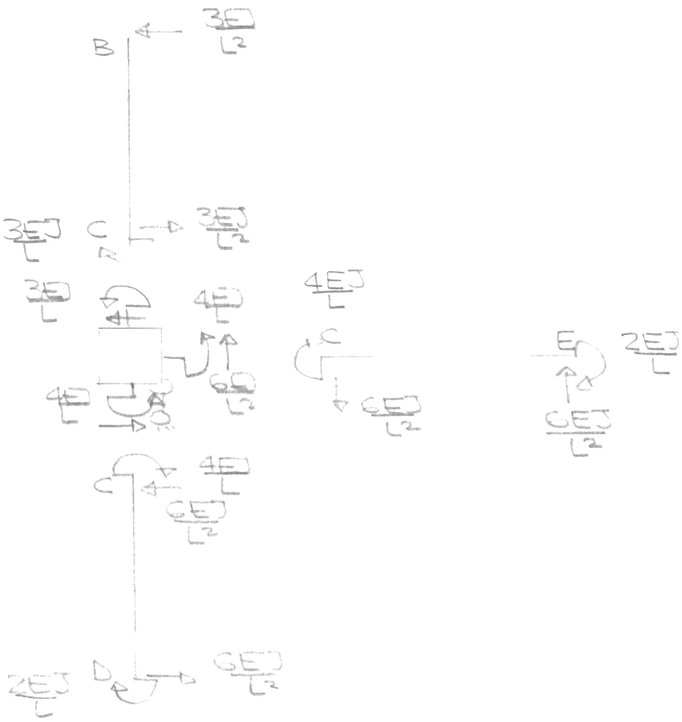






$\varphi = 1 \quad \eta = Q = p = \Delta T = 0$

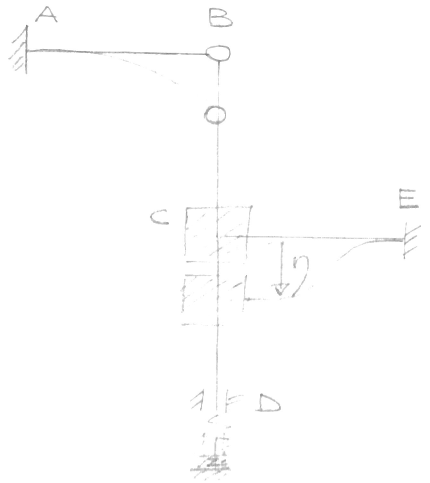


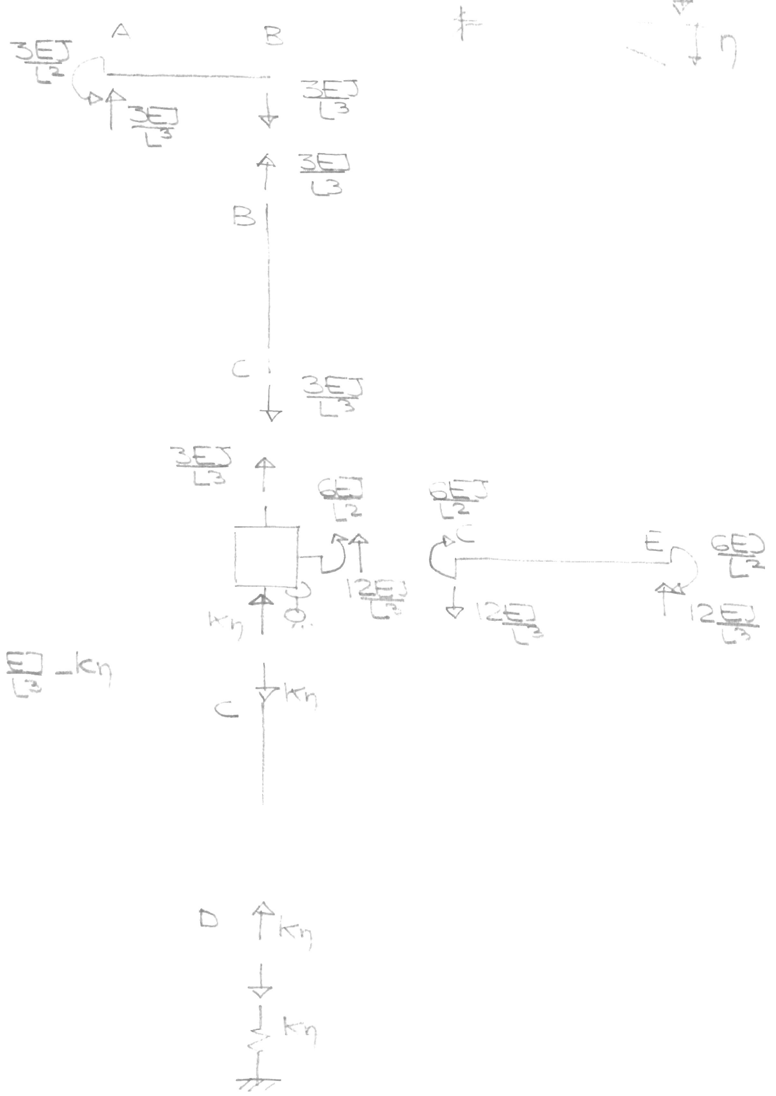


$m_{c\varphi} = 3\frac{EJ}{L} + 4\frac{EJ}{L} + 4\frac{EJ}{L} = 11\frac{EJ}{L}$

$h_{c\varphi} = -6\frac{EJ}{L^2}$

$\eta = 1 \quad \varphi = Q = p = \Delta T = 0$





$\eta = \frac{FL^3}{3EJ}$

$F = \frac{3EJ}{L^3}\eta$

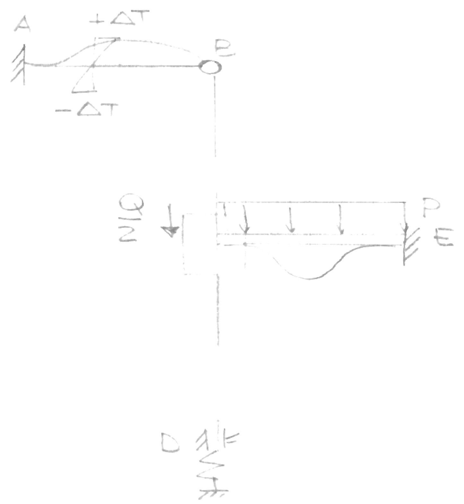
$\eta = 1$

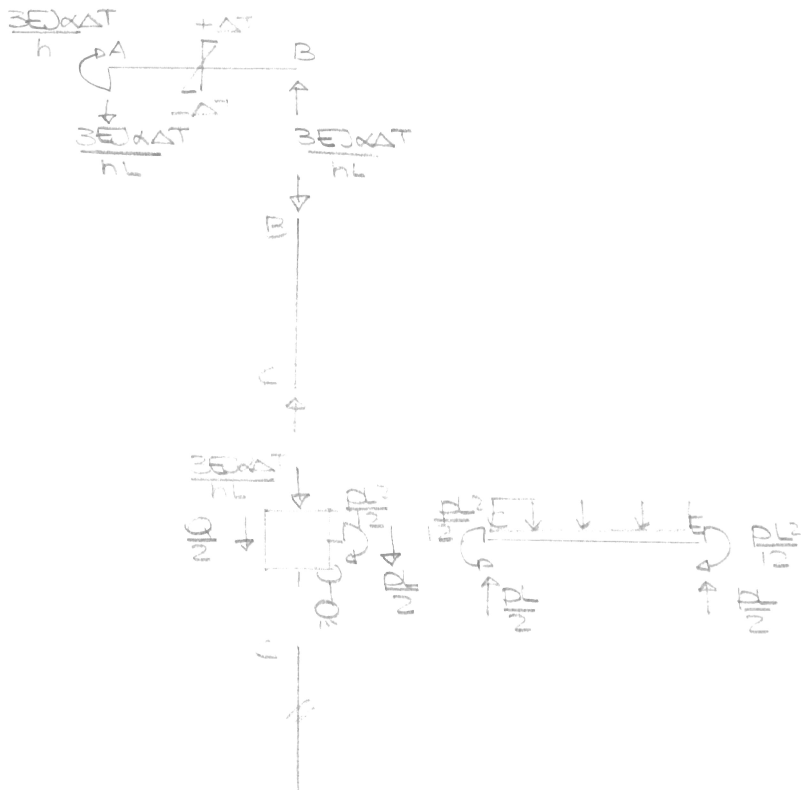
$F = \frac{3EJ}{L^3}$

$m_{c\eta} = \frac{6EJ}{L^2}$

$h_{c\eta} = -\frac{3EJ}{L^3} - \frac{12EJ}{L^3} - k\eta = -\frac{15EJ}{L^3} - k\eta$

$\Delta T \neq 0 \quad p \neq 0 \quad Q \neq 0 \quad \eta = \varphi = 0$





$m_{c0} = -\frac{pL^2}{12}$

$h_{c0} = \frac{Q}{2} + \frac{3EJ\alpha\Delta T}{hL} + \frac{pL}{2}$

$$11\frac{EJ}{L}\varphi + \frac{6EJ}{L^2}\eta - \frac{pL^2}{12} = 0$$

$$-\frac{6EJ}{L^2}\varphi - \left(15\frac{EJ}{L^3} + k_\eta\right)\eta + \frac{Q}{2} + \frac{3EJ\alpha\Delta T}{hL} + \frac{pL}{2} = 0$$

$$\begin{cases} 11\frac{EJ}{L}\varphi + \frac{6EJ}{L^2}\eta - \frac{pL^2}{12} = 0 \quad \times 3 \\ -\frac{6EJ}{L^2}\varphi - 18\frac{EJ}{L^3}\eta + \frac{pL}{2} + \frac{3}{2}pL + \frac{pL}{2} = 0 \end{cases}$$

$$-\frac{6EJ}{L^2}\varphi - 18\frac{EJ}{L^3}\eta + \frac{5}{2}pL = 0$$

$$33\frac{EJ}{L^2}\varphi + 18\frac{EJ}{L^3}\eta - \frac{pL}{4} = 0 \quad (+)$$

$$27\frac{EJ}{L^2}\varphi + \frac{9}{4}pL = 0 \qquad 27\frac{EJ}{L^2}\varphi = -\frac{9}{4}pL \qquad \boxed{\varphi = -\frac{1}{12}\frac{pL^3}{EJ}}$$

$$11\frac{EJ}{L}\left(-\frac{pL^3}{12EJ}\right) + \frac{6EJ}{L^2}\eta - \frac{pL^2}{12} = 0$$

$$-\frac{11}{12}pL^2 - \frac{pL^2}{12} = -\frac{6EJ}{L^2}\eta$$

$$+pL^2 = \frac{6EJ}{L^2}\eta \qquad \boxed{\eta = \frac{1}{6}\frac{pL^4}{EJ}}$$

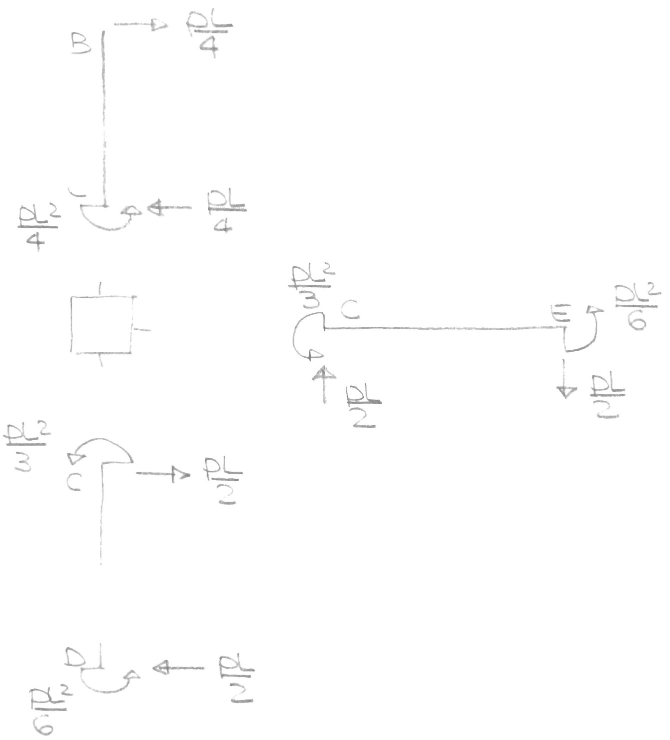
$$\varphi = -\frac{1}{12}\frac{pL^3}{EJ}$$

$4\cdot\frac{1}{12} = \frac{1}{3}$

$2\cdot\frac{1}{12} = \frac{1}{6}$

$6\cdot\frac{1}{12} = \frac{1}{2}$

$3\cdot\frac{1}{12} = \frac{1}{4}$



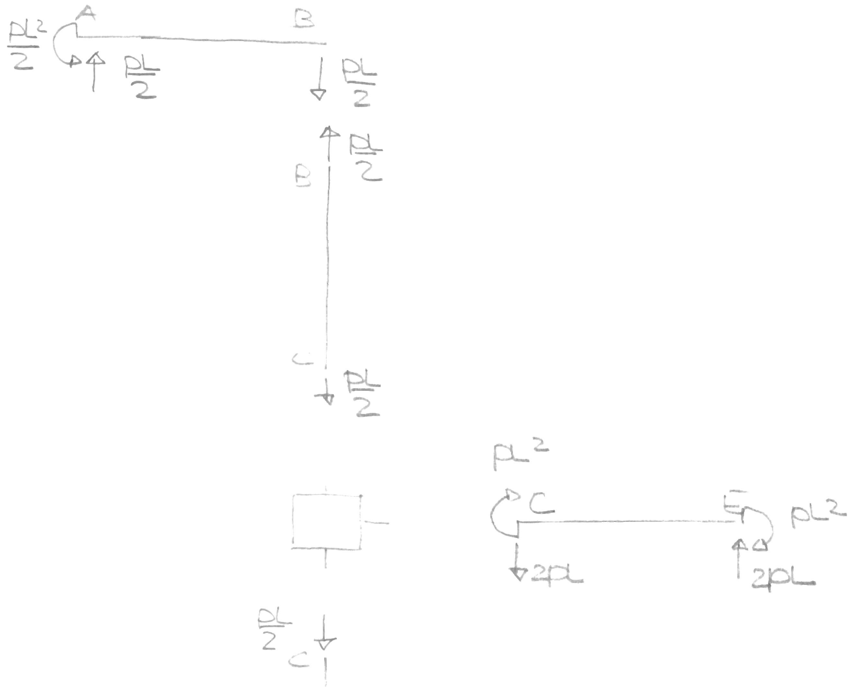
$\eta = \frac{1}{6}$

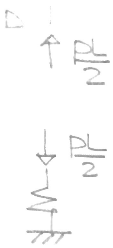
$\eta = 3 \cdot \frac{1}{6} = \frac{1}{2}$

$3 \cdot \frac{1}{6_2} = \frac{1}{2}$

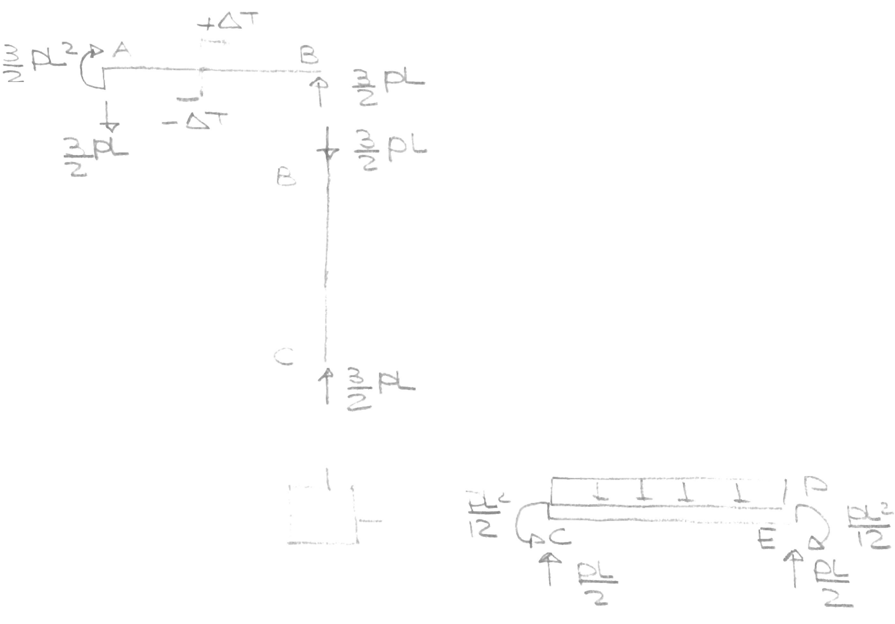
$6 \cdot \frac{1}{6} = 1$

$12 \cdot \frac{1}{6} = 2$

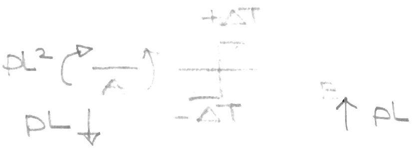




$\frac{\alpha\Delta T}{h} = \frac{pL^2}{2EJ}$ $p \neq 0$ $Q = pL$ $3EJ \cdot \frac{pL^2}{2EJ} = \frac{3}{2}pL^2$



ASTA **AB**



↻ $\frac{3}{2} - \frac{1}{2} = 1$

$+\downarrow \ \frac{3}{2} - \frac{1}{2} =$

$+\uparrow \ \frac{3}{2} - \frac{1}{2} = 1$

$M - pL^2 + pLx = 0$

$M = pL^2 - pLx$

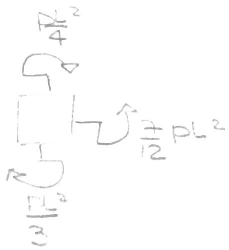
$y'' = -\frac{M}{EJ} = \frac{pLx}{EJ} - \frac{pL^2}{EJ} + \frac{2 \cdot pL^2}{2EJ} \geq 0$

$pLx - pL^2 + pL^2 \geq 0$

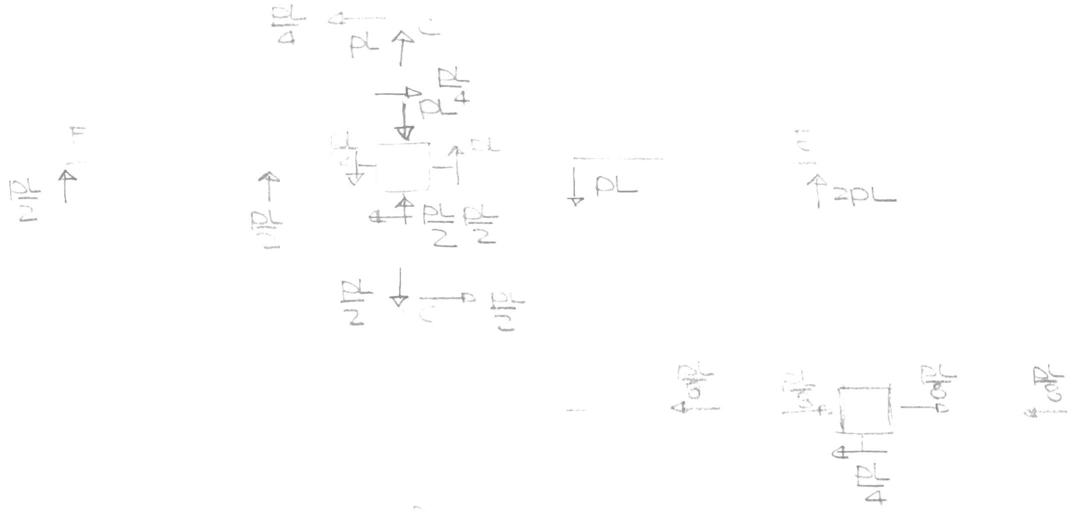
$x \geq 0$ (+) sempre

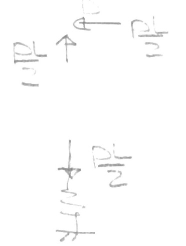
ASTA **BC**

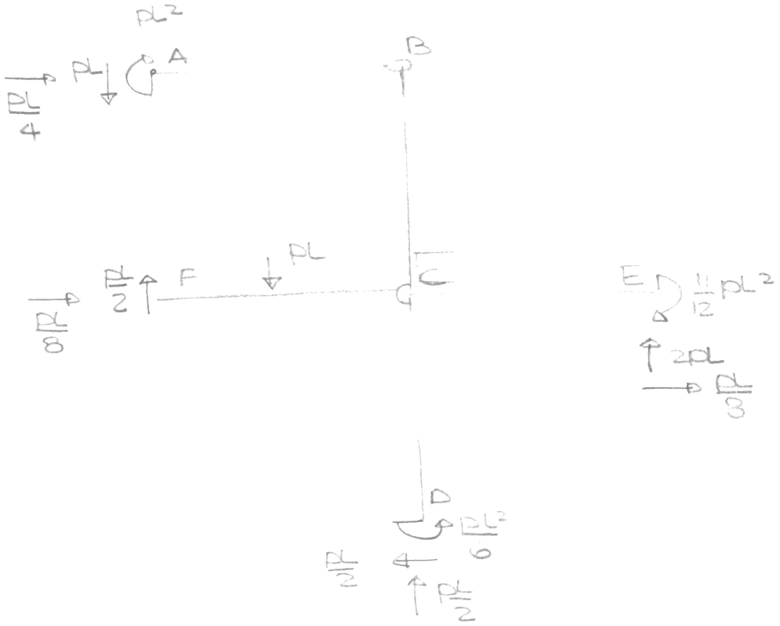
Verif al nodo C



$$\frac{1}{4}+\frac{1}{3}-\frac{7}{12}=\frac{4+3-7}{12}=0$$







$\Sigma F_y = 0$

$\uparrow + \quad pL - \frac{pL}{2} \quad \cancel{pL} + \cancel{pL} = \cancel{2}pL - \frac{pL}{2}$

$= 0$

$\Sigma F_x = 0$

$\overset{+}{\rightarrow} \quad \frac{pL}{4} + \frac{pL}{8} + \frac{pL}{8} - \frac{pL}{2} = 0$

$\Sigma M_A = 0$

$\circlearrowleft +$

$-pL^2 + \frac{pL^2}{8} - \cancel{\frac{pL^2}{2}} - \frac{3}{2}pL^2 - \frac{11}{12}pL^2$

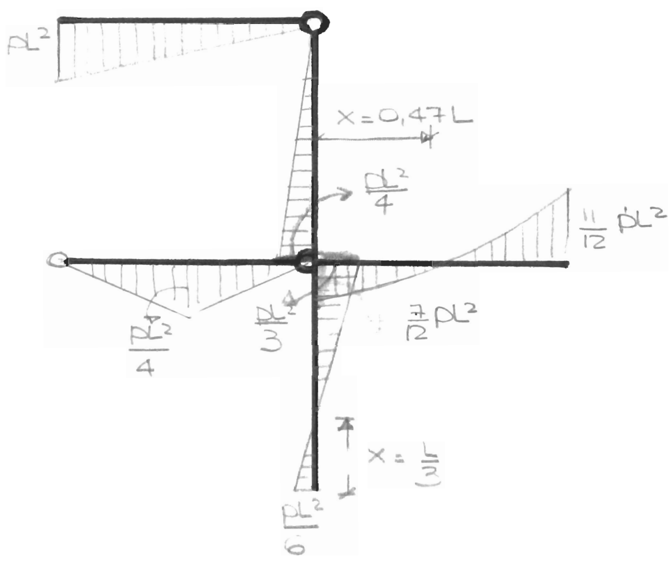
$+ 4pL^2 + \frac{pL^2}{8} + \frac{pL^2}{6} + \cancel{\frac{pL^2}{2}} -$

$- pL^2 =$

$2 - \frac{3}{2} + \frac{1}{4} + \frac{1}{6} - \frac{11}{12} =$

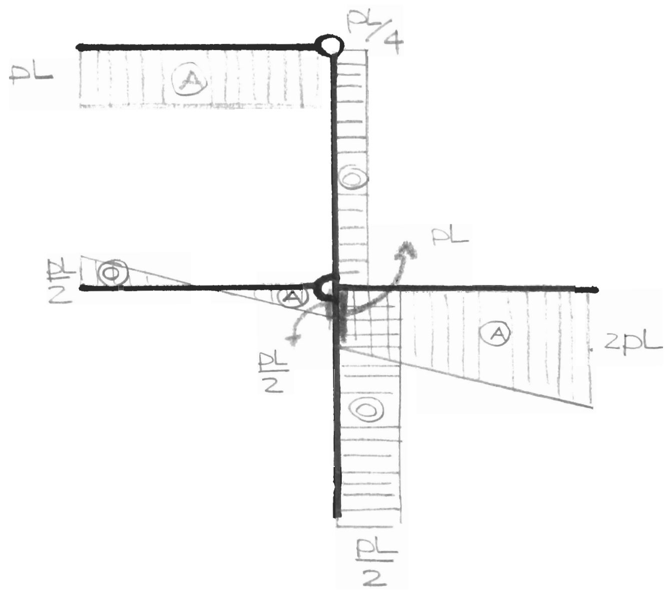
$= \frac{24 - 18 + 3 + 2 - 11}{12} = 0$

M

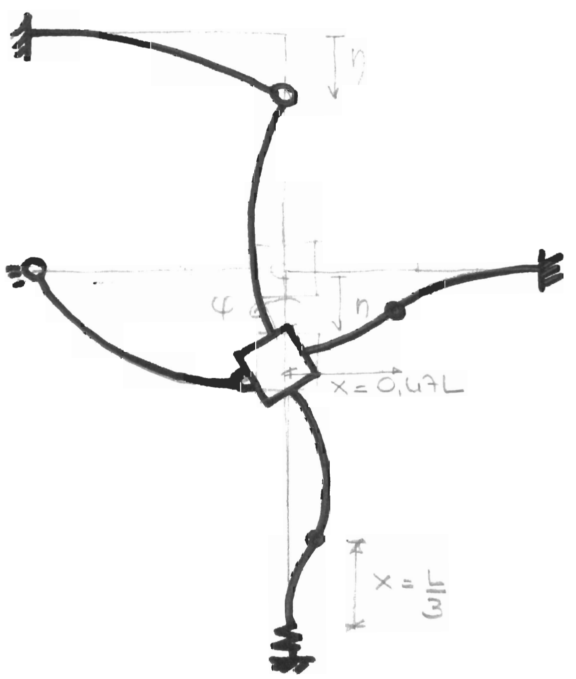


pL²
x = 0,47L
pL²/4
11/12 pL²
pL²/4
pL²/3
7/12 pL²
x = L/3
pL²/6

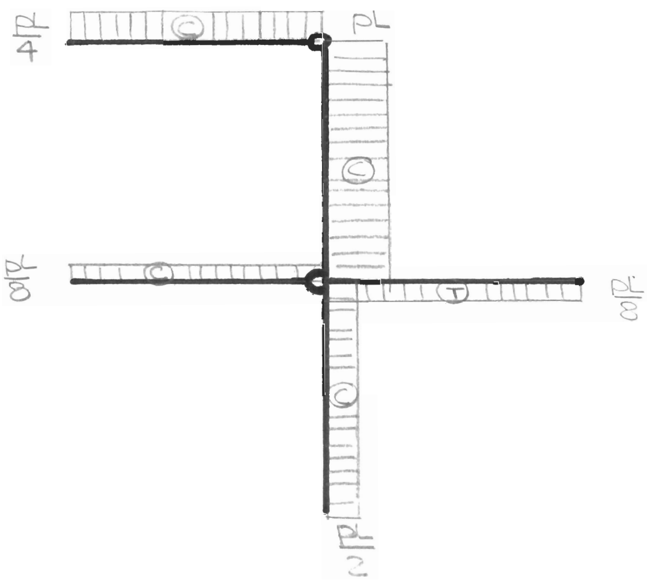
V



pL/4
pL
pL
2pL
pL/2
pL/2



x = 0,47L
x = L/3

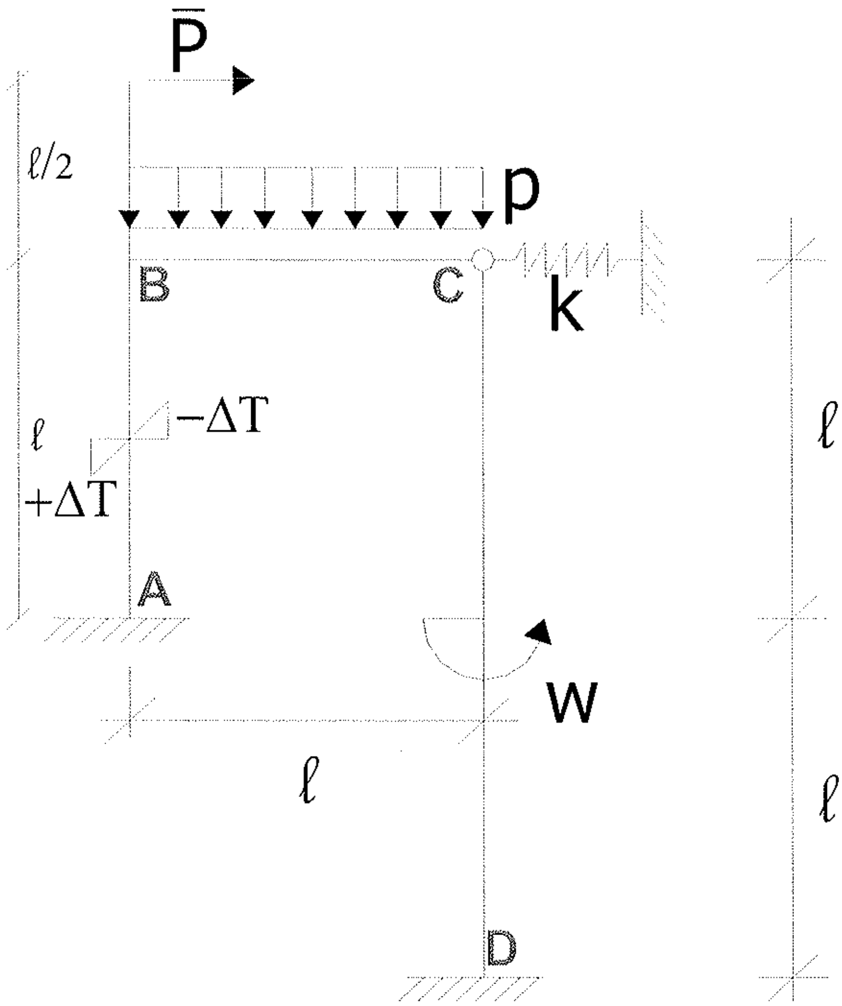


pL/4
pL
pL/2

Esame di
FONDAMENTI di
PROGETTAZIONE STRUTTURALE

(Corso di laurea in Ingegneria Civile N.O.)

7 – 09 – 2009



$$k = \frac{5EJ}{8l^3}$$

$$\frac{\alpha\Delta T}{h} = \frac{p\ell^2}{16EJ}$$

$$W = 2p\ell^2$$

$$\overline{P} = \frac{p\ell}{2}$$

Si richiedono i grafici di:

- Momento flettente (con il valore e la posizione dei massimi)
- Taglio
- Azione assiale
- Deformata qualitativa con posizione dei flessi

SOLUZIONE

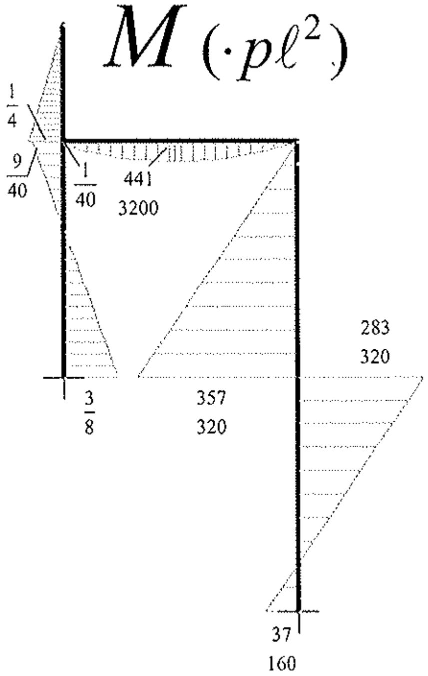
Sistema risolvente

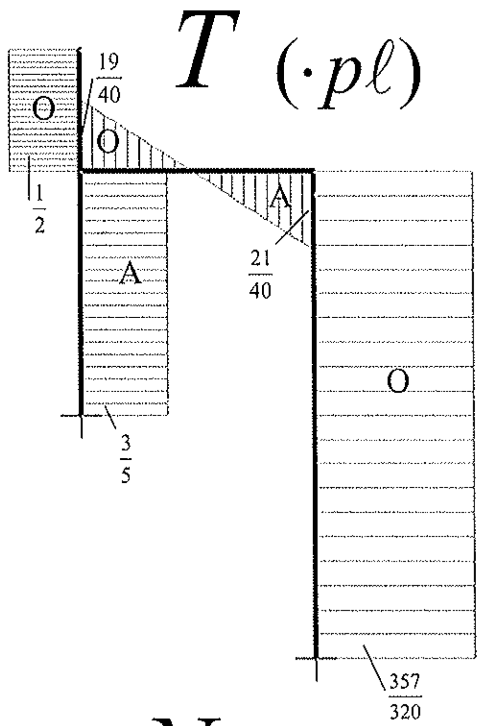
$$\begin{cases} \dfrac{7EJ}{\ell}\varphi_B - \dfrac{6EJ}{\ell^2}\eta_B - \dfrac{p\ell^2}{2} = 0 \\ -\dfrac{6EJ}{\ell^2}\varphi_B + \dfrac{13EJ}{\ell^3}\eta_B + \dfrac{5p\ell}{8} = 0 \end{cases}$$

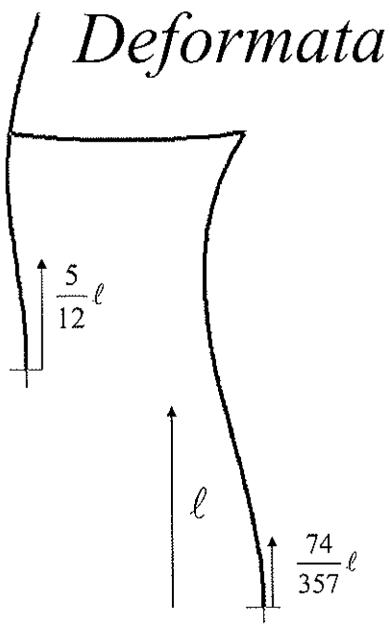
Soluzioni

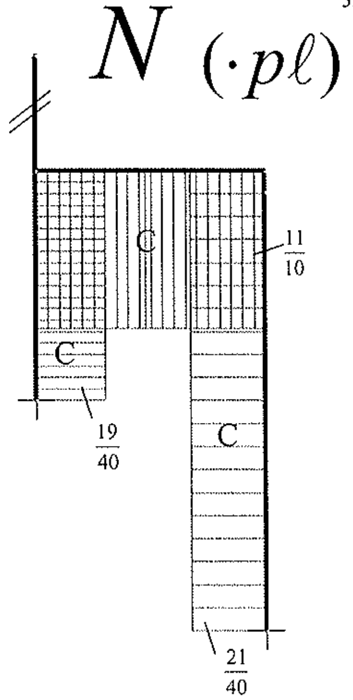
$$\begin{cases} \varphi_B = \dfrac{p\ell^3}{20EJ} \\ \eta_B = -\dfrac{p\ell^4}{40EJ} \end{cases}$$

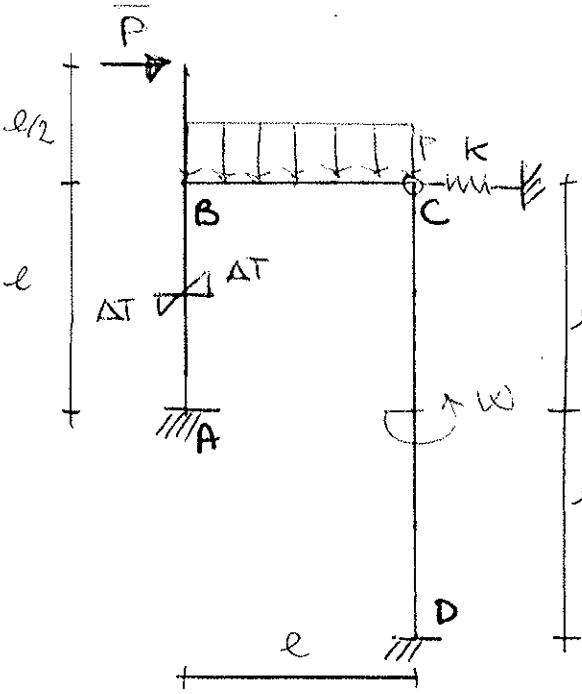
Diagrammi delle azioni interne e Deformata











$K = \frac{5}{8}\frac{EJ}{\ell^3}$

$\frac{d\Delta T}{h} = \frac{1}{16}\frac{p\ell^2}{EJ}$

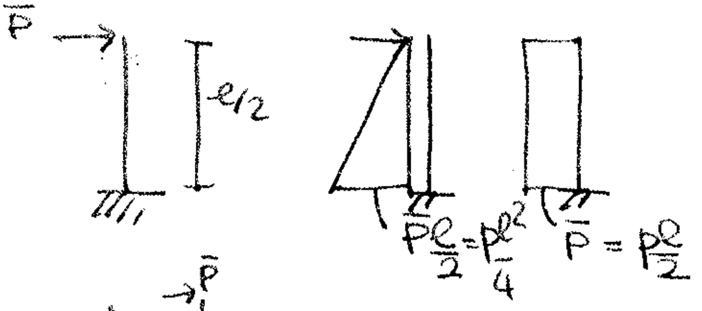
$\omega = 2p\ell^2$

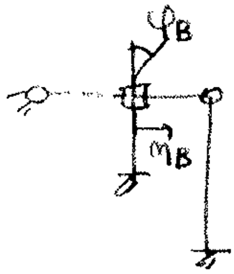
$\bar{P} = p\ell/2$

$m_{BB}\varphi_B + m_{B\eta}\eta + m_{B0} = 0$

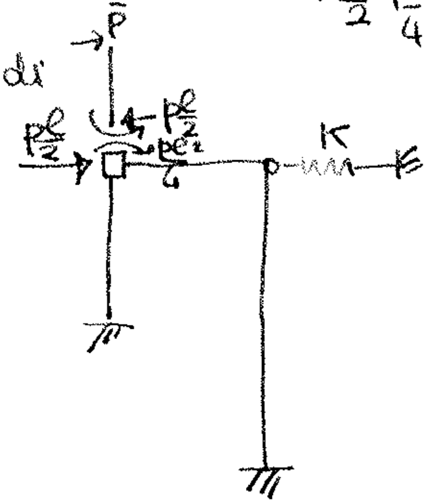
$h_{\eta B}\varphi_B + h_{\eta\eta}\eta + h_{\eta 0} = 0$

Tolgo l'appendice isostatica

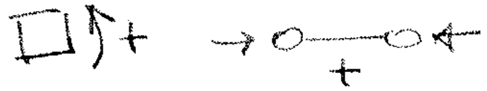




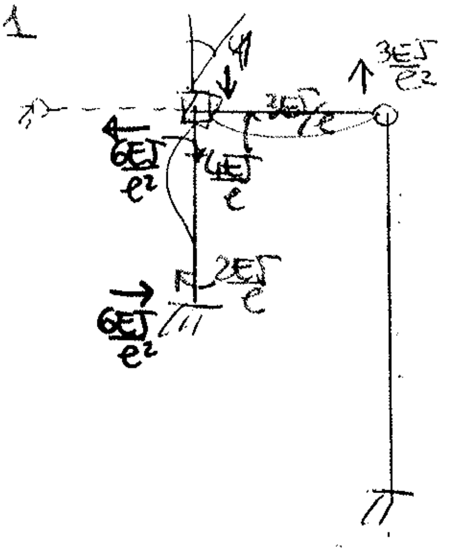
quindi



CONVENZIONI

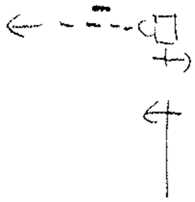


$\varphi_B = 1$

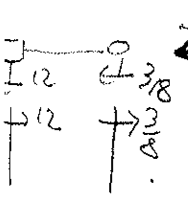


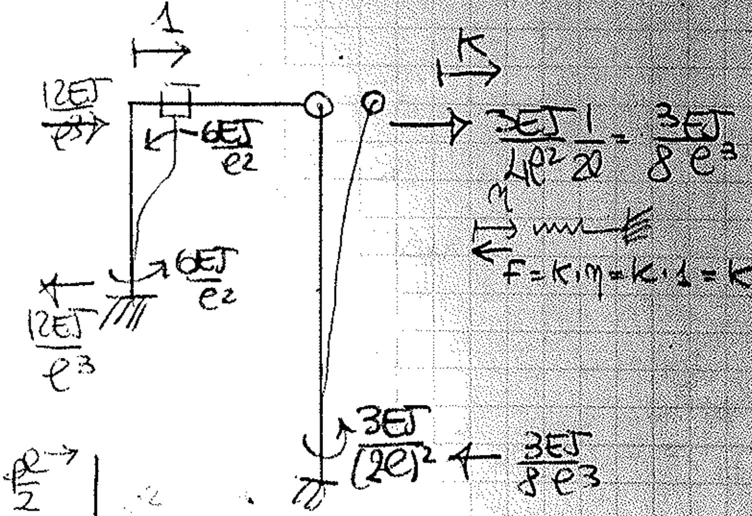
$m_{BB} = \frac{7EJ}{\ell}$

$h_{\eta B} = -\frac{6EJ}{\ell^2}$



$\eta_B = 1$

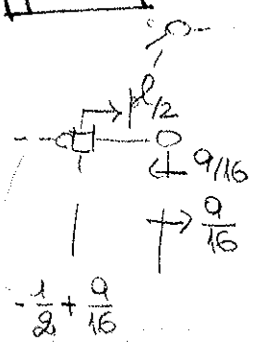


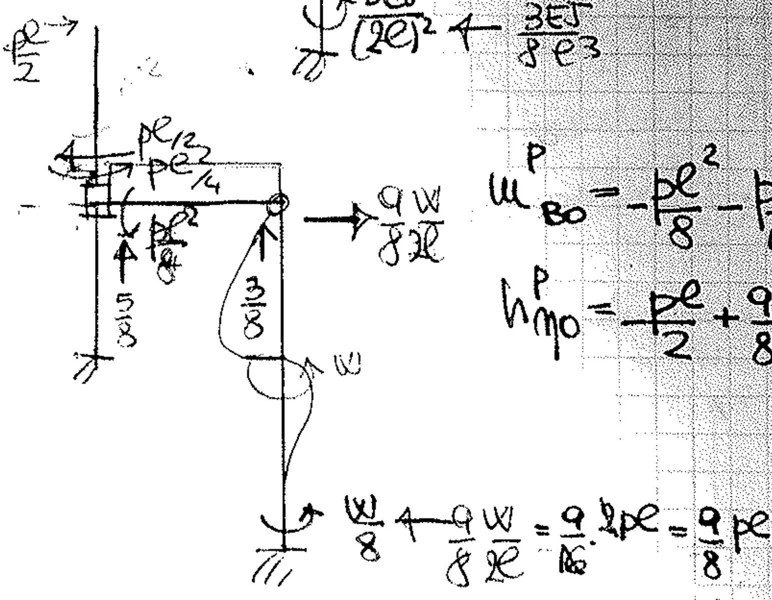


$$m_{B\eta} = -\frac{6EJ}{l^2}$$

$$h_{\eta\eta} = \frac{12EJ}{l^3} + \frac{3EJ}{8l^3} + K = \frac{99EJ}{8l^3} + K = \frac{99}{8} + \frac{5}{8}\frac{EJ}{l^3} = 13\frac{EJ}{l^3}$$

$p \neq 0$

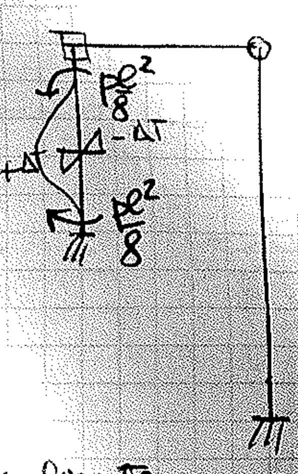




$$m^{p}_{B0} = -\frac{pl^2}{8} - \frac{pl^2}{4} = -\frac{3}{8}pl^2$$

$$h^{p}_{\eta 0} = -\frac{pl}{2} + \frac{9}{8}\frac{W}{2l} = -\frac{pl}{2} + \frac{9}{16}\frac{2pl^2}{l} = \frac{10}{16} = \frac{5}{8}pl$$

$\Delta T \neq 0$



$$h^{\Delta T}_{\eta 0} = 0$$

$$m^{\Delta T}_{B0} = -\frac{2EJ\alpha\Delta T}{h} = -2EJ\cdot\frac{1}{16}\frac{pl^2}{EJ} = \frac{pl^2}{8}$$

sistema risolvente

$$\frac{7EJ}{l}\varphi_B - \frac{6EJ}{l^2}\eta - \underbrace{\frac{3}{8}pl^2}_{m^{p}_{B0}} - \underbrace{\frac{pl^2}{8}}_{m^{\Delta T}_{B0}} = 0$$

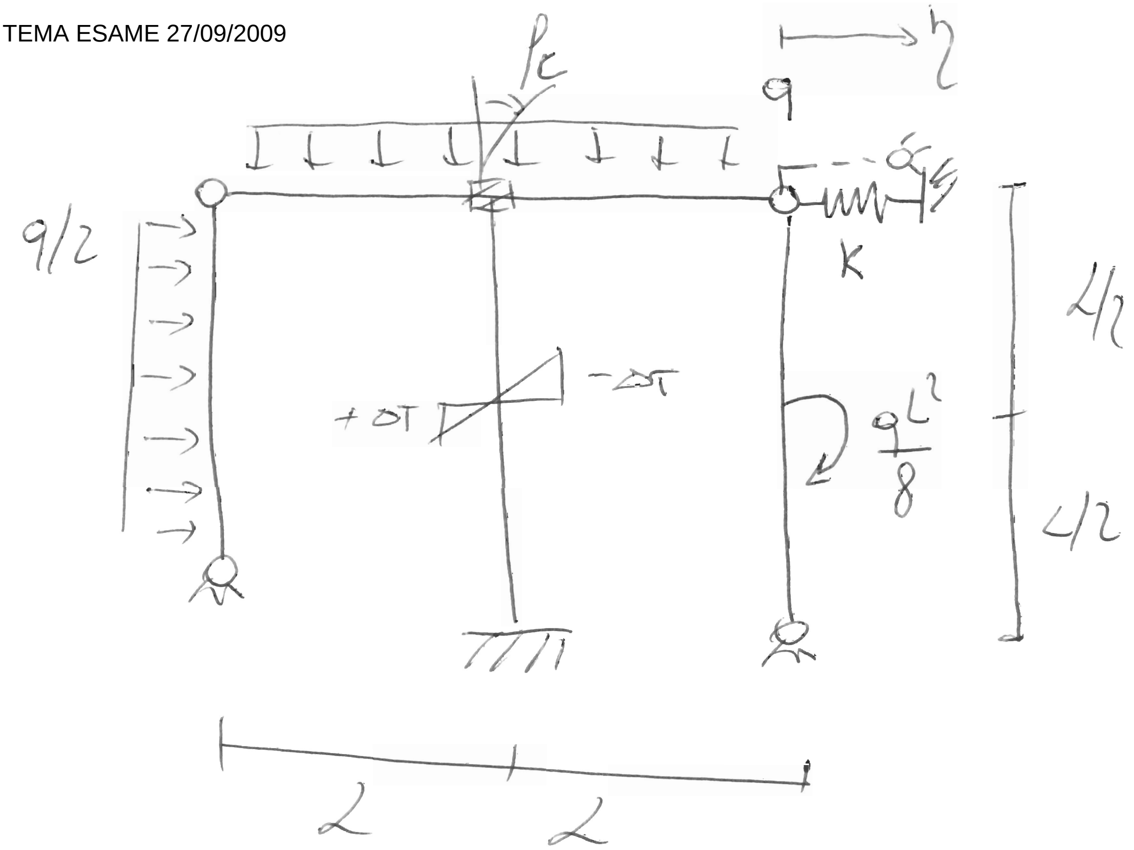
$$-\frac{6EJ}{l^2}\eta + \frac{13EJ}{l^3}\eta + \frac{5}{8}pl = 0$$

$$\begin{cases} \frac{7EJ}{l}\varphi_B - \frac{6EJ}{l^2}\eta - \frac{pl^2}{2} = 0 \\ -\frac{6EJ}{l^2} + \frac{13EJ}{l^3} + \frac{5}{8}pl = 0 \end{cases}$$

soluzione del sistema risolvente

$$\begin{cases} \varphi_B = \frac{pl^3}{20EJ} \\ \eta = -\frac{pl^4}{40EJ} \end{cases}$$

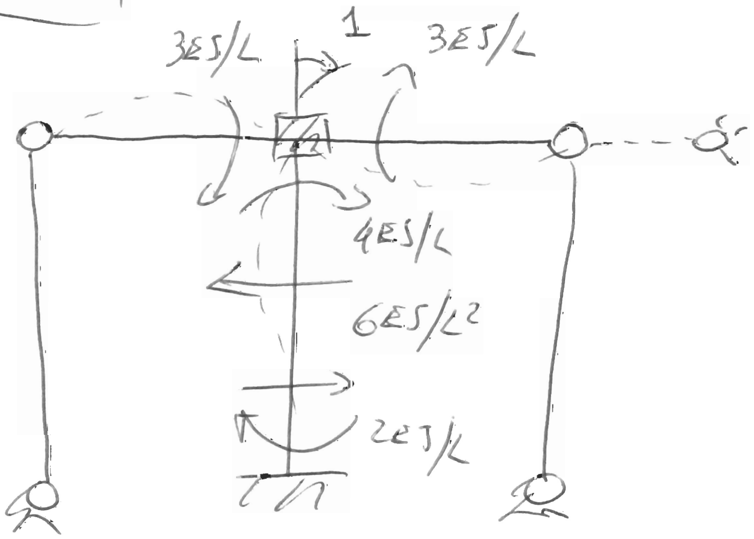
TEMA ESAME 27/09/2009



$$k = \frac{21}{5}\frac{EJ}{L^3}$$

$$\frac{\alpha \Delta T}{t} = 1 \, \frac{qL^2}{EJ}$$

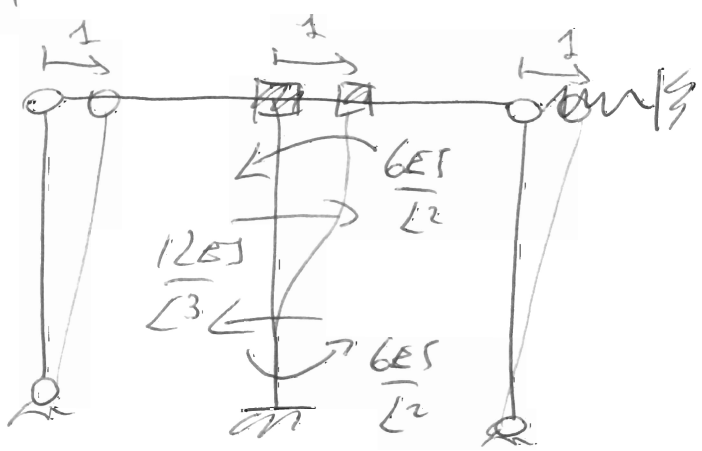
$\varphi_c = 1$



$$m_{cc} = 10\,\frac{EJ}{L}$$

$$h_{\eta c} = \frac{6EJ}{L^2}$$

$\eta = 1$

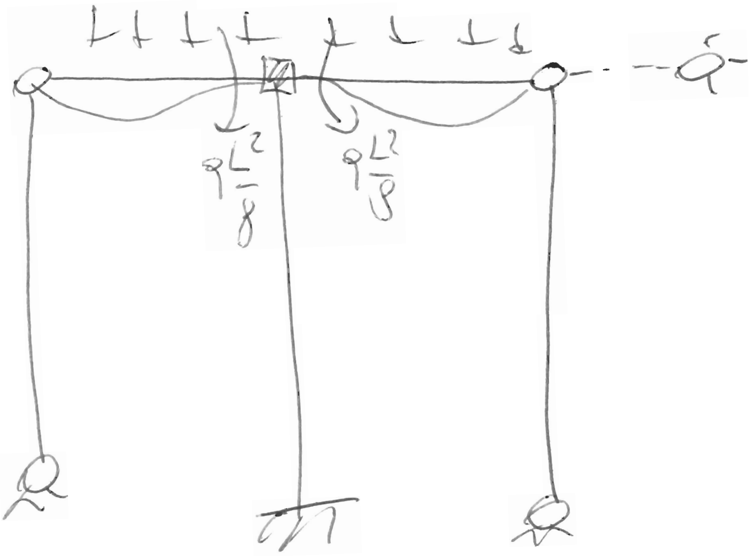


$$m_{c\eta} = -\frac{6EJ}{L^2}$$

$$h_{\eta\eta} = -\frac{12EJ}{L^3} - k$$

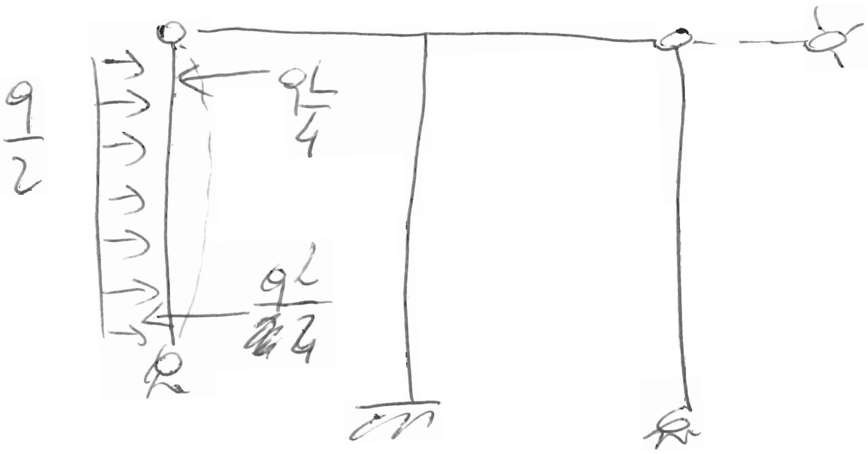
$$= -\frac{81}{5}\,\frac{EJ}{L^3}$$

$q \neq 0$



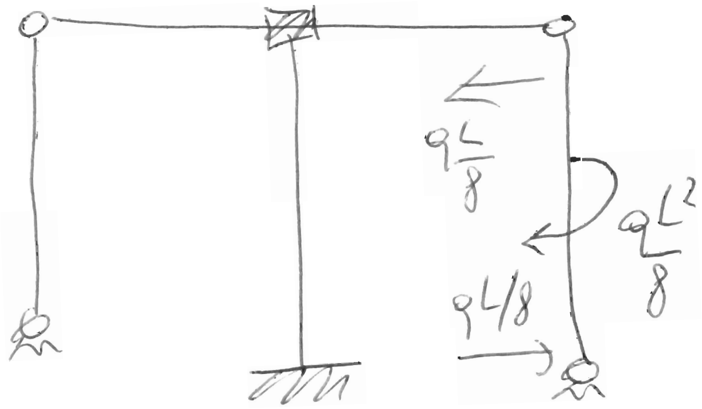
$m_{B0}^{q} = 0$

$h_{h0}^{q} = 0$



$m_{B0}^{q} = 0$

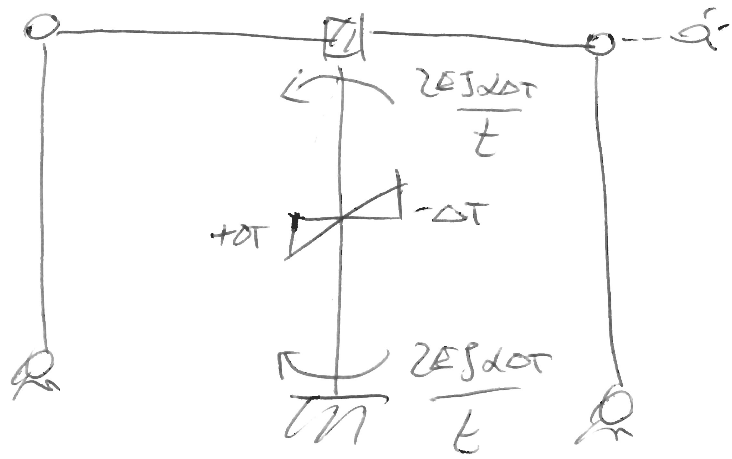
$h_{h0}^{q} = \frac{qL}{4}$



$m_{B0}^{q} = 0$

$h_{h0}^{q} = + \frac{qL}{8}$

$\boxed{\Delta T \neq 0}$



$$m_{BO}^{\Delta T} = -\frac{2EJ\alpha\Delta T}{t}$$

$$h_{HO}^{\Delta T} \overset{!}{=} 0 \qquad -2qL^2$$

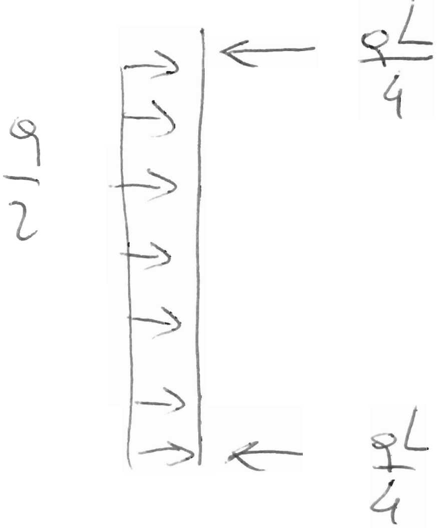
$$\begin{cases} \frac{10EJ}{L}\varphi_c - \frac{6EJ}{L^2}\eta - \frac{2EJ\alpha\Delta T}{t} = 0 \\ \frac{6EJ}{L^2}\varphi_c - (12+k)\frac{EJ}{L^3}\eta + \frac{qL}{4} + \frac{qL}{8} = 0 \end{cases}$$

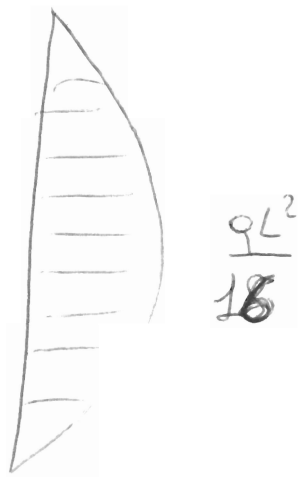
$$\begin{cases} \varphi_c = \frac{11}{40}\frac{qL^3}{EJ} \\ \eta = \frac{1}{8}\frac{qL^4}{EJ} \end{cases}$$

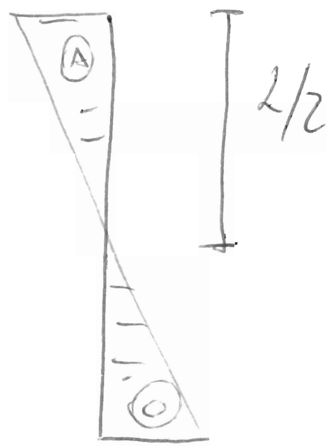
Azioni interne

aste AB

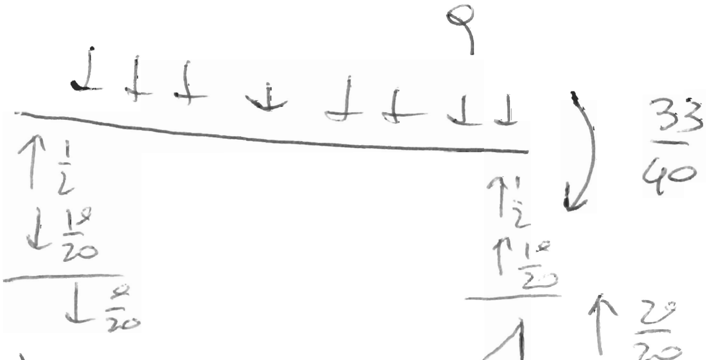
M(x) V(x)







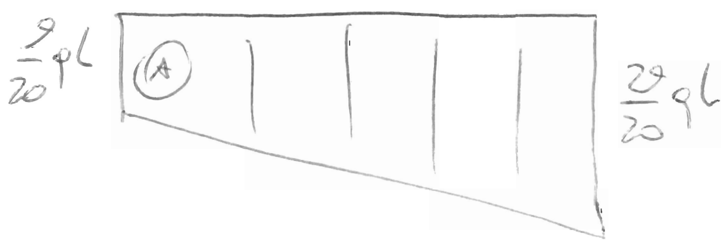
Asta BC



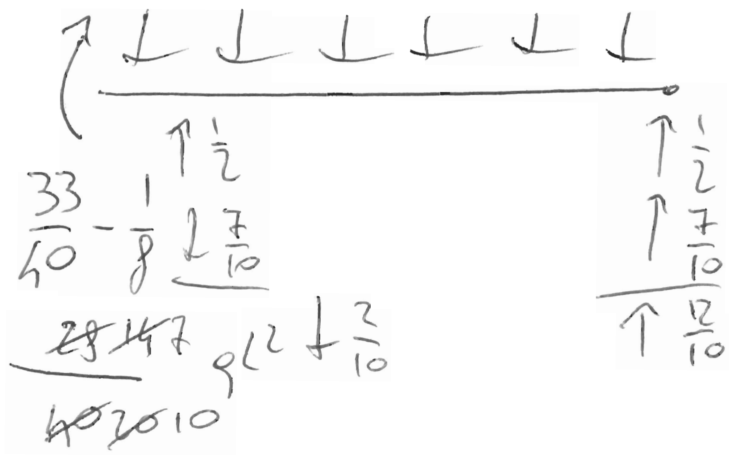
$$\frac{33}{40} q L^2 + \frac{q L^2}{8} = \frac{\cancel{38}\ 19}{\cancel{40}\ 20} q L^2$$

M(x)

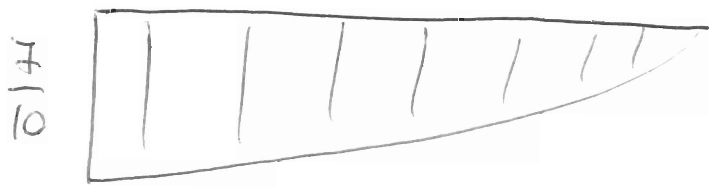
V(x)



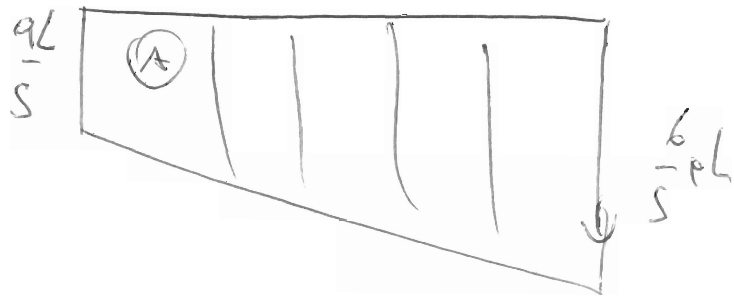
Asta BC



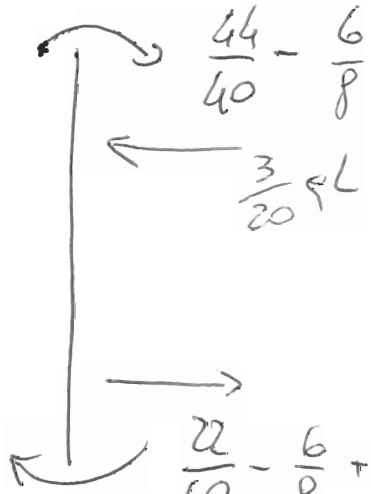
M(x)



V(x)

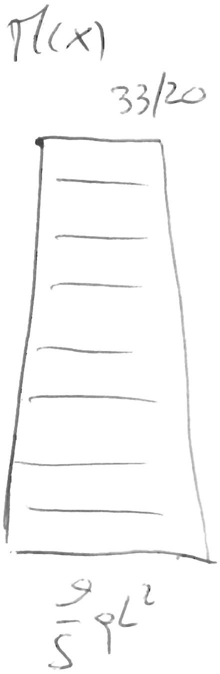


Asta CD

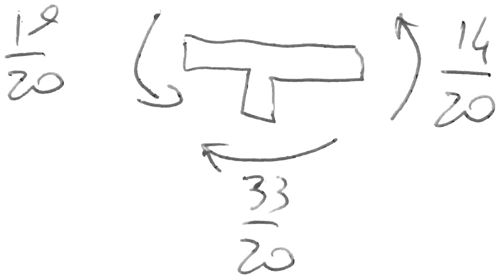


$$\frac{44}{40} - \frac{6}{8} - 2 = \frac{44-30-80}{40} = -\frac{66}{40} = -\frac{33}{20}\rho L^2$$

$$\frac{22}{40} - \frac{6}{8} + 2 = \frac{22-30+80}{40} = \frac{72}{40} = \frac{9}{5}\rho L^2 = \frac{36}{20}\rho L^2$$

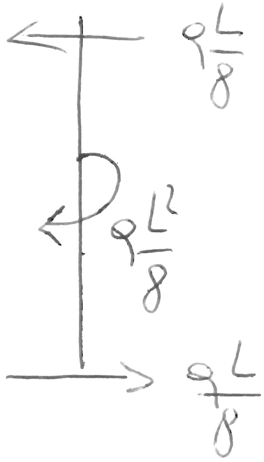


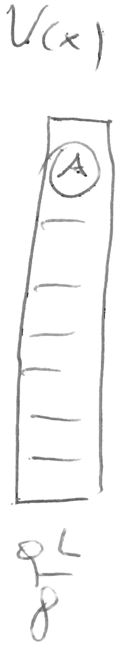
Nodo C



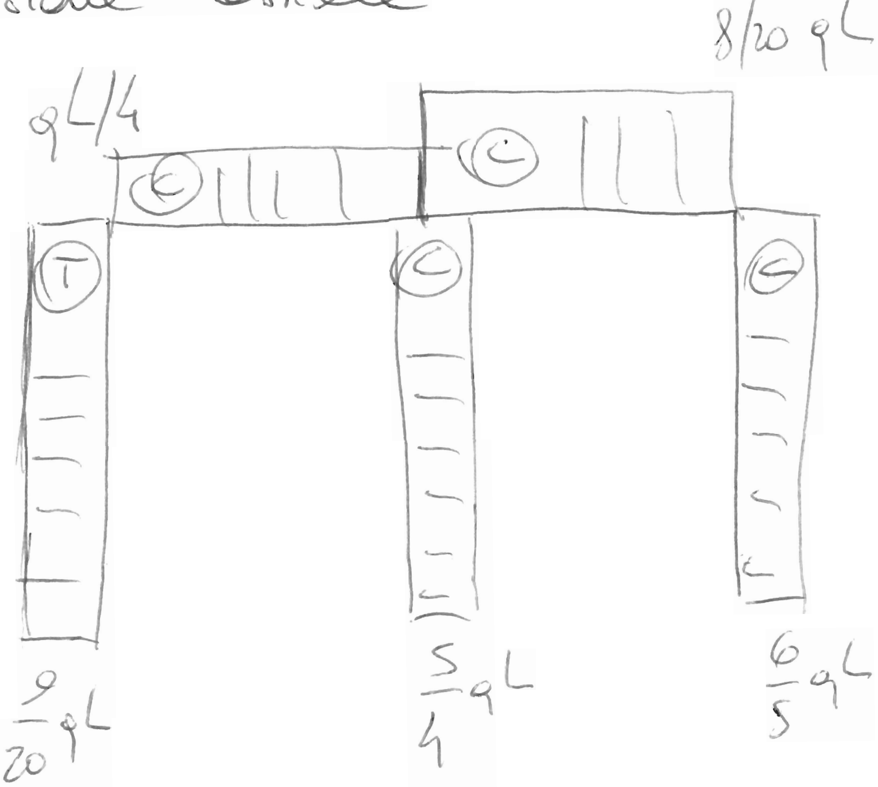
OK

Asta

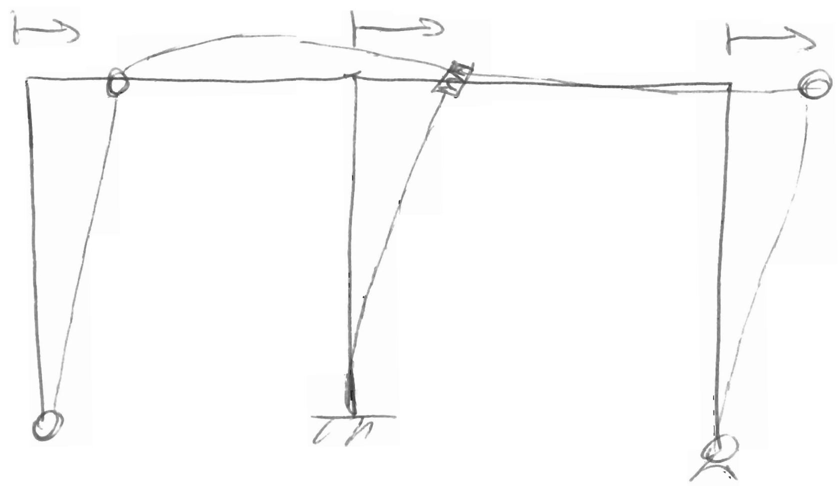




Azione assiale



Deformata qual.



FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 30/11/2009, 3h

ing. F. Minelli; ing. F. Germano

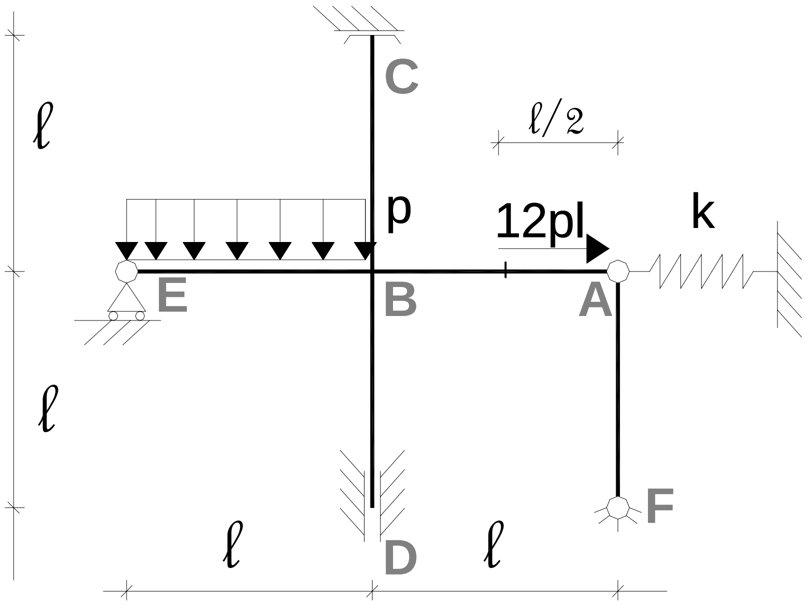
***Nome:*……………….. *Cognome:* ………………………….. *n.matr.:* ….……….**

Superato ELEMENTI STRUTTURALI A? Sì ☐ NO ☐

ESERCIZIO 1 (22 PUNTI)

$$k = 4\frac{EJ}{\ell^3};$$

$$EA \to \infty$$



Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi

ESERCIZIO 2 (10 PUNTI)

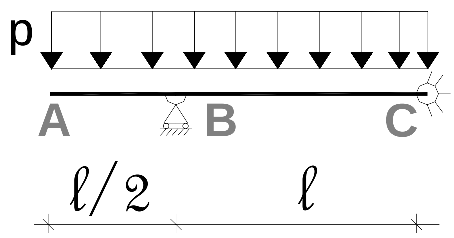
Si consideri la trave AC

Siano:

p = 4000 kg/ml

ℓ = 6 mt

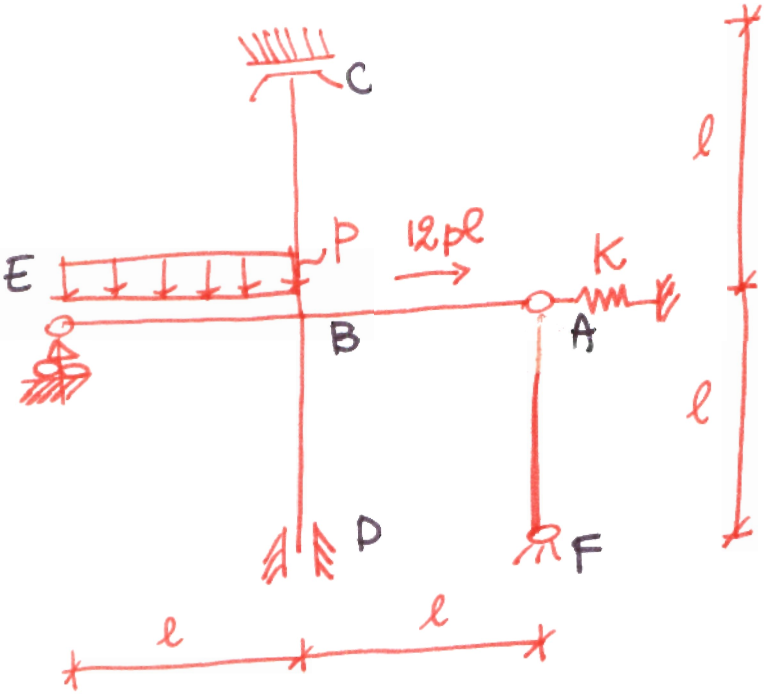
$$f_{MAX} = \frac{3}{384}\frac{p\ell^4}{EJ}$$



1- Si dimensioni la trave AC (2);
2- Si verifichi la trave nella sezione più sollecitata (flessione e taglio) (4)
3- Si verifichi la trave a deformabilità $f_{max} < \ell/400$ (1)
4- Si dimensioni il pilastro che insiste sull'appoggio B con luce di libera inflessione h=5mt (3);

N.B. Assumere **S275** (Fe430)

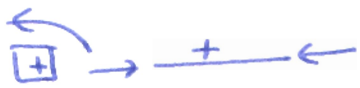
Tema esame 30/11/09 1° APPELLO

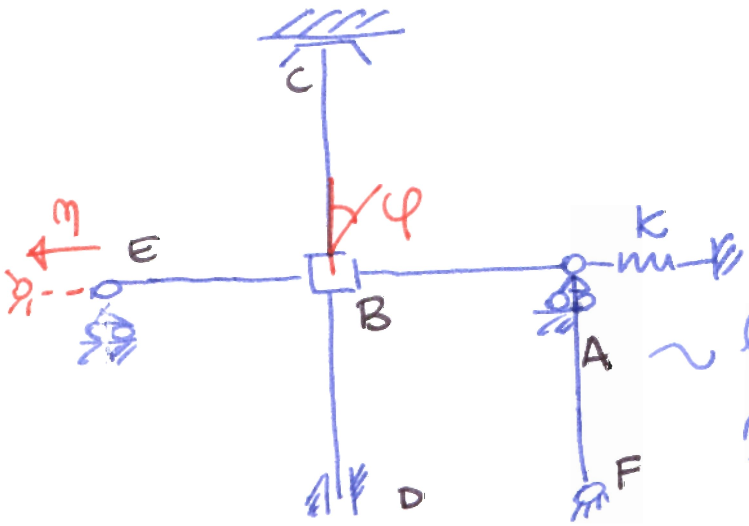


$EA \to \infty$

$K = \frac{4EJ}{l^3}$

Telaio a NODI SPOSTABILI



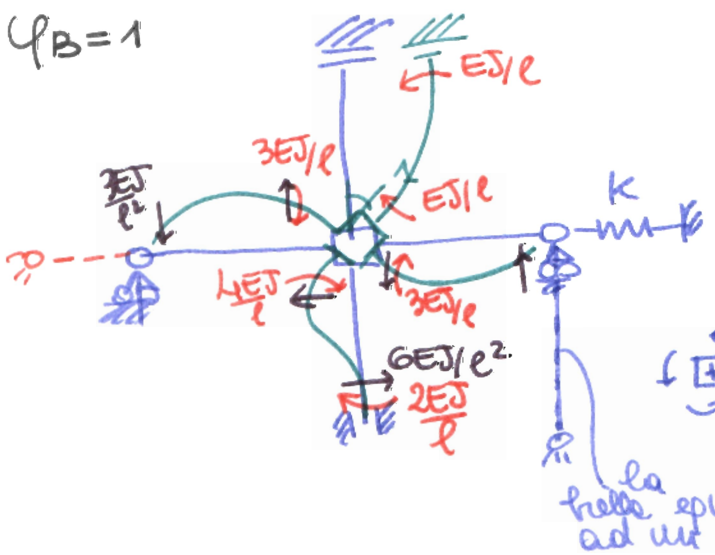


$$\begin{cases} m_{BB}\varphi_B + m_{B\eta}\cdot\eta + m_{B0} = 0 \\ l_{\eta B}\varphi_B + l_{\eta\eta}\,\eta + l_{\eta 0} = 0 \end{cases}$$

la biella AF equivale ad un carrello

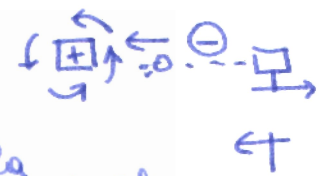
M AF M e T saranno nulli

$\varphi_B = 1$

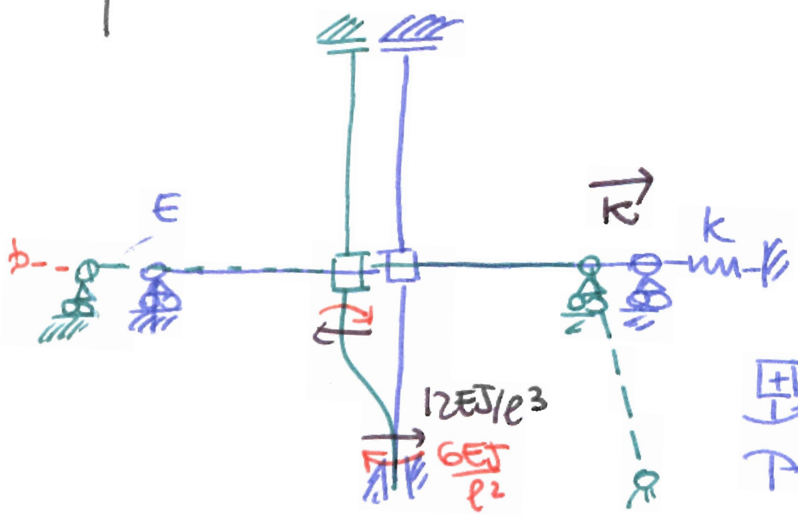


$$m_{BB} = \frac{3EJ}{l}\cdot 2 + \frac{EJ}{l} + \frac{4EJ}{l} = \frac{11EJ}{l}$$

$$l_{\eta B} = -\frac{6EJ}{l^2}$$

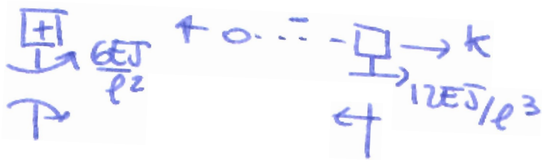


$\eta = 1$



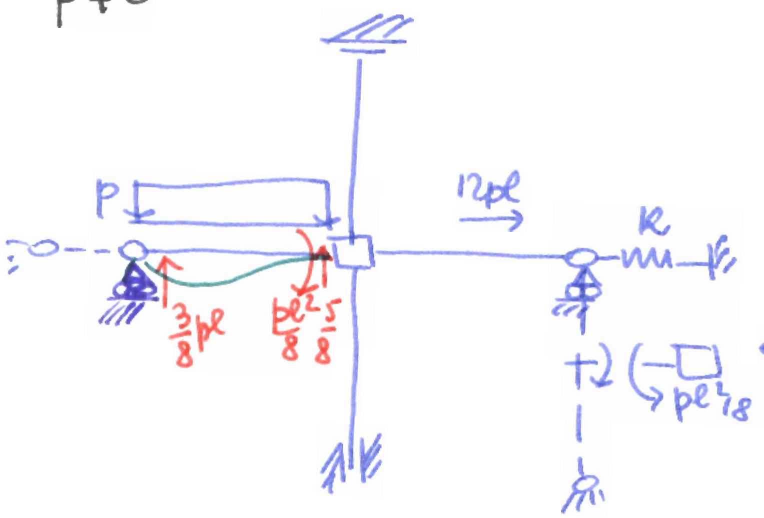
$m_{B\eta} = \frac{6EJ}{l^2}$

$l_{\eta\eta} = -\frac{12EJ}{l^3} - k = -\frac{16EJ}{l^3}$



N.B. EF. equivale ad un carrello

$p \neq 0$



$m_{B0} = + \frac{pl^2}{8}$

$l_{\eta 0} = -12pl$

SISTEMA RISOLVENTE

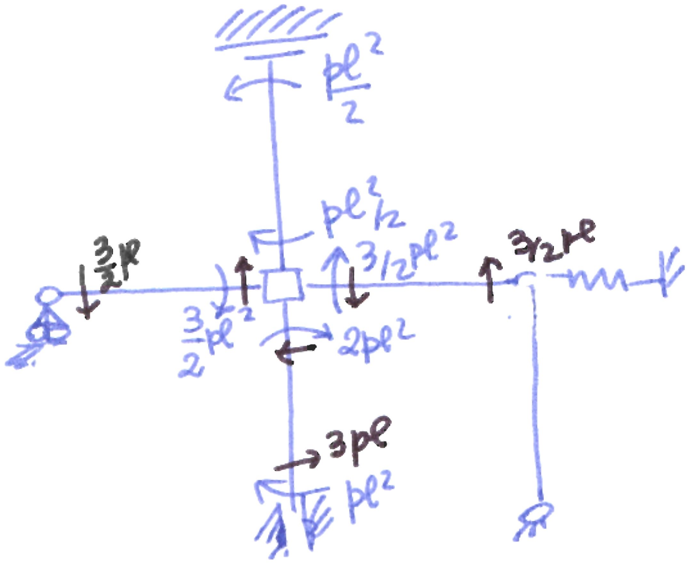
$$\begin{cases} \frac{11EJ}{l}\varphi_B + \frac{6EJ}{l^2}\eta + \frac{pl^2}{8} = 0 \\ -\frac{6EJ}{l^2}\varphi_B - \frac{16EJ}{l^3}\eta - 12pl = 0 \end{cases} \quad \left(\frac{11l}{6}\right)$$

$$(1) + \frac{11l}{6}(2) = \varphi_B\left(\frac{11EJ}{l} - \frac{11EJ}{l}\right) + \eta\left(\frac{6EJ}{l^2} - \frac{88}{3}\frac{EJ}{l^2}\right) + \left(\frac{1}{8} - 22\right)pl^2 = 0$$

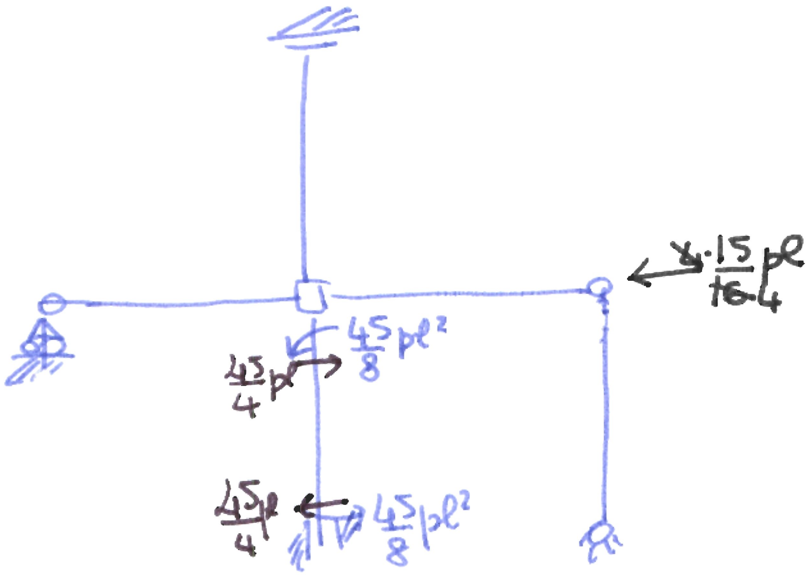
$$\eta \cdot \left(-\frac{70}{3}\frac{EJ}{l^2}\right) = \frac{175}{8}pl^2 \rightarrow \boxed{\eta = -\frac{15}{16}\frac{pl^4}{EJ}}$$

$$-\left(6 \cdot \left(-\frac{15}{16}p\frac{l^4}{EJ}\right) + \frac{pl^2}{8}\right) \cdot \frac{l}{11EJ} = \varphi_B \rightarrow \boxed{\varphi_B = \frac{1}{2}\frac{pl^3}{EJ}}$$

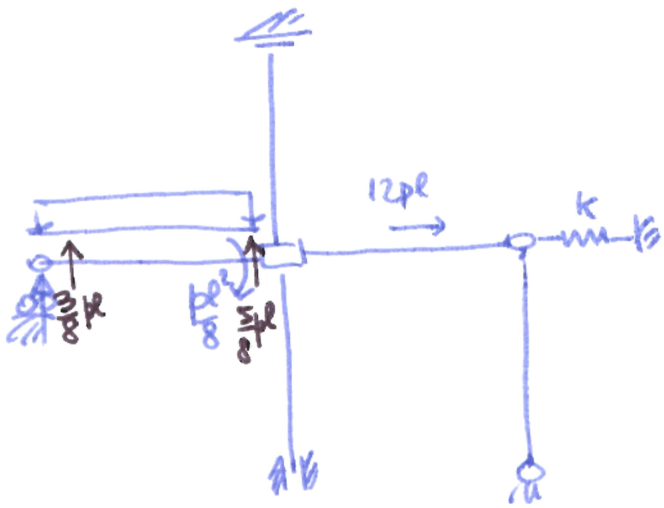
$$\varphi_B = \frac{1}{2}\frac{pl^3}{EJ}$$

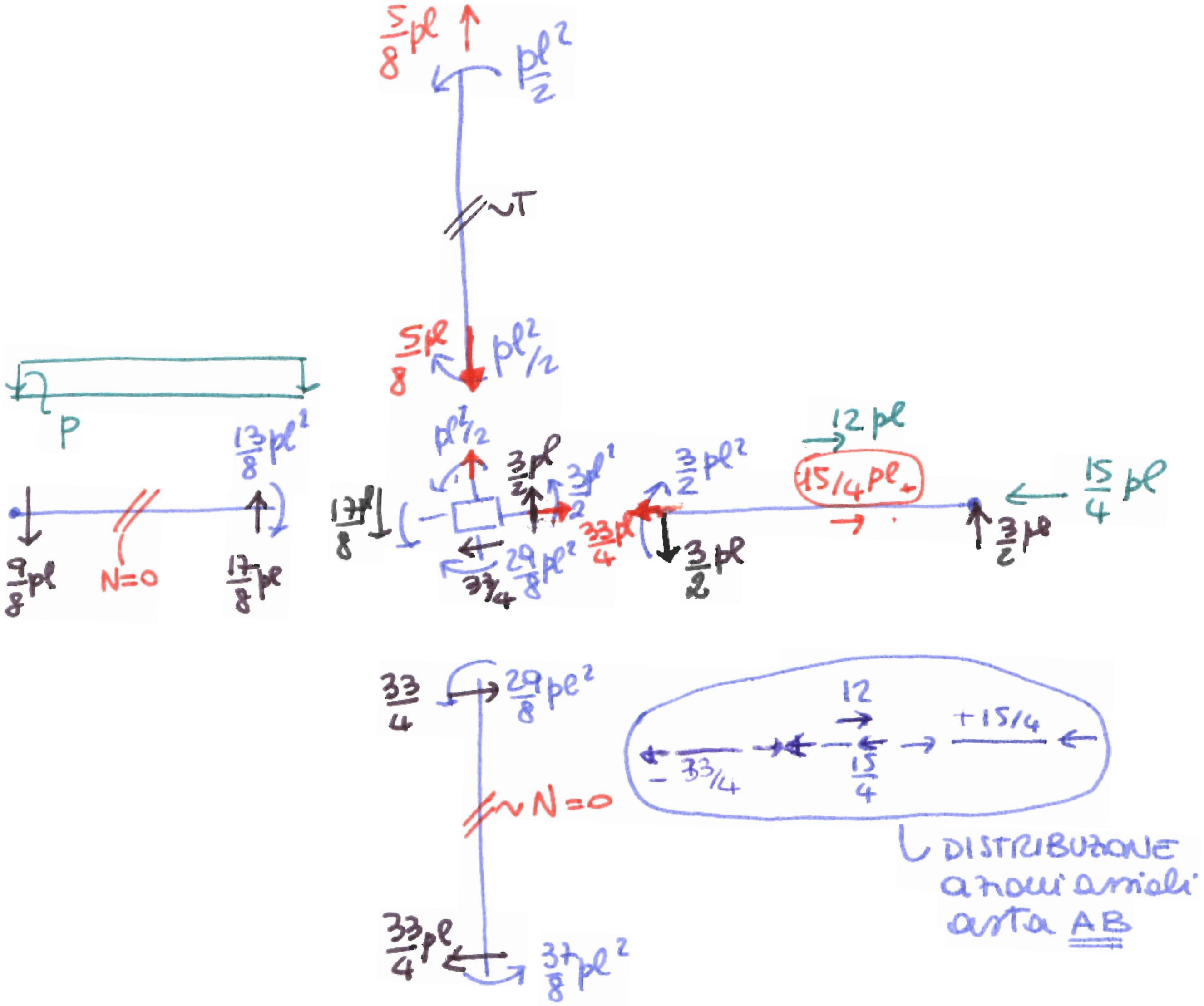


$$\eta = -\frac{15}{16}\frac{pl^4}{EJ}$$



$p \neq 0$





Equilibri globali

↑+ $-\frac{9}{8}pl - pl + \frac{3}{2}pl + \frac{5}{8}pl = 0$ ok!

→+ $-\frac{33}{4}pl + 12pl - \frac{15}{4}pl = 0$ ok!

↺+ A $\frac{9}{8}pl\cdot 2l + pl\cdot\frac{3}{2}l - \frac{5}{8}pl^2 + \frac{pl^2}{2} + \frac{37}{8}pl^2 - \frac{33}{4}pl^2 = 0$ ok!

Eq. NODO B

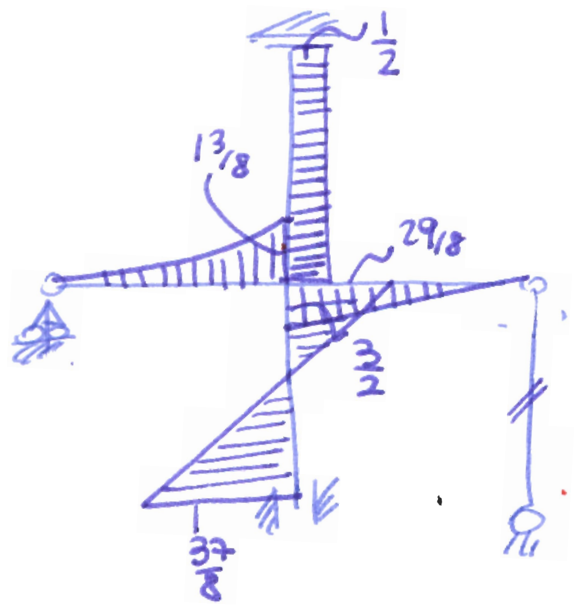
→+ $\frac{33}{4}pl - \frac{33}{4}pl = 0$ ok!

↑+ $\frac{17}{8}pl - \frac{3}{2}pl + \frac{5}{8}pl = 0$ ok!

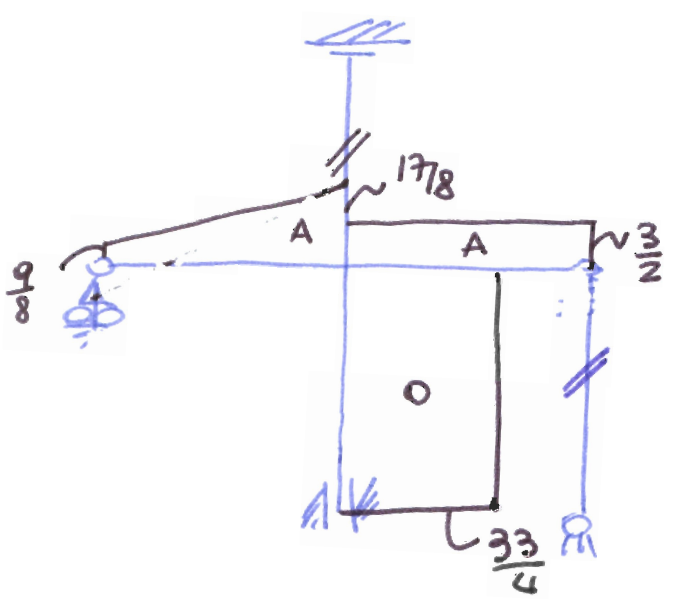
↺+ $\frac{3}{2}pl^2 + pl^2/2 + \frac{13}{8}pl^2 - \frac{29}{8}pl^2 = 0$ ok!

Diagrammi

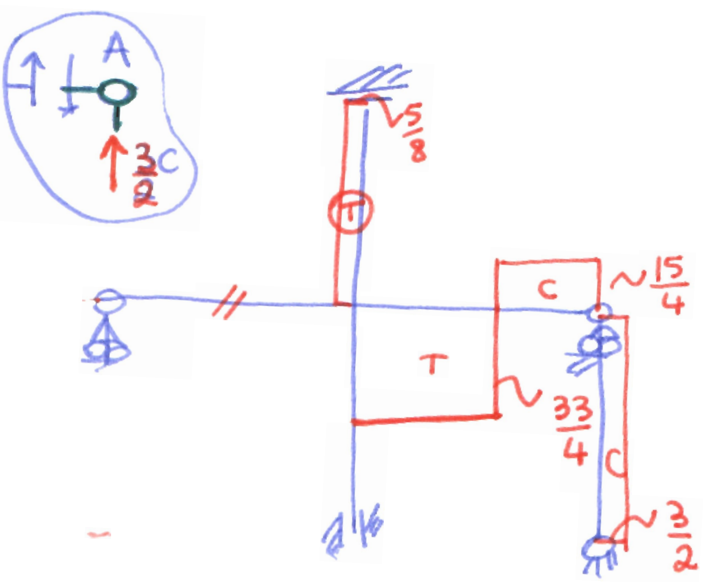
Ⓜ $\cdot pl^2$



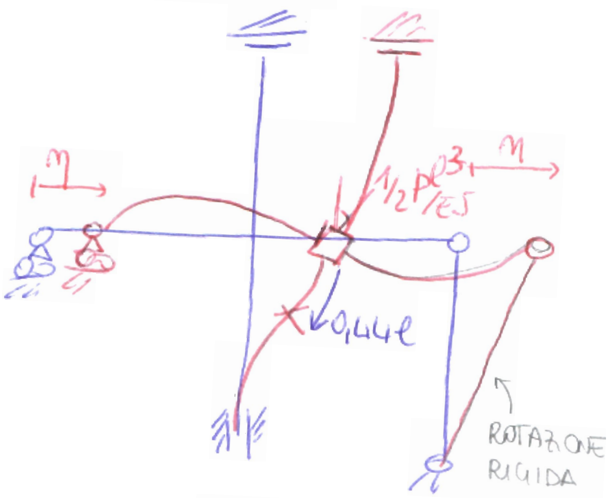
Ⓣ $\cdot pl$



Ⓝ $\cdot pl$



Deformata



$$\frac{29}{8}pl^2 : x = \frac{37}{8}pl^2 : (l - x)$$

$$\frac{37}{8}pl^2 x = \frac{29}{8}pl^2 (l - x)$$

$$37x + 29x = 29l$$

$$x = \frac{29}{66}l$$

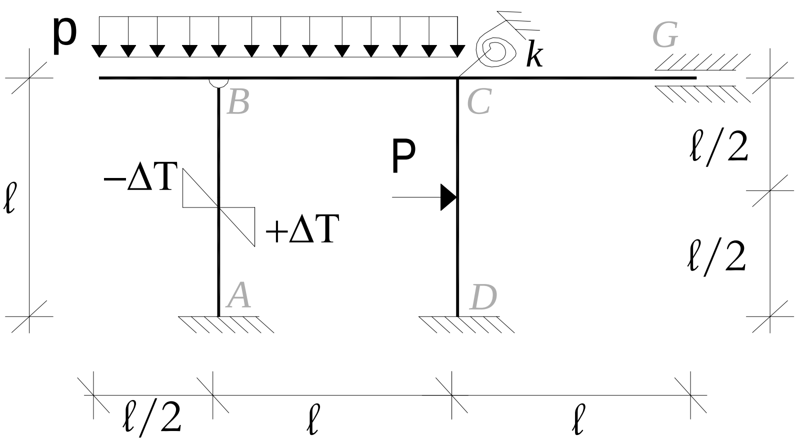
FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 17/12/2009, 3h

ing. F. Minelli; ing. F. Germano

***Nome:*……………….. *Cognome:* ……………………….. *n.matr.:* ………….**

Superato ELEMENTI STRUTTURALI A? Sì ☐ NO ☐

ESERCIZIO 1 (20 PUNTI)



$$P = 2p\ell;\quad k = \frac{2EJ}{\ell};\quad \frac{\alpha \Delta T}{h} = \frac{5p\ell^2}{8EJ};\quad EA \to \infty;\quad EJ = \cos t$$

Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi);
2- Taglio;
3- Azione assiale;
4- Deformata qualitativa con posizione dei flessi.

ESERCIZIO 2 (10 PUNTI)

Le due aste AB e BC (**figura 1**) sono delle IPE 550. In B si realizza il collegamento disegnato in **figura 2.**

Siano:

P = 16000 kg; ℓ = 5 m

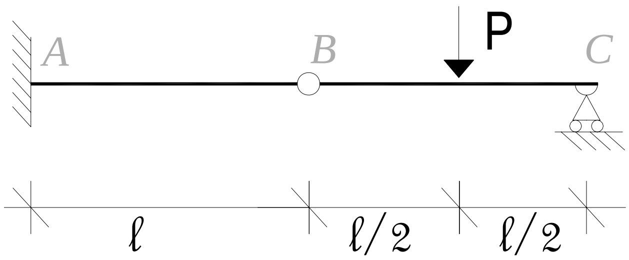


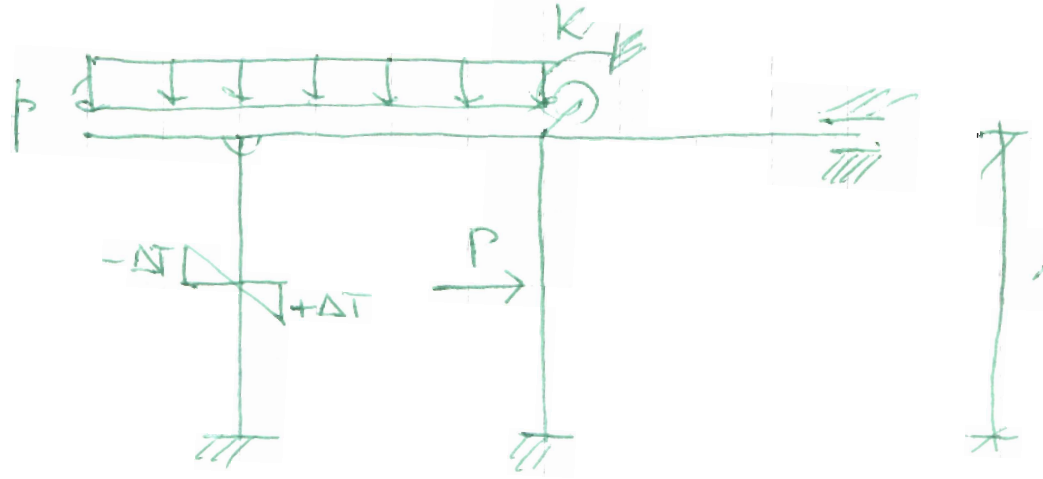
figura 1

Si richiedono:

1- Il tracciamento delle azioni interne della struttura (1);
2- Il dimensionamento del <u>diametro dei bulloni</u> e dello <u>spessore (**s**) dei fazzoletti</u> (4);
3- Le <u>verifiche del collegamento</u> (2)
4- Si dimensioni il pilastro che insiste sull'appoggio C. Si assuma una luce di libera inflessione **l =7m** (3);

N.B. Assumere **S275** (Fe430) e **bulloni di classe 6.6**

Tema Esame



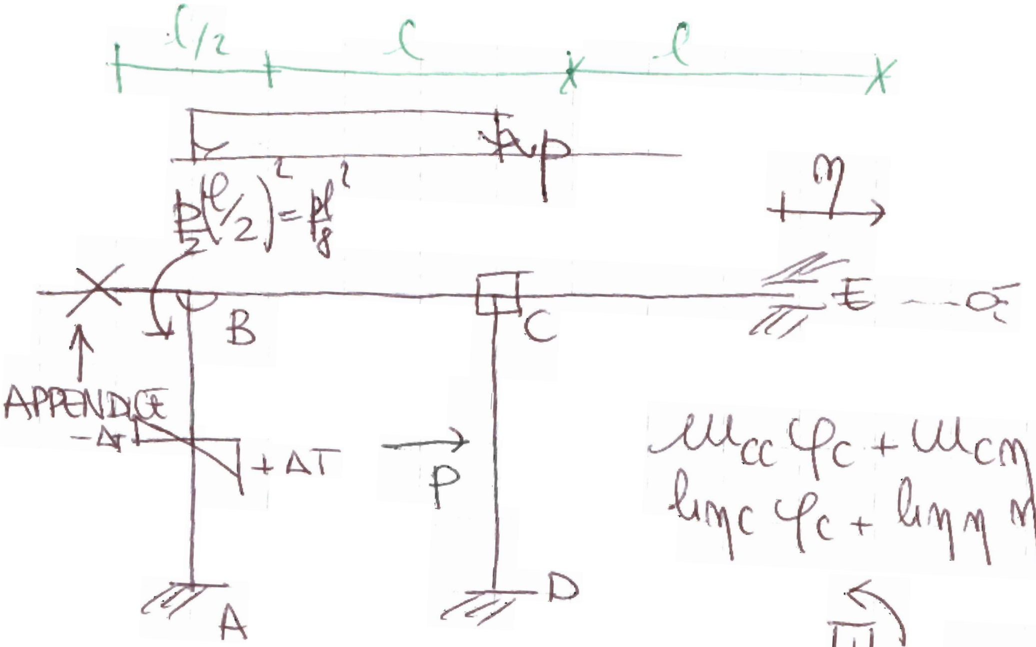
$P = 2pl$

$k = \frac{2EJ}{l}$

$\frac{\alpha \Delta T}{h} = \frac{5}{8}\frac{pl^2}{EJ}$

$EA \to \infty$

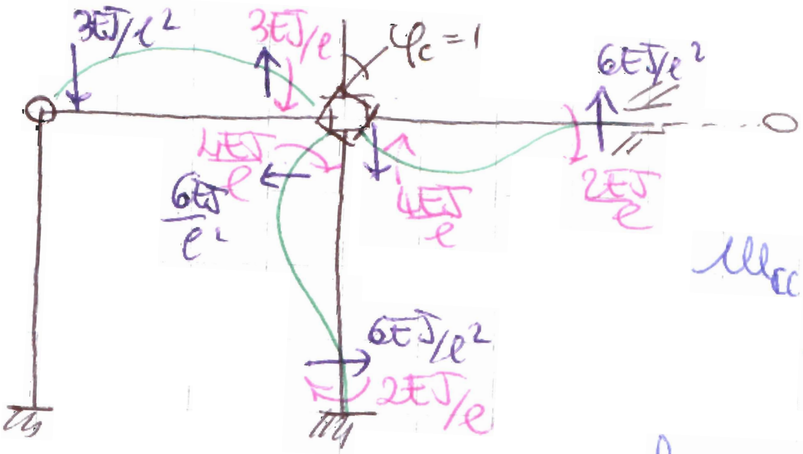
EJ cost



$$\mathcal{M}_{cc}\varphi_c + \mathcal{M}_{c\eta}\,\eta + \mathcal{M}_{c0} = 0$$

$$\mathcal{L}_{\eta c}\varphi_c + \mathcal{L}_{\eta\eta}\,\eta + \mathcal{L}_{\eta 0} = 0$$

$\varphi_C = 1$

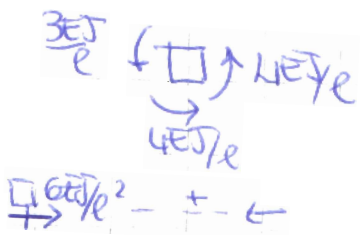


$\mathcal{M}_{cc} = \frac{11EJ}{l} + k$

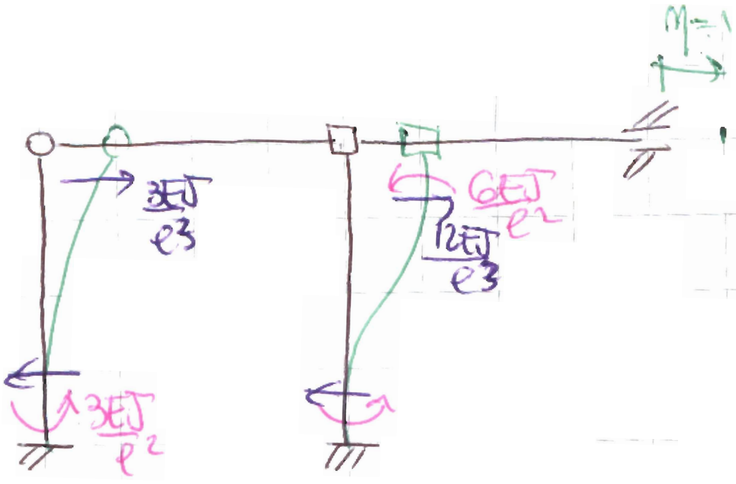
$= \frac{13EJ}{l}$

$\mathcal{L}_{\eta c} = \frac{6EJ}{l^2}$

NODO C



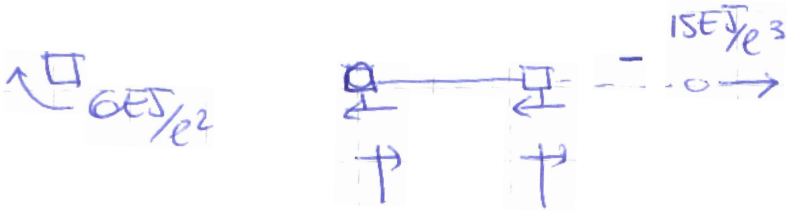
$\eta = 1$



$M_{C\eta} = -\frac{6EJ}{\ell^2}$

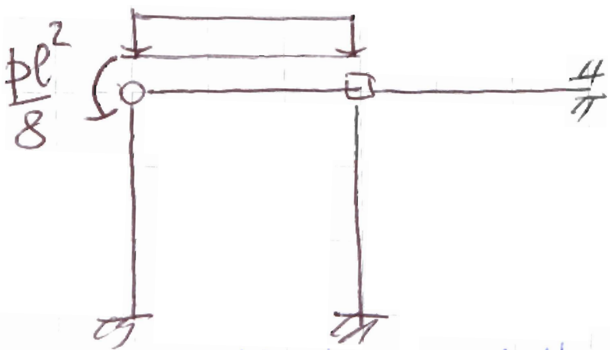
$L_{\eta\eta} = -\frac{15EJ}{\ell^3}$

NODO C



Carichi esterni

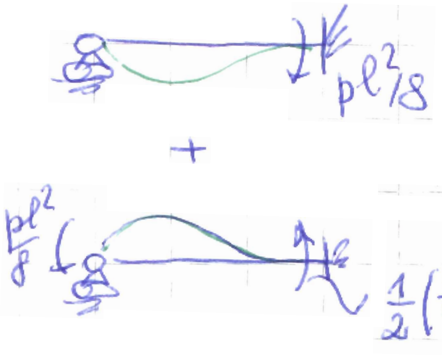
① p distribuito ≠ 0



$L^{(1)}_{\eta 0} = 0$

$M^{(1)}_{C0} = +\frac{1}{16}p\ell^2$

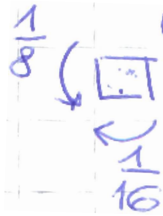
è somma di due contributi



$\frac{1}{2}\left(\frac{p\ell^2}{8}\right) = \frac{p\ell^2}{16}$

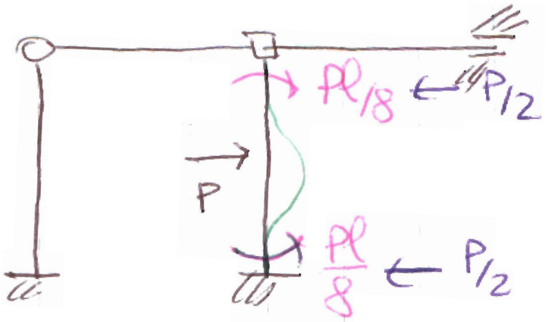
=

NODO C



$M_{TOT} = \frac{1}{16}p\ell^2$

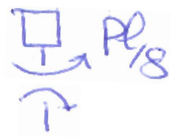
② $P \neq 0$

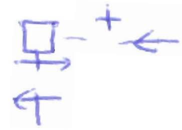


$$\mathcal{M}_{co}^{(2)} = +\frac{Pl}{8} = \frac{l \cdot 2pl}{8} = \frac{pl^2}{4}$$

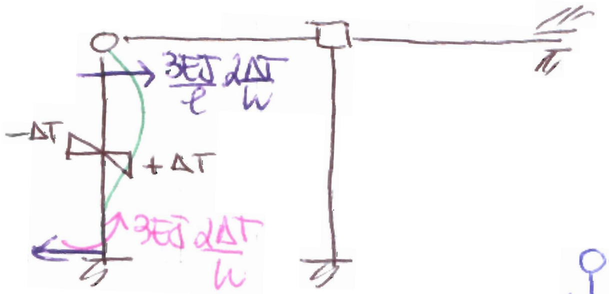
$$T_{mo}^{(2)} = +\frac{P}{2} = \frac{2pl}{2} = pl$$

Nodo C



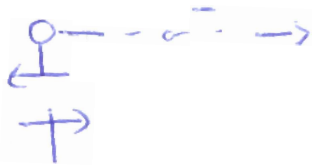


③ $\Delta T \neq 0$



$$\mathcal{M}_{co}^{(3)} = 0$$

$$T_{mo}^{(3)} = -\frac{3EJ}{l} \cdot \frac{\alpha \Delta T}{w} = -\frac{15}{8} pl$$



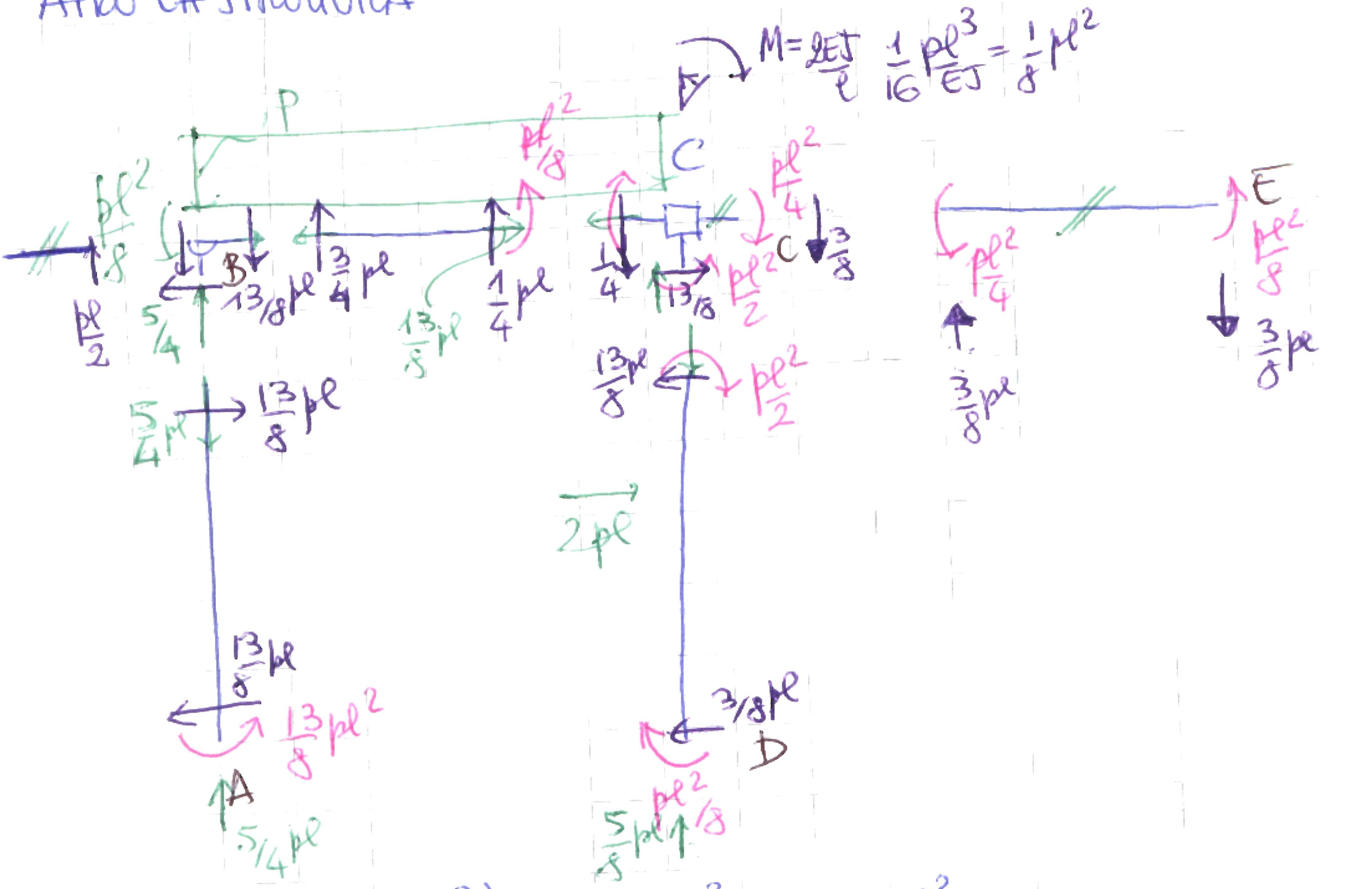
$$\begin{cases} \left(\frac{11EJ}{l} + K\right)\varphi_c - \frac{6EJ}{l^2} \cdot \eta + \frac{5}{16} pl^2 = 0 \\ \frac{6EJ}{l^2} \varphi_c - \frac{15EJ}{l^3} \eta + \left(pl - \frac{3EJ}{l} \cdot \frac{\alpha \Delta T}{w}\right) = 0 \end{cases}$$

$$\begin{cases} \frac{13EJ}{l} \varphi_c - \frac{6EJ}{l^2} \eta + \frac{5}{16} pl^2 = 0 \\ \frac{6EJ}{l^2} \varphi_c - \frac{15EJ}{l^3} + \frac{7}{8} pl = 0 \end{cases}$$

$$\varphi_c = -\frac{1}{16} \frac{pl^3}{EJ}$$

$$\eta = -\frac{1}{12} \frac{pl^4}{EJ}$$

APRO LA STRUTTURA

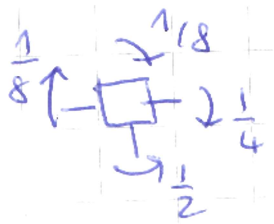


$\curvearrowright_{+} M_C^{sx} = \frac{3EJ}{l}\left(-\frac{1}{16}\frac{pl^3}{EJ}\right) + 0 + \frac{pl^2}{16} = -\frac{1}{8}pl^2$

$\curvearrowleft_{+} M_C^{dx} = \frac{4EJ}{l}\left(-\frac{1}{16}\frac{pl^3}{EJ}\right) = -\frac{pl^2}{4}$

$\curvearrowright M_C^{sotto} = \frac{4EJ}{l}\left(-\frac{1}{16}\frac{pl^3}{EJ}\right) - \frac{6EJ}{l^2}\left(-\frac{1}{12}\frac{pl^4}{EJ}\right) + \frac{pl^2}{4} = \left(-\frac{1}{4} + \frac{1}{2} + \frac{1}{4}\right) = \frac{pl^2}{2}$

Eq. NODO C



ok!

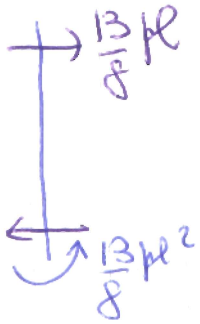
$\curvearrowleft_{+} M_A = \frac{3EJ}{l^2}\left(-\frac{1}{12}\frac{pl^4}{EJ}\right) + \frac{15}{8}pl^2 = \frac{13}{8}pl^2$

$\curvearrowleft_{+} M_D = \frac{2EJ}{l}\left(-\frac{1}{16}\frac{pl^3}{EJ}\right) - \frac{6EJ}{l^2}\left(-\frac{1}{12}\frac{pl^4}{EJ}\right) - \frac{pl^2}{4} = \left(-\frac{1}{8} + \frac{1}{2} - \frac{1}{4}\right)pl^2 = \frac{pl^2}{8}$

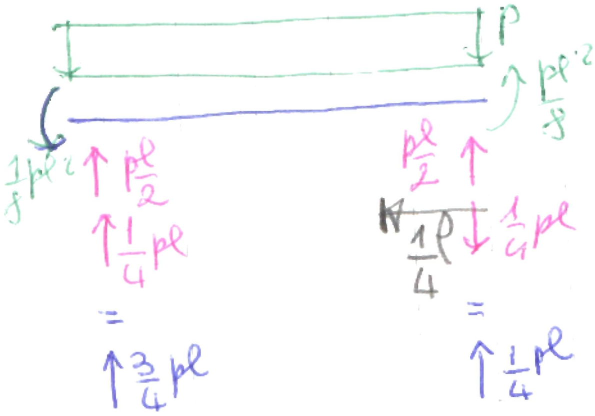
$\curvearrowright M_E = \frac{2EJ}{l}\left(-\frac{1}{16}\frac{pl^3}{EJ}\right) = -\frac{1}{8}pl^2$

Equilibrio ASTA x ASTA

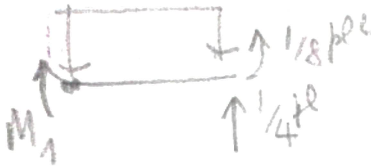
AB



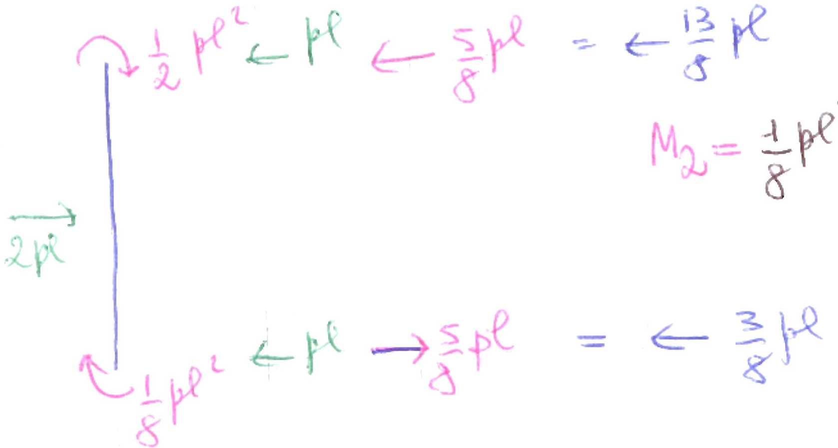
BC



$$M_1 = \frac{1}{8}pl^2 + \frac{1}{16}pl^2 - \frac{pl}{4}\frac{l}{8} = \frac{5}{32}pl^2$$

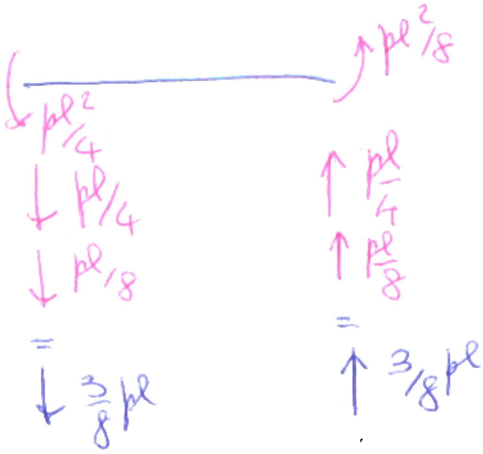


CD



$$M_2 = \frac{1}{8}pl^2 + \frac{3}{8}pl\frac{l}{2} = \frac{5}{16}pl^2$$

CE



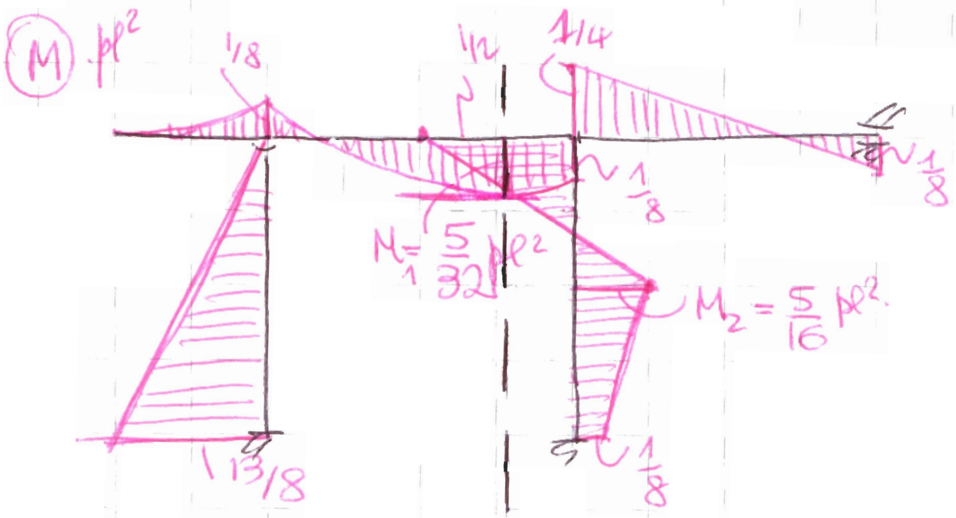
Eq. trasl. orizz.

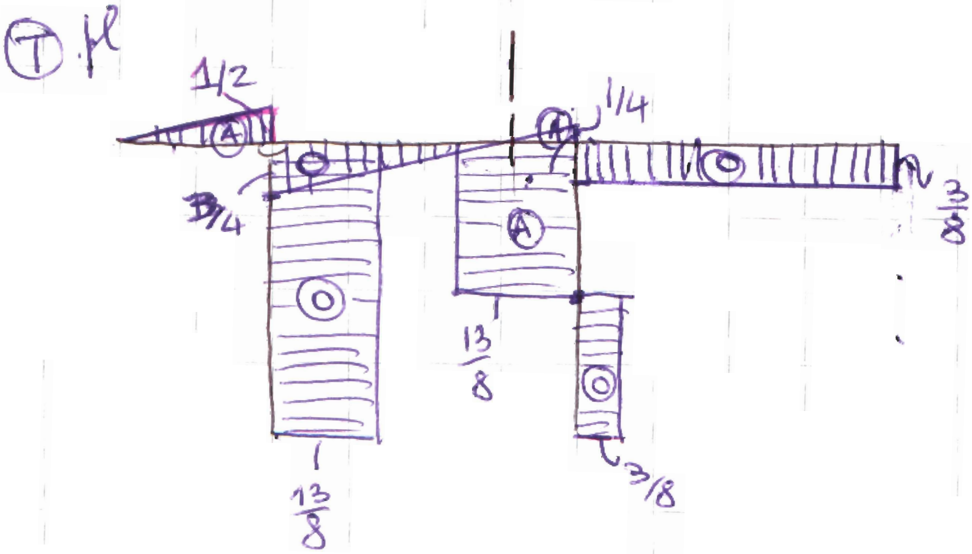
$\xrightarrow{+}$ $\quad -\frac{13}{8}pl - \frac{3}{8}pl + 2pl \stackrel{?}{=} 0$ ok!

$\uparrow +$ $\quad \frac{5}{4}pl + \frac{5}{8}pl - \frac{3}{8}pl - \frac{3}{2}pl \stackrel{?}{=} 0$ ok!

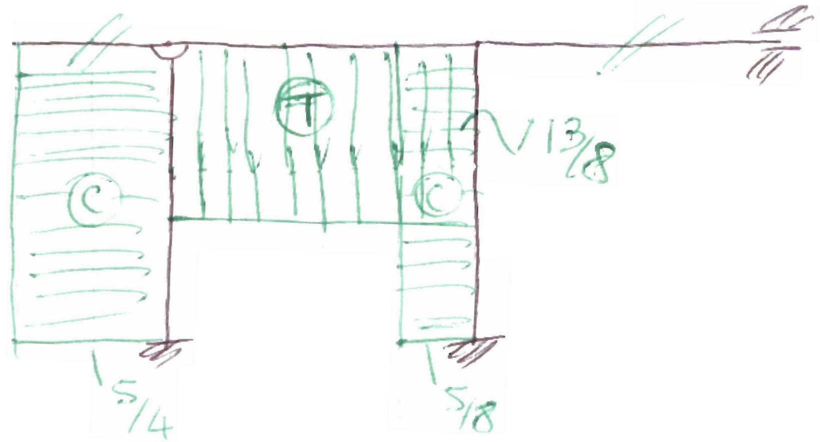
$\circlearrowleft_D +$ $\quad -\frac{1}{8}pl^2 + \frac{13}{8}pl^2 + \frac{pl^2}{8} - \frac{pl^2}{8} + \frac{pl}{2}\left(\frac{5}{4}l\right) + pl\frac{l}{2} - \frac{5}{4}pl^2 - \frac{3}{8}pl^2 - 2pl\frac{l}{2} \stackrel{?}{=} 0$ ok!

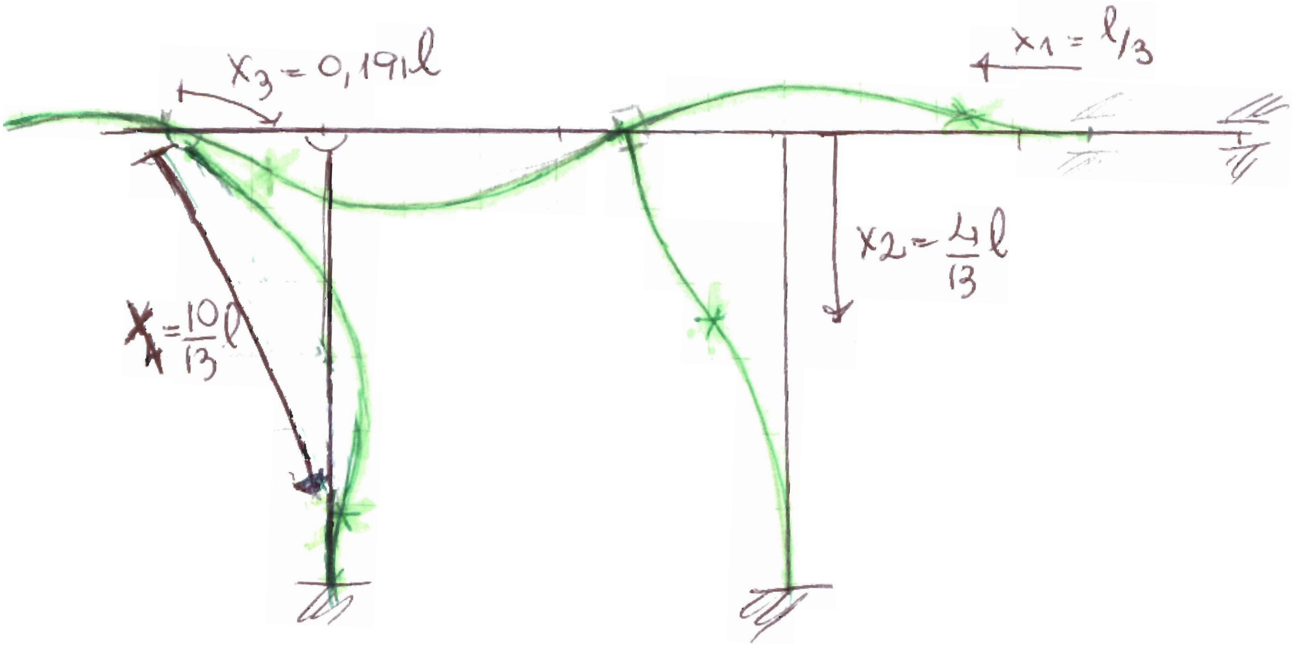
DIAGRAMMI





(N) $p\ell$



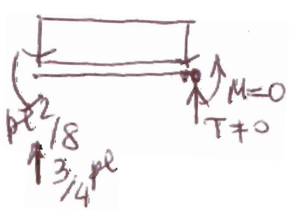


Determinazione dei flessi

(X1) $x_1 : \frac{1}{8} pl^2 = \ell : \frac{3}{8} pl^2 \rightarrow x_1 \frac{3}{8} pl^2 = \frac{1}{8} pl^2 \ell \qquad x_1 = \frac{\ell}{3}$

(X2) $x_2 : \frac{1}{2} pl^2 = \frac{\ell}{2} : \frac{13}{16} pl^2 \qquad x_2 \frac{13}{16} pl^2 = \frac{1}{2} pl^2 \frac{\ell}{2} \qquad x_2 = \frac{4}{13}\ell$

(X3)



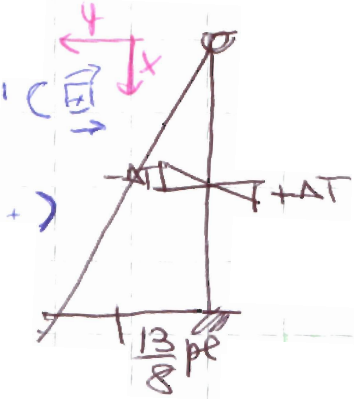
$\frac{pl^2}{8} - \frac{3}{4} p\ell x_3 + \frac{p x_3^2}{2} = 0 \qquad 4x_3^2 - 6\ell x_3 + \ell^2 = 0$

$\vec{v}$ diretto

$$4x_3^2 - 6\ell x_3 + \ell^2 = 0$$

$$x_{1,2} = \frac{+6 \mp \sqrt{6^2 - 16}}{8} = \frac{+6 \mp \sqrt{36-16}}{8} = \begin{cases} \frac{6-4{,}47}{8} = 0{,}1913\ell \\ \frac{6+4{,}47}{8} = 1{,}3\ell \quad \text{NA} \end{cases}$$

Devo determinare ipotetico flesso su AB

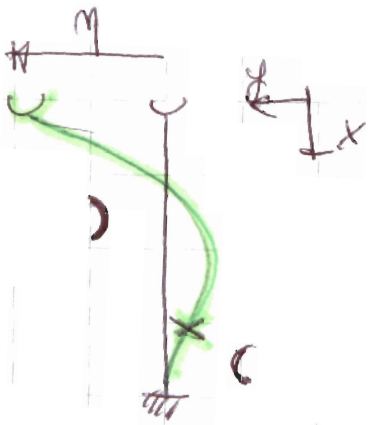


$$y''(x) = -\frac{M(x)}{EJ} \mp \frac{2\alpha\Delta T}{h}$$

$$y''(x) = -\frac{1}{EJ}\left(\frac{13}{8}px\right) + 2 \cdot \frac{5}{8}\frac{p\ell^2}{EJ} \leq 0$$

$$-13px + 10p\ell^2 \leq 0$$

$$x \geq \frac{10}{13}\ell$$



$$x_4 = \frac{10}{13}\ell$$

$x \geq \frac{10}{13}\ell$ $\quad y'' < 0$

$x < \frac{10}{13}\ell$ $\quad y'' > 0$

FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 17/02/2010, 2h

ing. F. Minelli; ing. F. Germano

***Nome:*……………….. *Cognome:* ……………………….. *n.matr.:* …..……….
Superato ELEMENTI STRUTTURALI A? Sì ☐ NO ☐

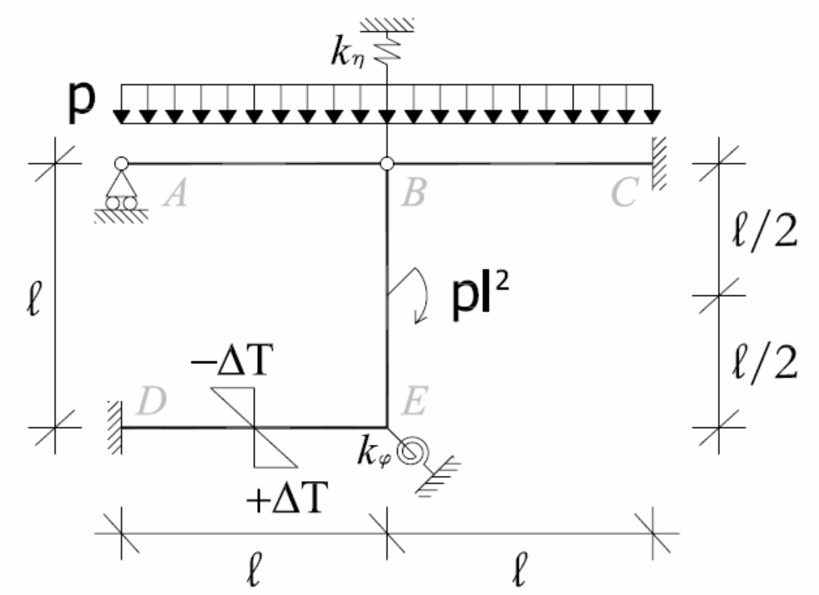
ESERCIZIO 1 (20 PUNTI)

$$k_\varphi = 1\frac{EJ}{\ell};$$

$$k_\eta = 17\frac{EJ}{\ell^3};$$

$$\frac{\alpha\Delta T}{h} = \frac{1}{16}\frac{Pl^2}{EJ}$$

$$EA \to \infty$$

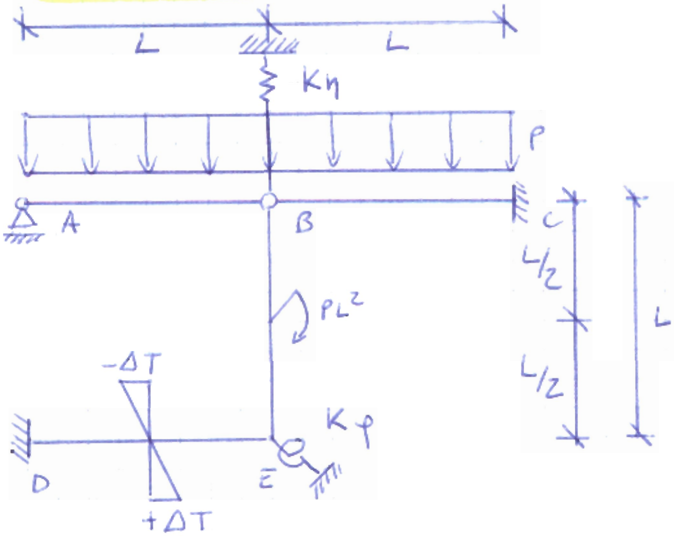


Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi

TEMA D'ESAME



$K_\varphi = 1 \cdot \frac{EJ}{L}$

$K_\eta = 17 \cdot \frac{EJ}{L^3}$

$\frac{\alpha \Delta T}{t} = \frac{1}{16} \frac{pL^2}{EJ}$

$EJ = cost$

$EA \to \infty$

ANALISI CINEMATICA

GDL $3 + 3 + 3 = 9$

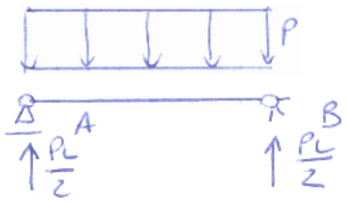
GDV $1 + 4 + 1 + 3 + 3 + 1 = 13$

I $13 - 9 = 4$

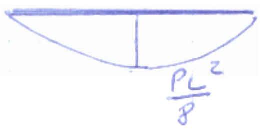
STRUTTURA A NODI SPOSTABILI

APPENDICE ISOSTATICA

ASTA AB



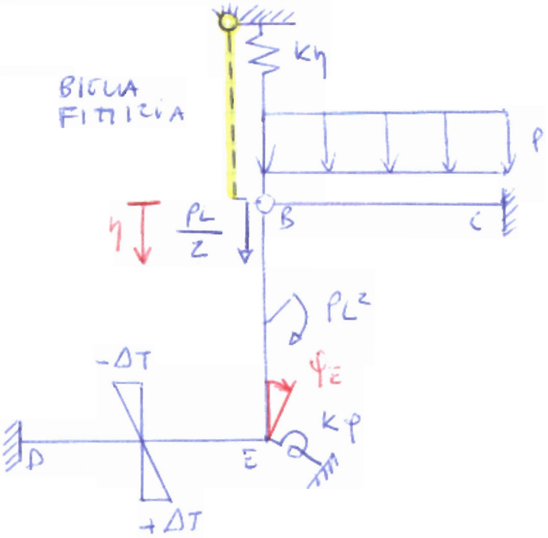
$V(x)$



$M(x)$

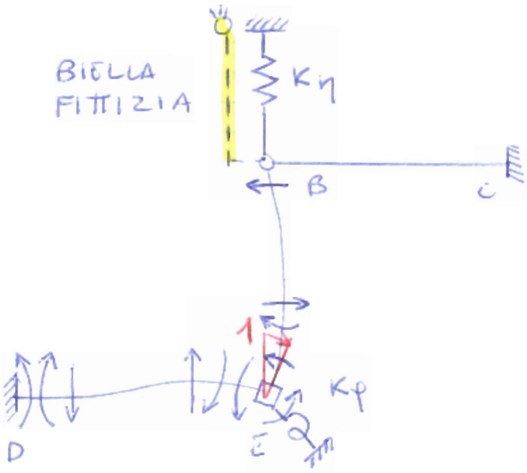
DEFORMATA QUALITATIVA

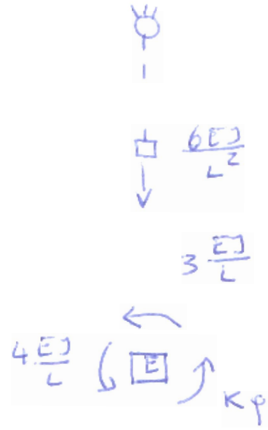
SISTEMA RISOLVENTE / CONVENZIONE DI SEGNO



$$\begin{cases} \Sigma m_E = 0 \\ \Sigma F_{biella} = 0 \end{cases} \qquad \begin{cases} m_{EE} * \varphi_E + m_{E\eta} * \eta + m_{E0} = 0 \\ h_{BE} * \varphi_E + h_{B\eta} * \eta + h_{B0} = 0 \end{cases}$$

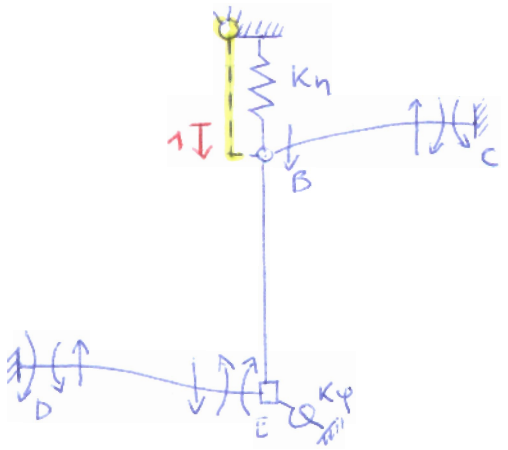
$\varphi_E = 1 \qquad \eta = 0 \qquad p = 0 \qquad \Delta T = 0$

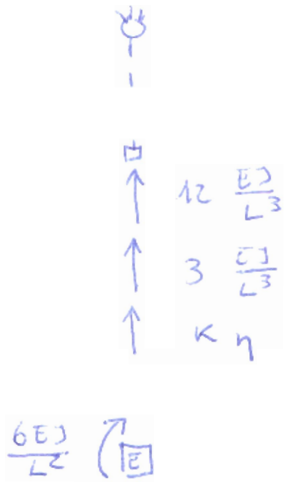




$$m_{EE} = -\frac{7EJ}{L} - K\varphi \qquad h_{BE} = -\frac{6EJ}{L^2}$$

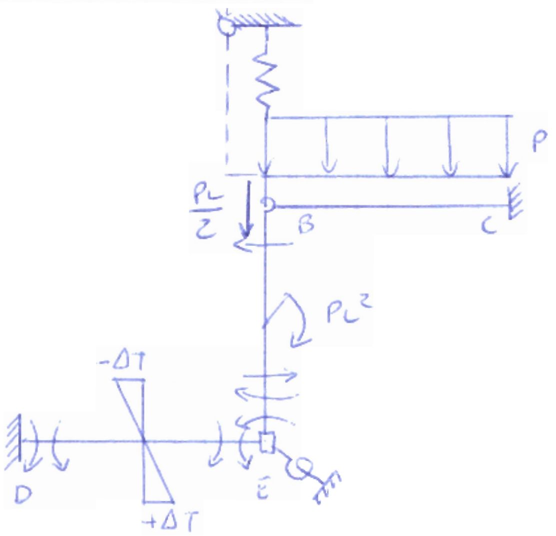
$$\varphi_E = \emptyset \qquad \eta = 1 \qquad P = \emptyset \qquad \Delta T = \emptyset$$

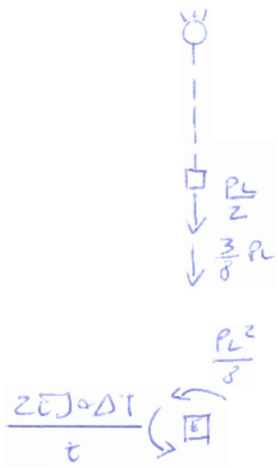




$$m_{BE} = \frac{6\,EJ}{L^2} \qquad h_{B\eta} = +15\,\frac{EJ}{L^3} + K_\eta$$

$$\varphi_E = \emptyset \qquad \eta = \emptyset \qquad P \neq \emptyset \qquad \Delta T \neq \emptyset$$





$$m_{E\phi} = -\frac{2EJ\,\alpha\,\Delta T}{t} - \frac{PL^2}{8} \qquad h_{B\phi} = -\frac{PL}{2} - \frac{3}{8}PL = -\frac{7}{8}PL$$

SISTEMA RISOLVENTE

$$\begin{cases} -\left(7\frac{EJ}{L}+K\varphi\right)*\varphi_E + \left(\frac{6EJ}{L^2}\right)*\eta - \left(\frac{2EJ\alpha\Delta T}{t}+\frac{PL^2}{8}\right)=\phi \\ -\left(\frac{6EJ}{L^2}\right)*\varphi_E + \left(15\frac{EJ}{L^3}+K\eta\right)*\eta - \frac{7}{8}PL = \phi \end{cases}$$

$$\begin{bmatrix} -\left(7\frac{EJ}{L}+K\varphi\right) & +\frac{6EJ}{L^2} \\ -\frac{6EJ}{L^2} & +\left(15\frac{EJ}{L^3}+K\eta\right) \end{bmatrix} * \begin{bmatrix} \varphi_E \\ \eta \end{bmatrix} = \begin{bmatrix} +\frac{2EJ\Delta T}{L}+\frac{PL^2}{8} \\ \frac{7}{8}PL \end{bmatrix}$$

DETERMINAZIONE DELLE COSTANTI $K\varphi$, $K\eta$, $\frac{\alpha\Delta T}{t}$

$$\begin{bmatrix} -8 & +6 \\ -6 & +21 \end{bmatrix} ; \begin{bmatrix} 1/4 \\ 7/8 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1/24 \end{bmatrix}$$

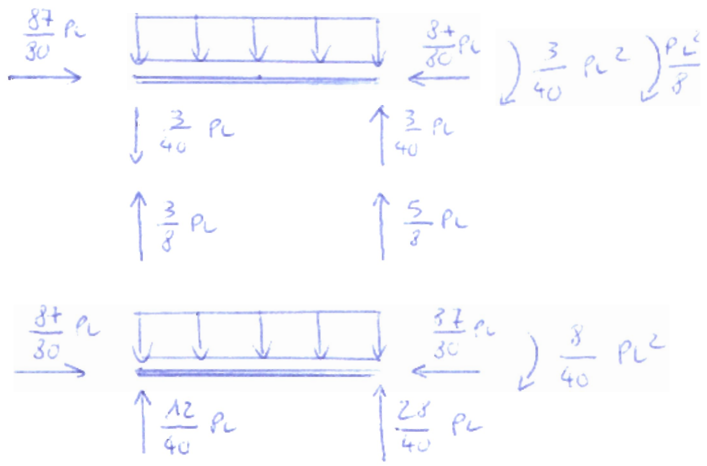
$$\begin{bmatrix} -8 & +6 \\ -6 & +32 \end{bmatrix} ; \begin{bmatrix} 1/4 \\ 7/8 \end{bmatrix} \rightarrow \begin{bmatrix} -1/80 \\ 1/40 \end{bmatrix}$$

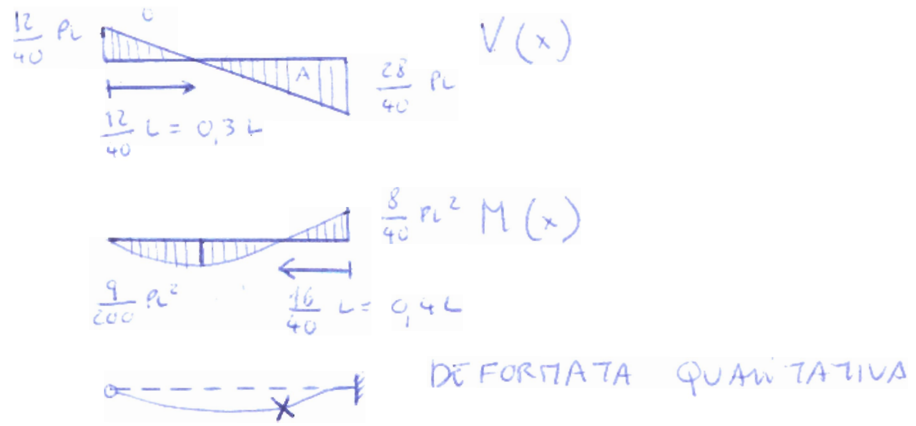
$$\varphi_E = -\frac{1}{80}\frac{PL^3}{EJ} \quad ; \quad \eta = \frac{1}{40}\frac{PL^4}{EJ}$$

$$K\varphi = 1 * \frac{EJ}{L} \qquad K\eta = 17\frac{EJ}{L^3} \qquad \frac{\alpha\Delta T}{t} = \frac{1}{16}\frac{PL^2}{EJ}$$

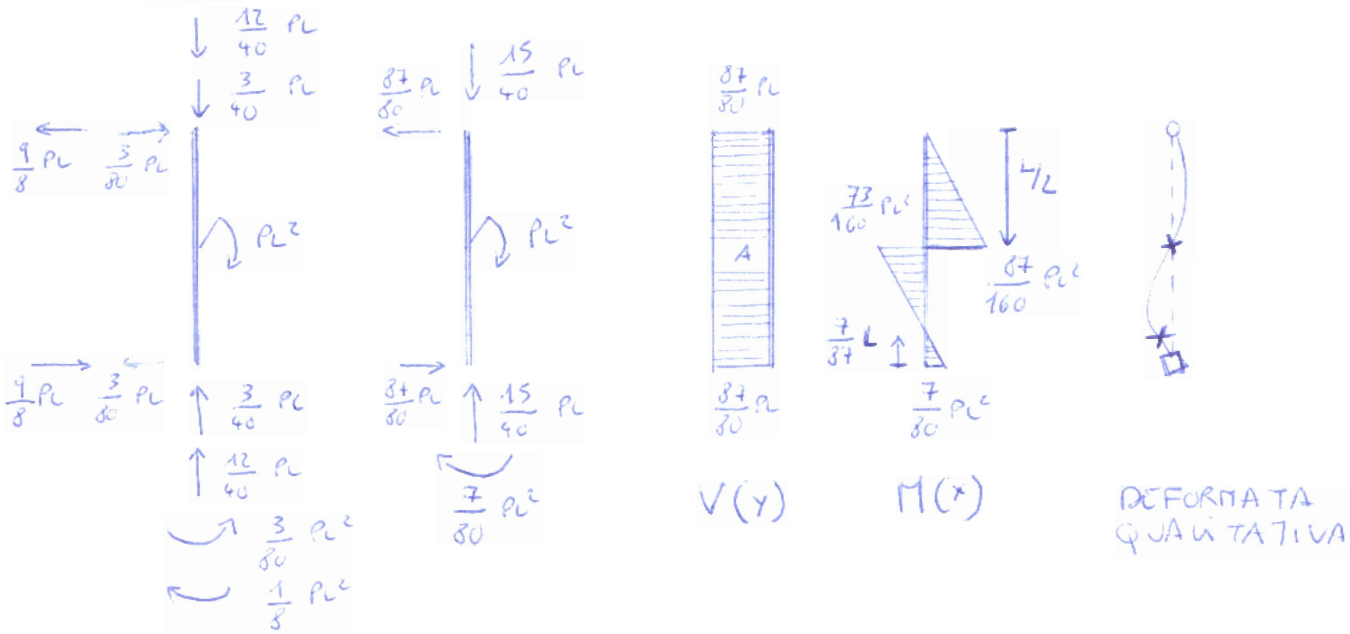
DETERMINAZIONE DELLE AZIONI INTERNE

ASTA BC

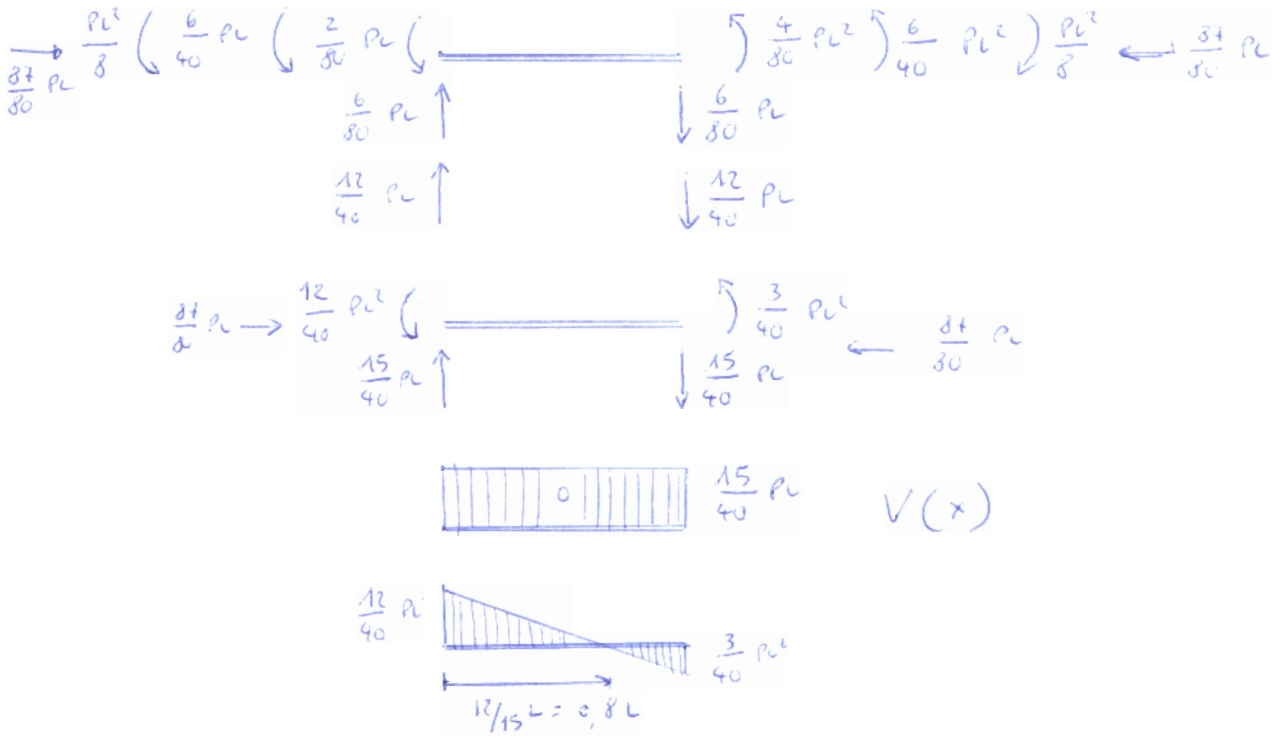




ASTA BE

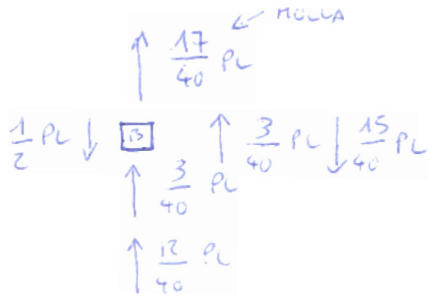


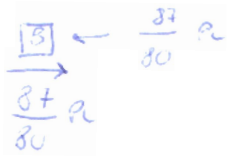
ASTA DE



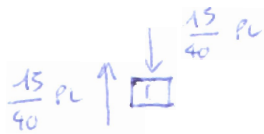
EQUILIBRIO DEI NODI

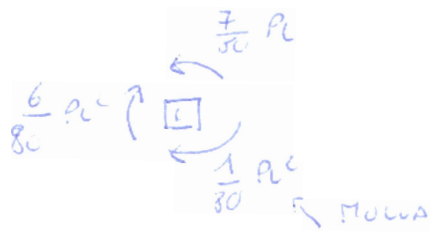
NODO B



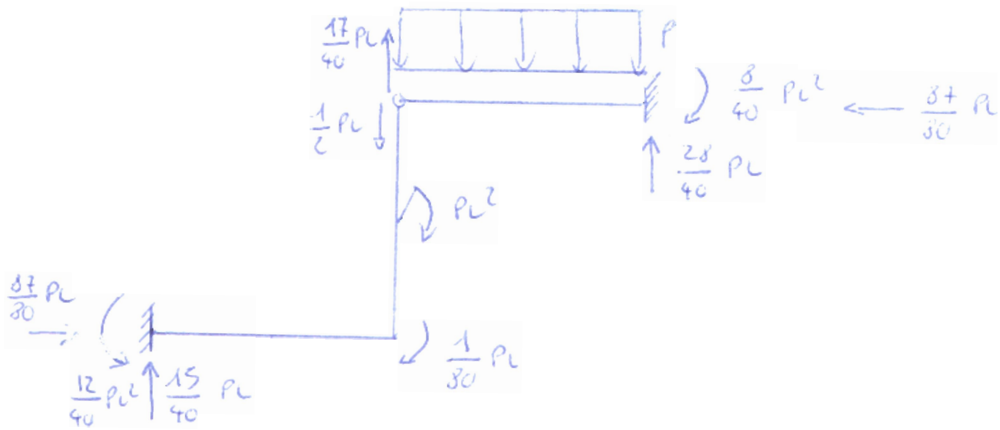


NODO I



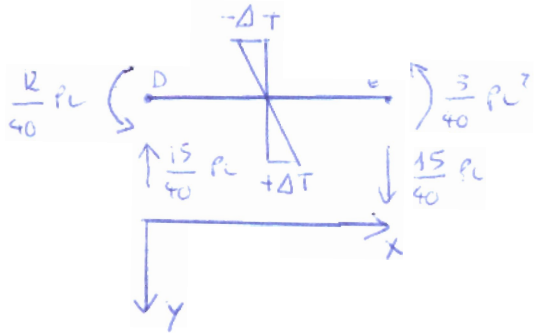


EQUILIBRIO GLOBALE



$$
\begin{cases}
\sum H = \phi \\
\sum V = \phi \\
\sum M_D = \phi
\end{cases}
\qquad
\begin{cases}
+\frac{87}{80} PL - \frac{87}{80} PL = \phi \\
+\frac{15}{40} PL + \frac{1}{2} PL + \frac{17}{40} PL + \frac{28}{40} - 2PL = \phi \\
-\frac{17}{40} PL^2 + \frac{1}{2} PL^2 + PL \cdot \frac{3}{2} L + \frac{8}{40} PL^2 - \frac{87}{80} PL^2 + \\
-\frac{28}{40} PL \cdot 2L + PL^2 + \frac{1}{80} PL^2 - \frac{12}{40} PL^2 = \phi
\end{cases}
$$

DETERMINAZIONE DELLA DEFORMATA DELL'ASTA DC



$$M(x) = \frac{15}{40} PL \cdot x - \frac{12}{40} P \cdot L^2$$

$$\frac{\alpha \Delta T}{t} = \frac{1}{16} \frac{PL^2}{EJ}$$

$$y''(x) = -\frac{M(x)}{EJ} - \frac{2\alpha\Delta T}{t}$$

$$y''(x) = \frac{15}{40} \frac{PLx}{EJ} + \frac{12}{40} \frac{PL^2}{EJ} - \frac{1}{8} \frac{PL^2}{EJ}$$

$$y''(x) = -\frac{15}{40} \frac{PLx}{EJ} + \frac{7}{40} \frac{PL^2}{EJ}$$

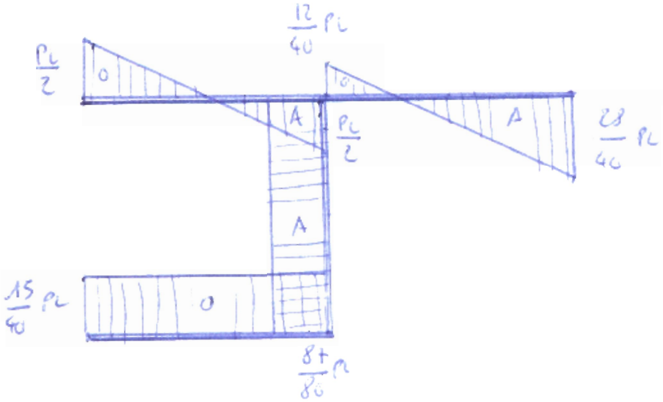
$$y''(x) > 0 \longrightarrow x < \frac{7}{15} L$$

SE $x < \frac{7}{15} L$ CONCAVITÀ VERSO IL BASSO

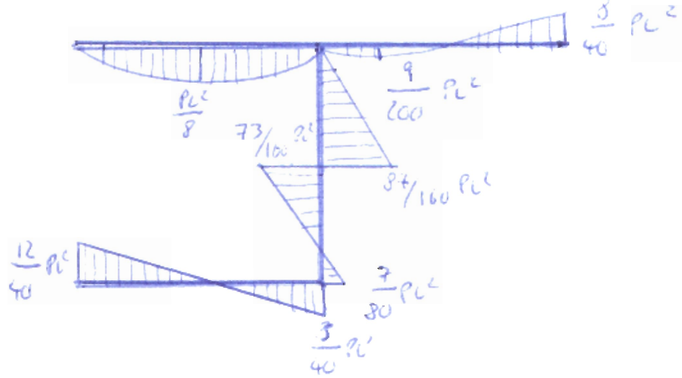
SE $x > \frac{7}{15} L$ CONCAVITÀ VERSO L'ALTO

TRACCIAMENTO AZIONI INTERNE

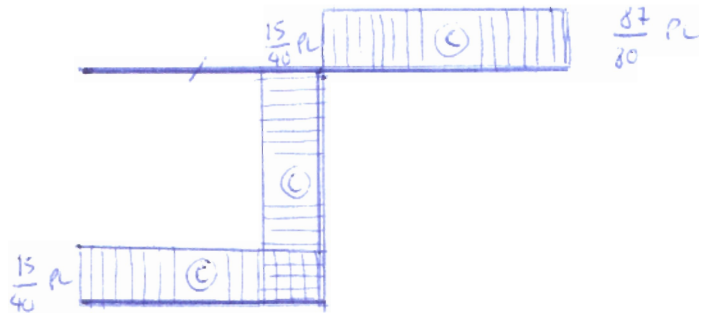
AZIONE DI TAGLIO V



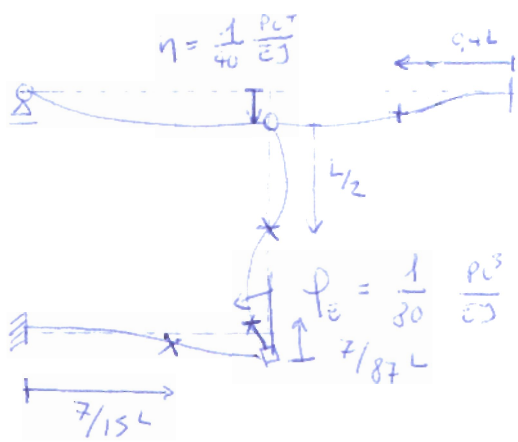
MOMENTO FLETTENTE M



AZIONE ASSIALE N



DEFORMATA QUALITATIVA

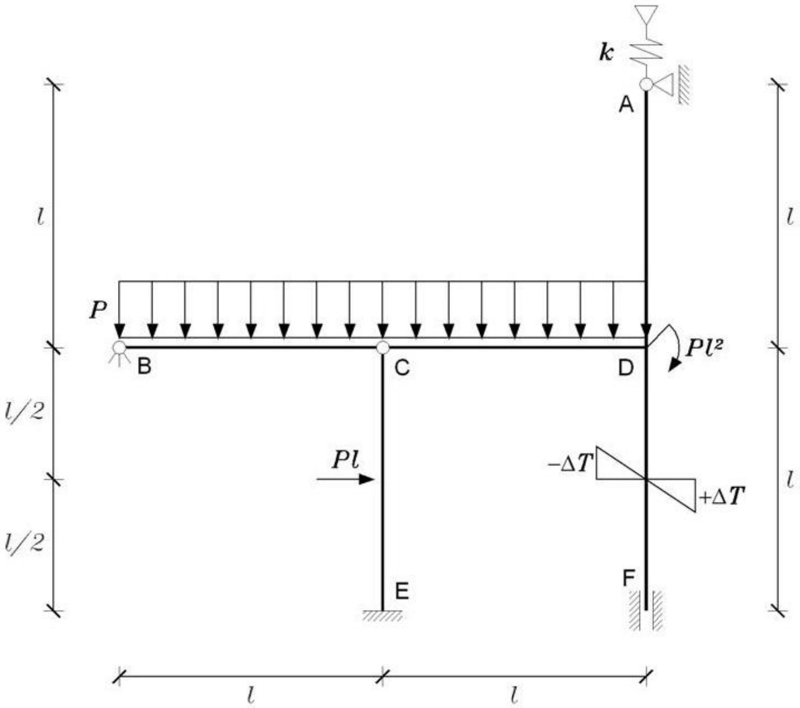


FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 23/03/2010, 3h

ing. F. Minelli; ing. F. Germano

***Nome:*……………………. *Cognome:* ……………………………. *n.matr.:* ….………..
Superato ELEMENTI STRUTTURALI A? Sì ☐ NO ☐

ESERCIZIO 1 (20 punti)



$$k = 3\frac{EJ}{\ell^3};$$

$$\frac{\alpha\Delta T}{h} = \frac{9}{8}\frac{Pl^2}{EJ}$$

$$EA \to \infty$$

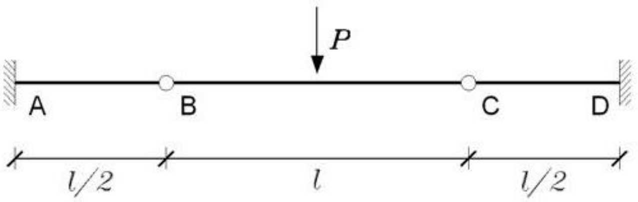
Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi

ESERCIZIO 2 (6 punti)

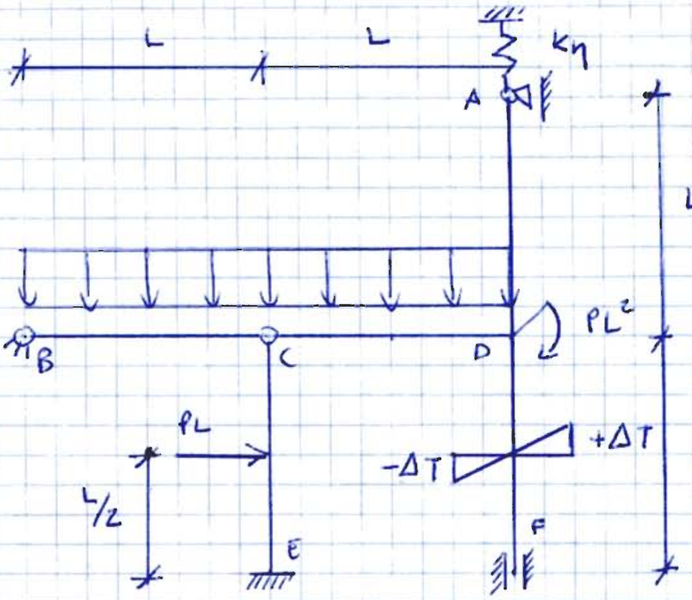
Dato lo schema statico in figura:



$$P = 5\,500\,kg$$

$$l = 5\,m$$

TEMA D'ESAME



$K_\eta = 3 \frac{EJ}{L^3}$

$\frac{\alpha \Delta T}{t} = \frac{9}{8} \frac{pL^2}{EJ}$

$EA \to \infty$

$EJ = \text{COST}$

ANALISI CINEMATICA

GDL $3 + 3 + 3 = 9$

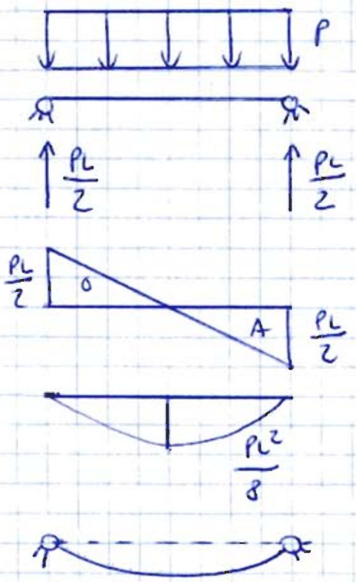
GDV $2 + 2 + 4 + 3 + 2 = 13$

I $13 - 9 = 4$

STRUTTURA A NODI SPOSTABILI

APPENDICE ISOSTATICA

ASTA BC

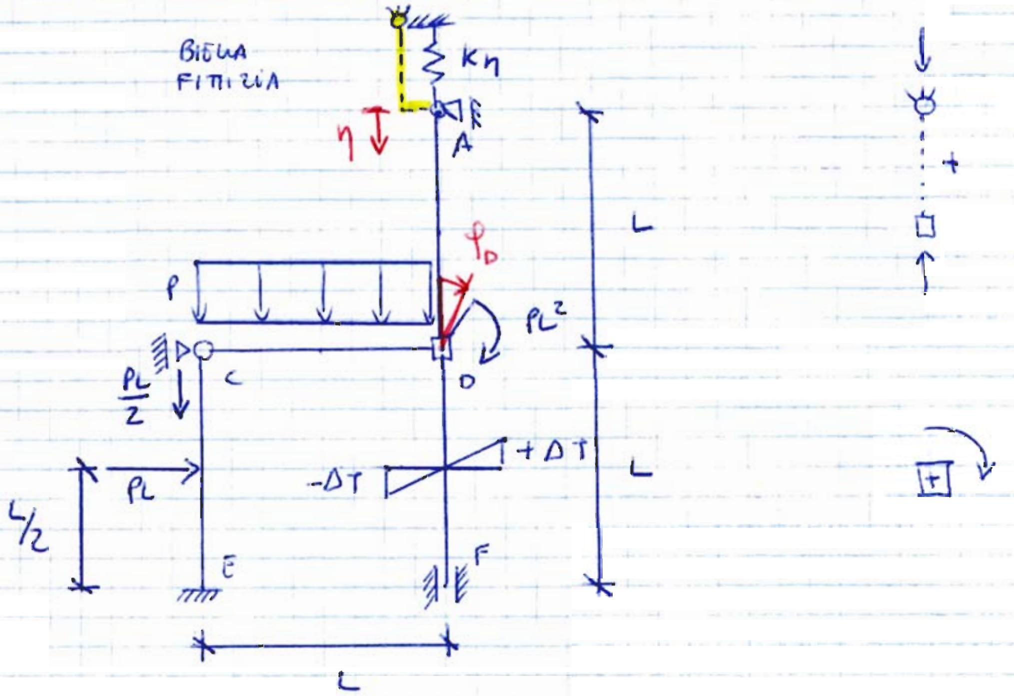


V(x)

M(x)

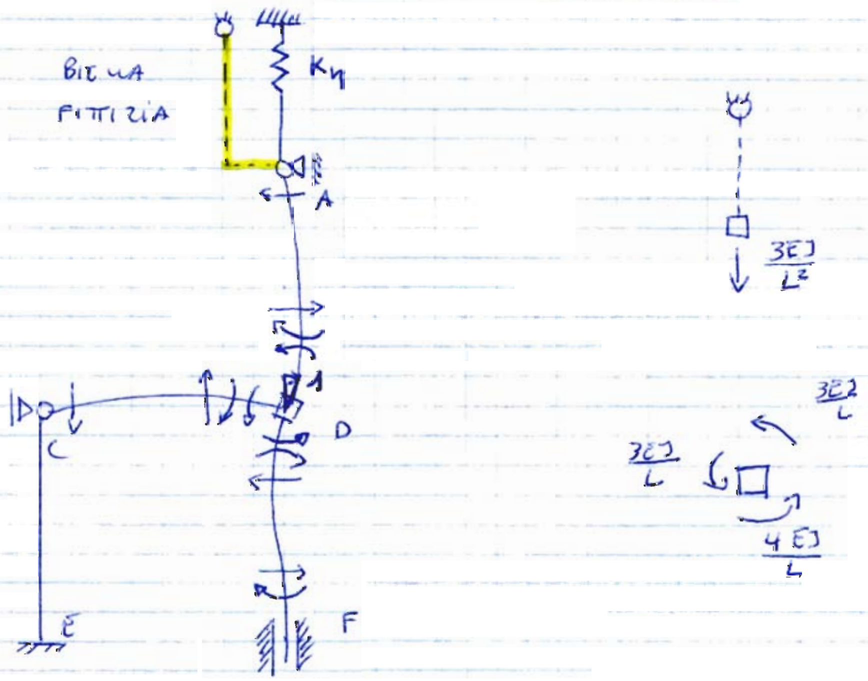
DEFORMATA QUALITATIVA

SISTEMA RISOLVENTE / CONVENZIONE DI SEGNO



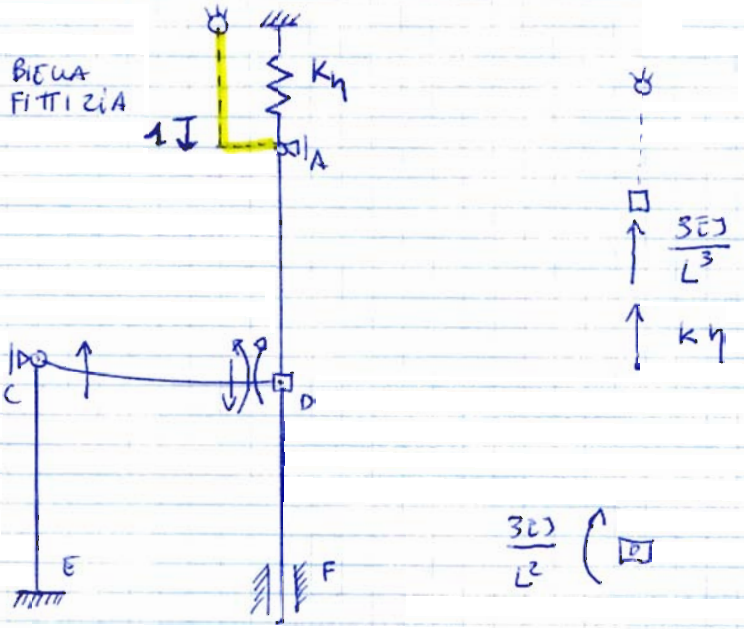
$$\begin{cases} \Sigma m_D = \phi \\ \Sigma F_{biella} = \phi \end{cases} \qquad \begin{cases} m_{DD} * \varphi_D + m_{D\eta} * \eta + m_{D\phi} = \phi \\ h_{AD} * \varphi_D + h_{A\eta} * \eta + h_{A\phi} = \phi \end{cases}$$

$\varphi_D = 1 \qquad \eta = \phi \qquad p = \phi \qquad \Delta T = \phi$



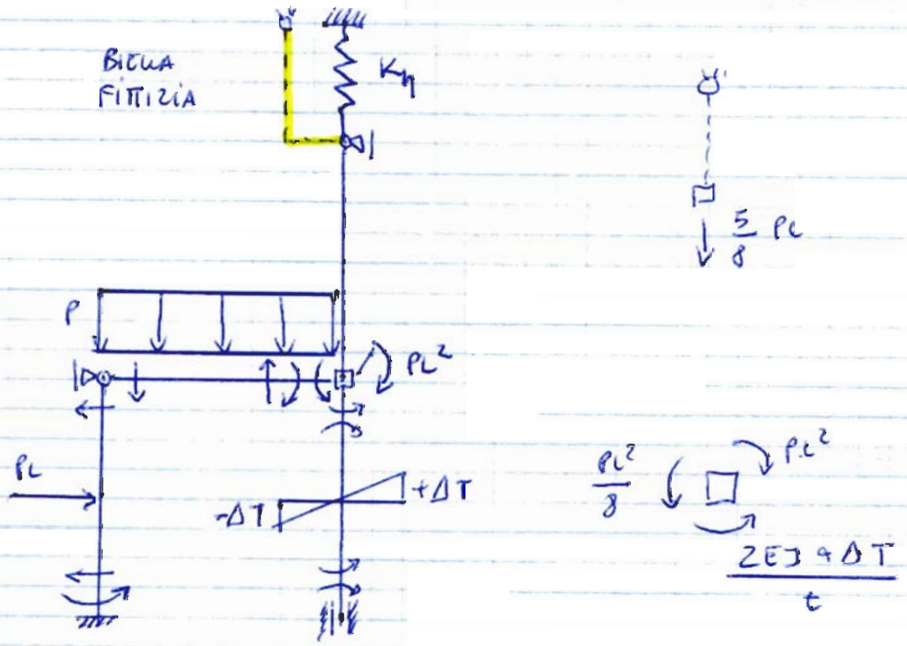
$$m_{DD} = -\frac{10\,EJ}{L} \qquad h_{AD} = -\frac{3EJ}{L^2}$$

$\varphi_D = \phi \qquad \eta = 1 \qquad p = \phi \qquad \Delta T = \phi$



$$m_{D\eta} = + \frac{3EJ}{L^2} \qquad h_{A\eta} = + \frac{3EJ}{L^3} + K_\eta$$

$\varphi_D = \phi \qquad \eta = \phi \qquad p \neq \phi \qquad \Delta T \neq \phi$



$$m_{D\phi} = pL^2 - \frac{pL^2}{8} - \frac{2EJ\,\alpha\Delta T}{t} \qquad h_{A\phi} = -\frac{5}{8} pL$$

SISTEMA RISOLVENTE

$$\begin{cases} \left(-10\,\frac{EJ}{L}\right) * \varphi_D + \left(\frac{3EJ}{L^2}\right) * \eta = \left(\frac{2EJ\alpha\Delta T}{t} - \frac{7}{8}PL^2\right) \\ \left(-3\,\frac{EJ}{L^2}\right) * \varphi_D + \left(\frac{3EJ}{L^3} + K_\eta\right) * \eta = \left(\frac{5}{8}PL\right) \end{cases}$$

$$\begin{bmatrix} -10\,\frac{EJ}{L} & +3\,\frac{EJ}{L^2} \\ -3\,\frac{EJ}{L^2} & +\frac{3EJ}{L^3} + K_\eta \end{bmatrix} * \begin{bmatrix} \varphi_D \\ \eta \end{bmatrix} = \begin{bmatrix} \frac{2EJ\alpha\Delta T}{t} - \frac{7}{8}PL^2 \\ \frac{5}{8}PL^2 \end{bmatrix}$$

DETERMINAZIONE DELLE COSTANTI K_η E $\frac{\alpha\Delta T}{t}$

$$\begin{bmatrix} -10 & +3 \\ -3 & +15 \end{bmatrix} ; \begin{bmatrix} 1/8 \\ 5/8 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1/24 \end{bmatrix}$$

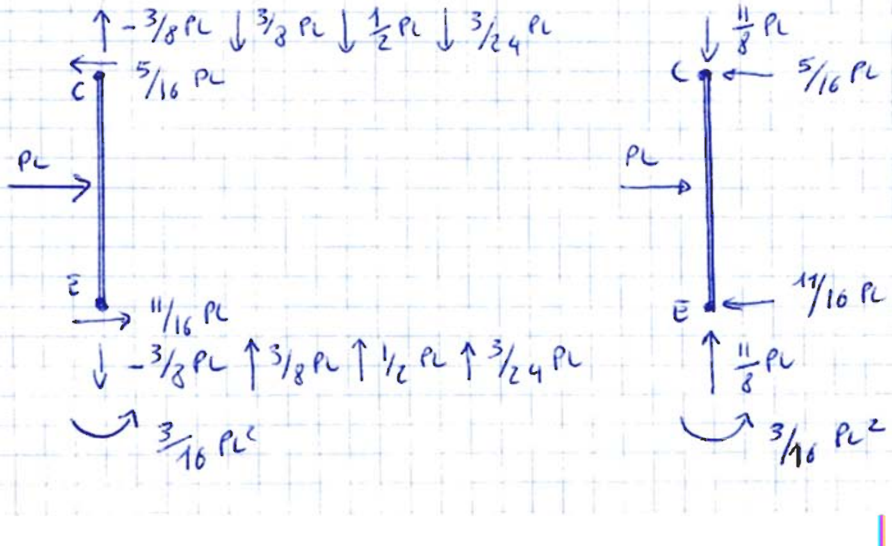
$$\begin{bmatrix} -10 & +3 \\ -3 & +6 \end{bmatrix} ; \begin{bmatrix} 11/8 \\ 5/8 \end{bmatrix} \rightarrow \begin{bmatrix} -1/8 \\ 1/24 \end{bmatrix}$$

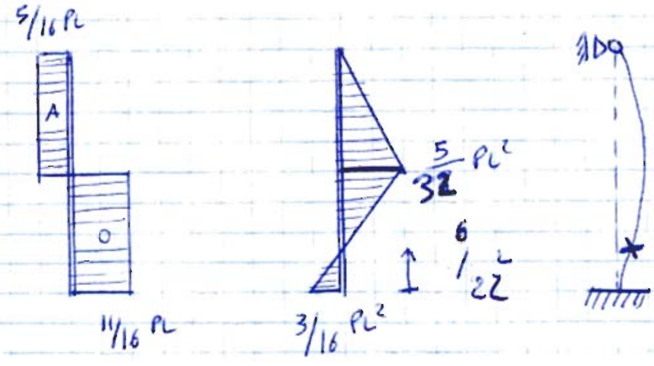
$$\varphi_D = -\frac{1}{8}\frac{PL^3}{EJ} \qquad \eta = \frac{1}{24}\frac{PL^4}{EJ}$$

$$K_\eta = 3\,\frac{EJ}{L^3} \qquad \frac{2EJ\alpha\Delta T}{t} = \frac{18}{8}PL^2 \rightarrow \frac{\alpha\Delta T}{t} = \frac{9}{8}\frac{PL^2}{EJ}$$

DETERMINAZIONE DELLE AZIONI INTERNE

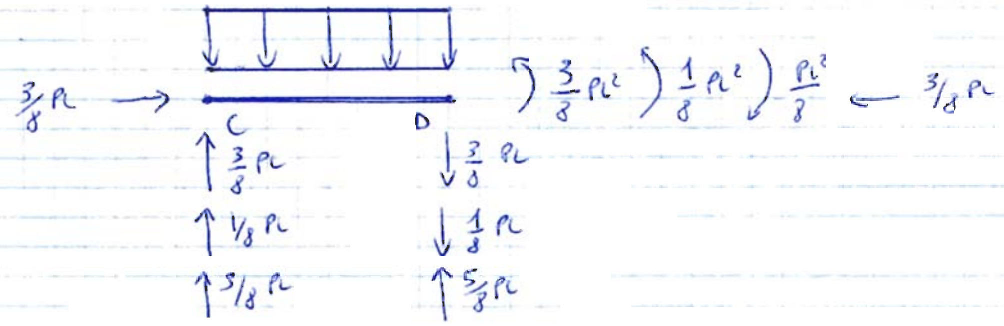
ASTA CE

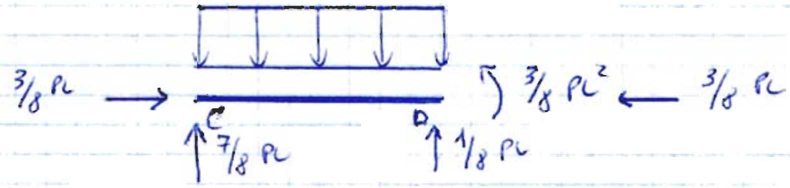


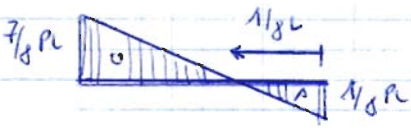


V(y) M(y) DEFORMATA QUALITATIVA

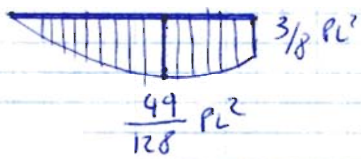
ASTA CD





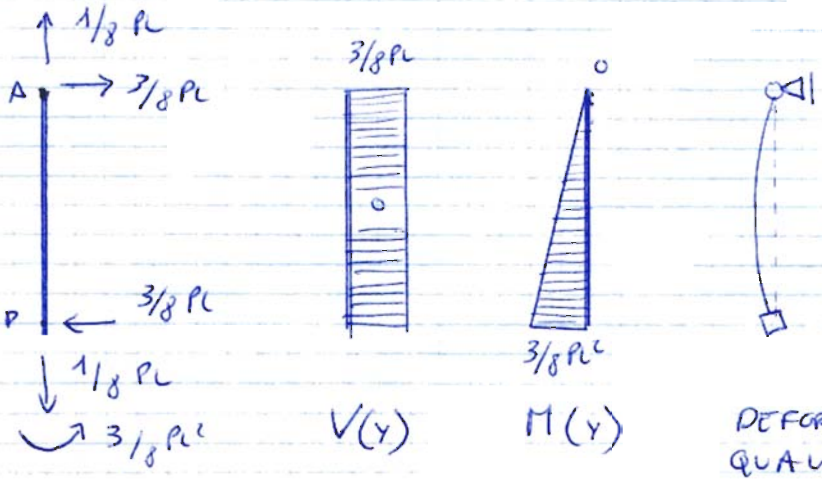


V(x)



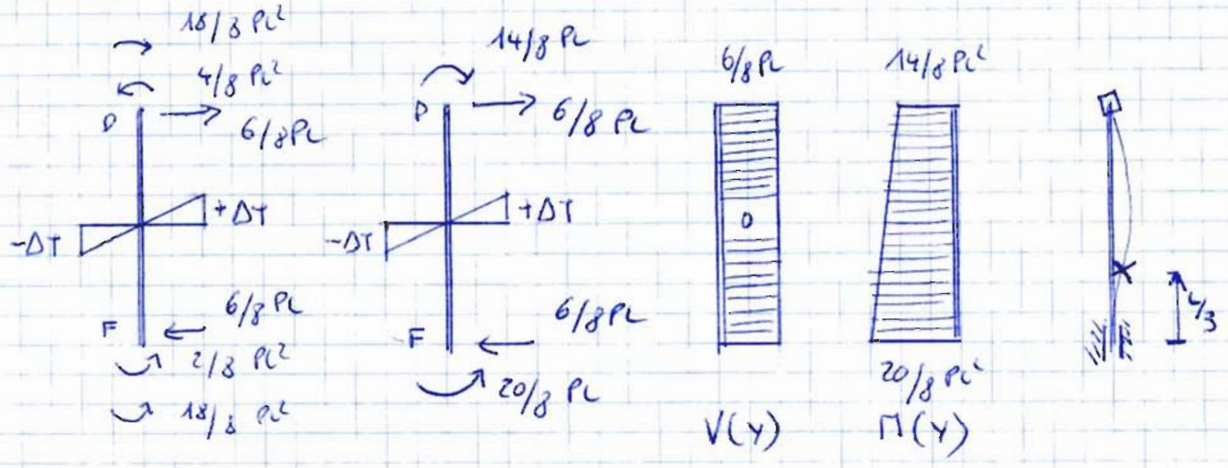
M(x)

ASTA AD

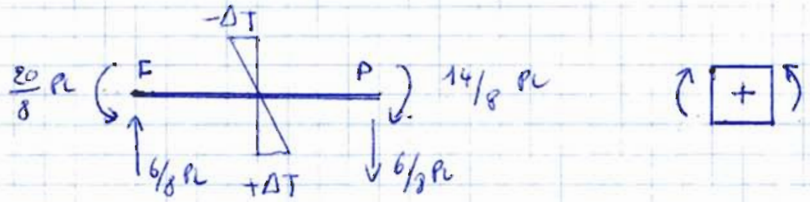


DEFORMATA QUALITATIVA

ASTA DF



ASTA DF : DETERMINAZIONE DELLA DEFORMATA



$$M(x) = \frac{6}{8} PLx - \frac{20}{8} PL^2$$

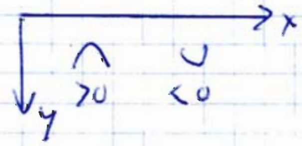
$$\frac{\alpha \Delta T}{t} = \frac{9}{8} \frac{PL^2}{EJ}$$

$$y''(x) = -\frac{M(x)}{EJ} - \frac{2\alpha \Delta T}{t}$$

$$y''(x) = -6/8 \frac{PLx}{EJ} + 20/8 \frac{PL^2}{EJ} - \frac{9}{4} \frac{PL^2}{EJ}$$

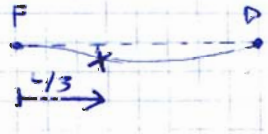
$$y''(x) = -\frac{6}{8} \frac{PLx}{EJ} + \frac{2}{8} \frac{PL^2}{EJ}$$

$$y''(x) > 0 \longrightarrow x < 1/3 L$$



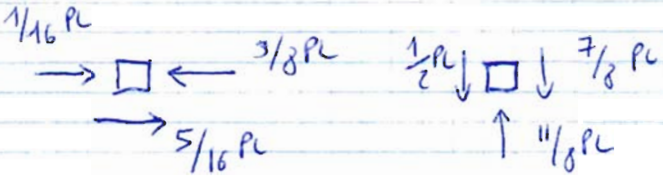
SE $x < 1/3 L$ CONCAVITA' VERSO IL BASSO

SE $x > 1/3 L$ CONCAVITA' VERSO L'ALTO

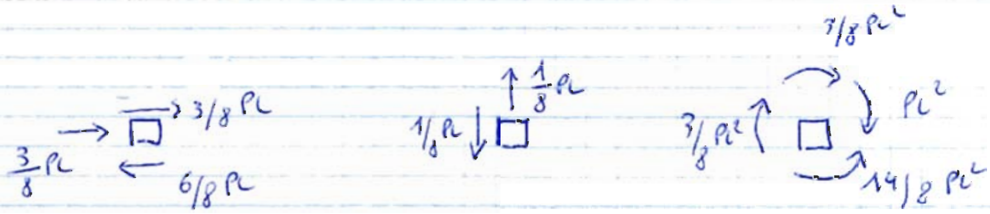


EQUILIBRIO DEI NODI

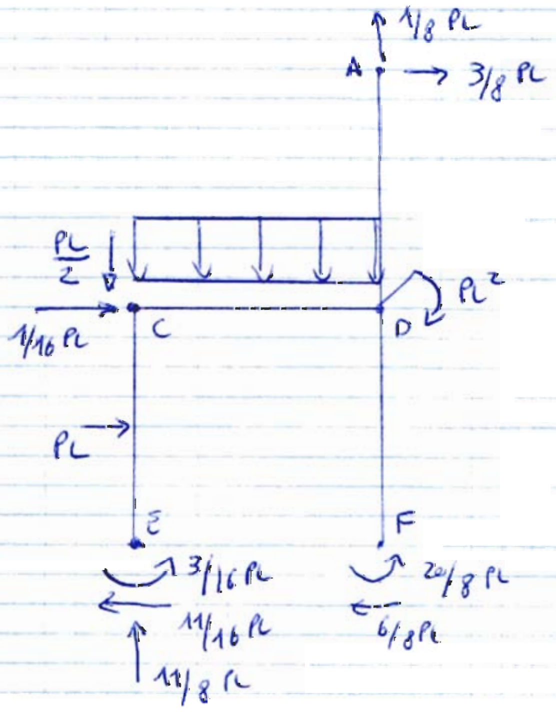
NODO C



NODO D



EQUILIBRIO GLOBALE



$\Sigma V = \phi \quad +\uparrow \quad PL \cdot \left(\frac{1}{8} - \frac{8}{8} - \frac{4}{8} + \frac{11}{8} \right) = \phi$

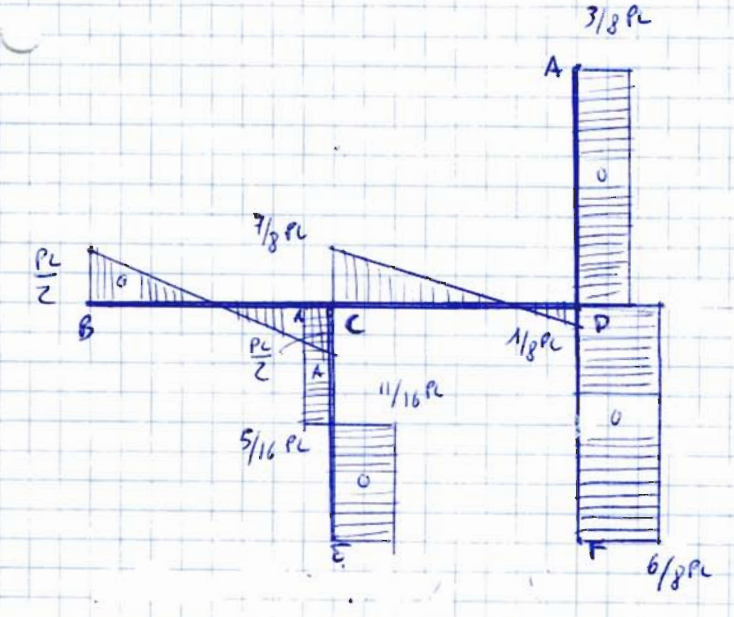
$\Sigma H = \phi \quad \xrightarrow{+} \quad PL \cdot \left(\frac{3}{8} - \frac{6}{8} + \frac{1}{16} - \frac{11}{16} + 1 \right) =$

$= \frac{PL}{16} \cdot \left(6 - 12 + 1 - 11 + 16 \right) = \phi$

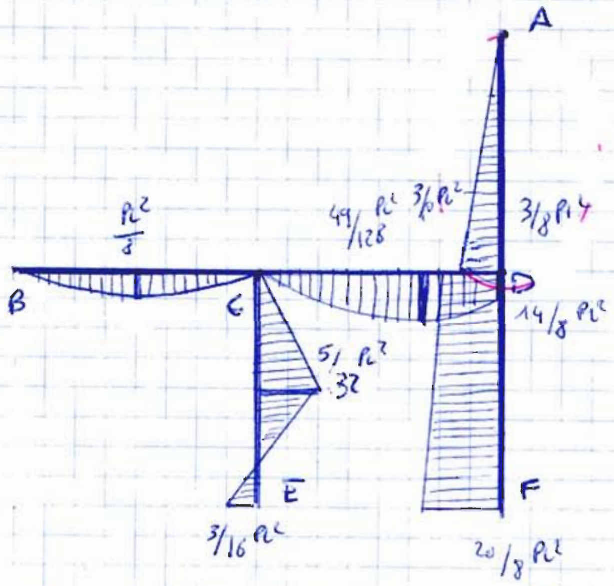
$\Sigma M_D = \phi \quad (+) \quad -\frac{1}{8} PL^2 + \frac{3}{4} PL^2 + PL^2 + \frac{PL^2}{2} + \frac{1}{16} PL^2 - \frac{20}{8} PL^2 + \frac{PL^2}{2} - \frac{3}{16} PL^2 =$

$= \phi$

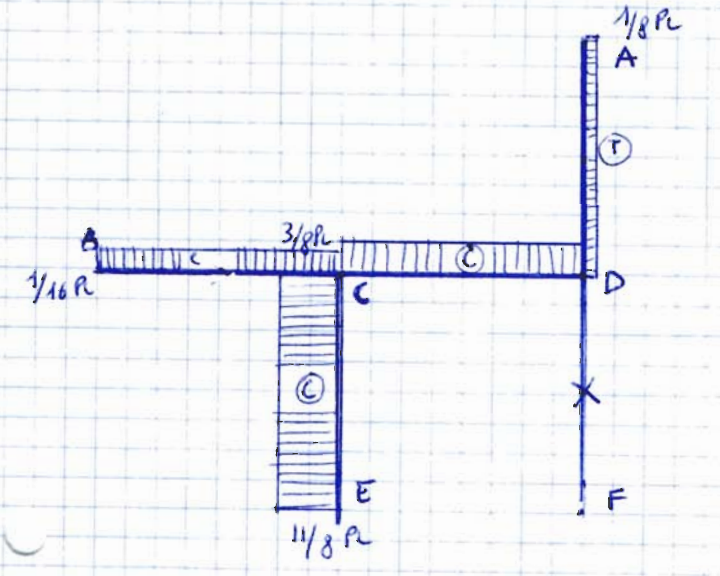
TRACCIAMENTO DELLE AZIONI INTERNE E DEFORMATA



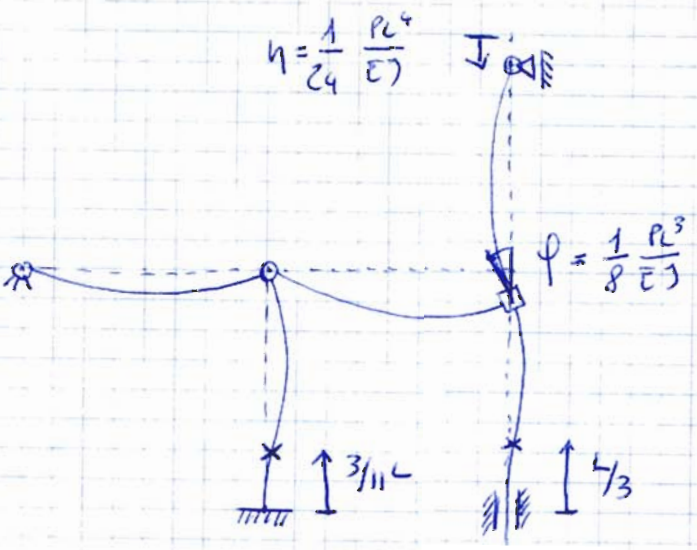
AZIONE DI TAGLIO V



MOMENTO FLETTENTE M



AZIONE ASSIALE N



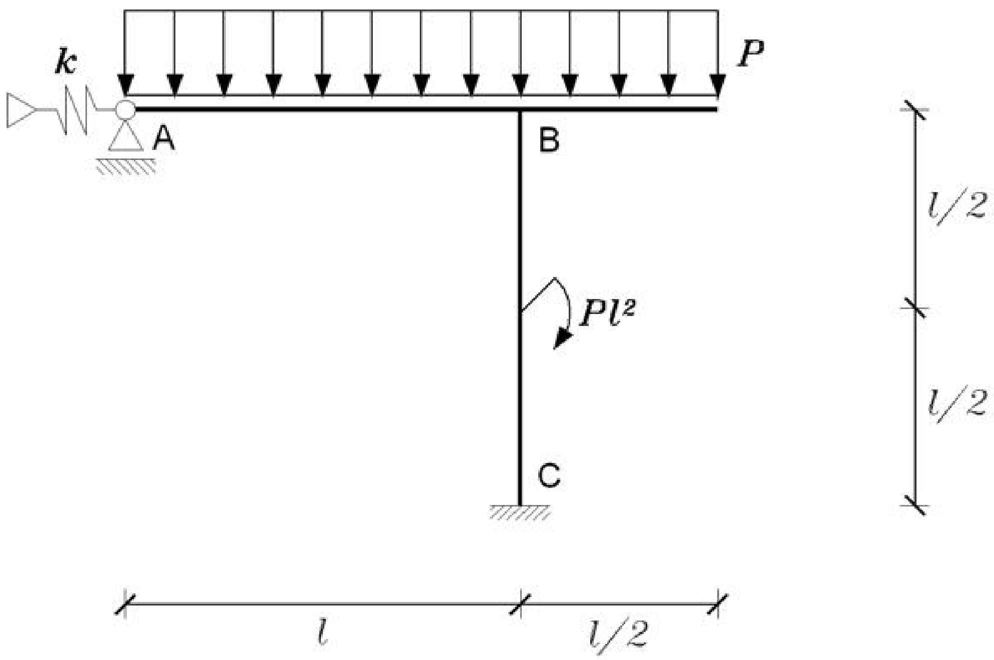
FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 12/04/2010, 3h

ing. F. Minelli; ing. F. Germano

***Nome:*………………….. *Cognome:* ………………………….. *n.matr.:* …..……….

Superato ELEMENTI STRUTTURALI A? Sì □ NO □

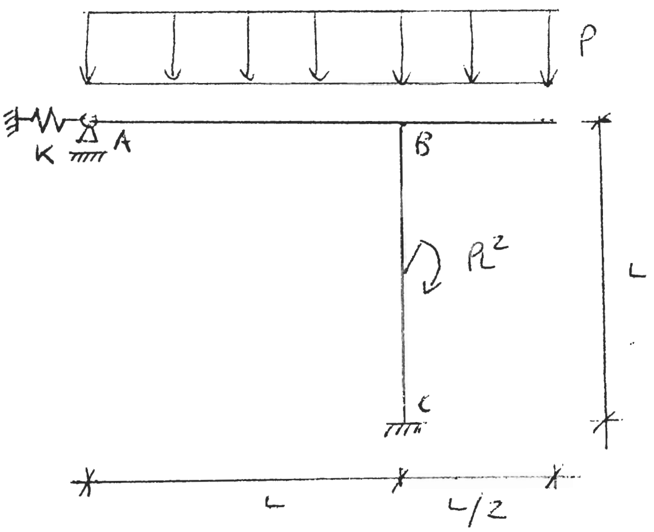
ESERCIZIO 1 (20 punti)



$$k = 6\frac{EJ}{\ell^3}; \quad EJ = \cos t \quad EA \to \infty$$

Dato il telaio in figura
Si richiedono:
1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi

TEMA D'ESAME 12 APRILE 2010



DATI:

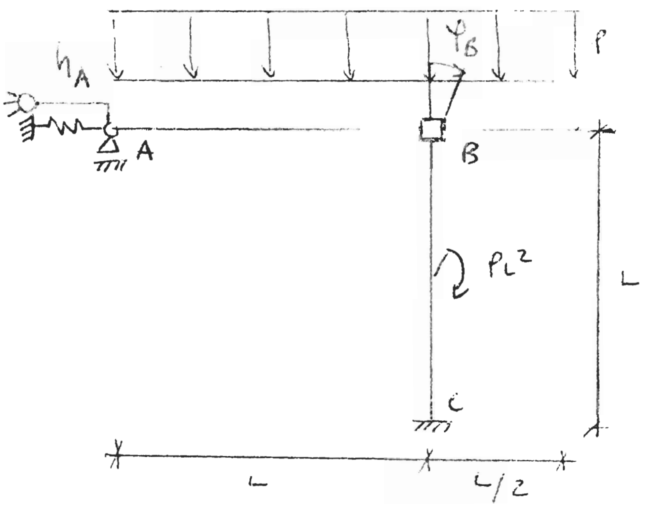
* $EJ = cost$
* $EA = \infty$
* $K = 6 \frac{EJ}{L^3}$

ANALISI CINEMATICA

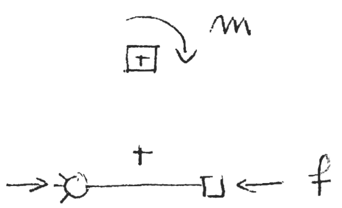
* GRADI DI LIBERTA' $L = 3$
* GRADI DI VINCOLO $V = 4$
* GRADO DI IPERSTATICITA' $I = V - L = 1$

STRUTTURA 1 VOLTA IPERSTATICA A NODI SPOSTABILI

METODO DEGLI SPOSTAMENTI



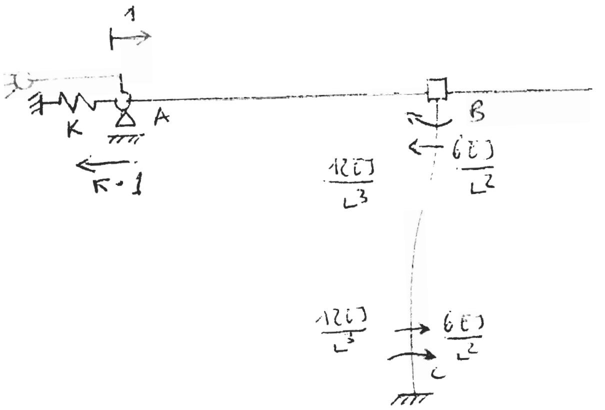
* CONVENZIONE DI SEGNO



* INCOGNITE

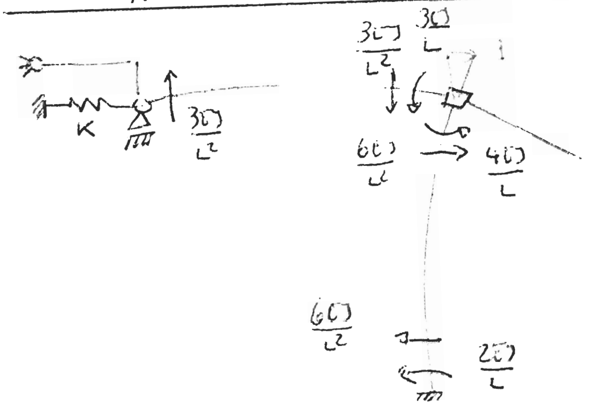
$\eta_A \; ; \; \varphi_B$

□ $\eta_A = 1 \qquad \varphi_B = \phi \qquad P = \phi$



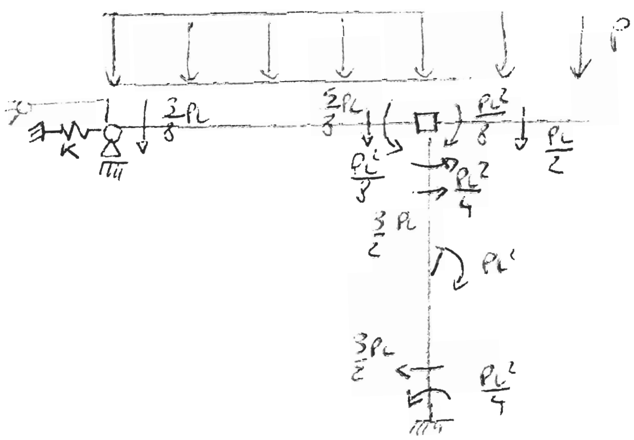
$$\begin{cases} f_{AA} = \dfrac{12EJ}{L^3} + K \\[2ex] m_{BA} = 6\dfrac{EJ}{L^2} \end{cases}$$

□ $\eta_A = \phi \qquad \varphi_B = 1 \qquad P = \phi$



$$\begin{cases} f_{AB} = -\dfrac{6EJ}{L^2} \\[2ex] m_{BB} = -7\dfrac{EJ}{L} \end{cases}$$

□ $\eta_A = \phi \qquad \varphi_B = \phi \qquad P \neq \phi$



$$\begin{cases} f_{A\phi} = -\dfrac{3}{2}PL \\[2ex] m_{B\phi} = -\dfrac{PL^2}{4} \end{cases}$$

□ SISTEMA RISOLVENTE

$$\begin{cases} \left(12\dfrac{EJ}{L^3}+k\right)\cdot \eta_A + \left(-6\dfrac{EJ}{L^2}\right)\cdot \varphi_B - \dfrac{3}{2}PL = 0 \\ \left(6\dfrac{EJ}{L^2}\right)\cdot \eta_A + \left(-7\dfrac{EJ}{L}\right)\cdot \varphi_B - \dfrac{PL^2}{4} = 0 \end{cases}$$

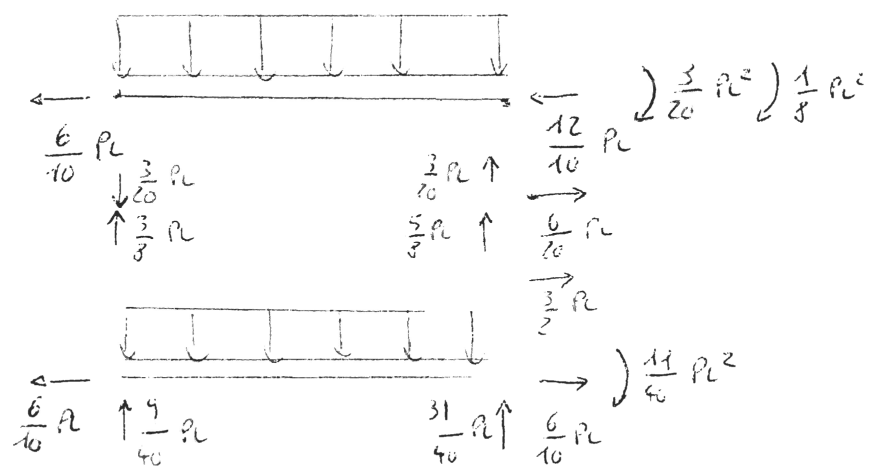
con

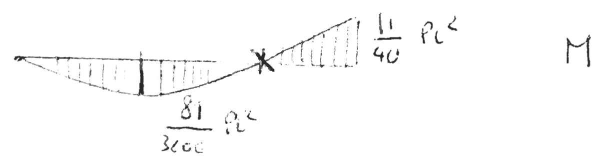
$$K = 6\frac{EJ}{L^3}$$

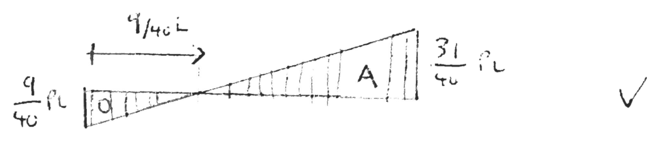
si hanno

$$\begin{cases} \eta_A = \dfrac{1}{10}\dfrac{PL^4}{EJ} \\ \varphi_B = \dfrac{1}{20}\dfrac{PL^3}{EJ} \end{cases}$$

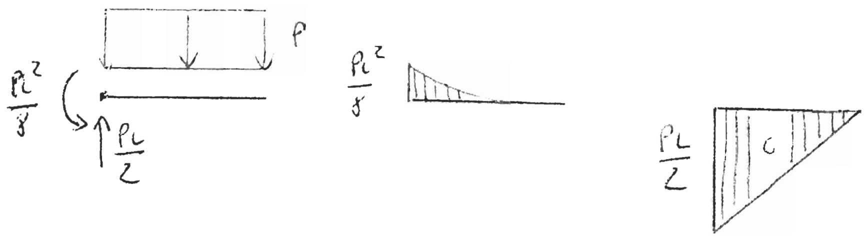
□ ASTA AB



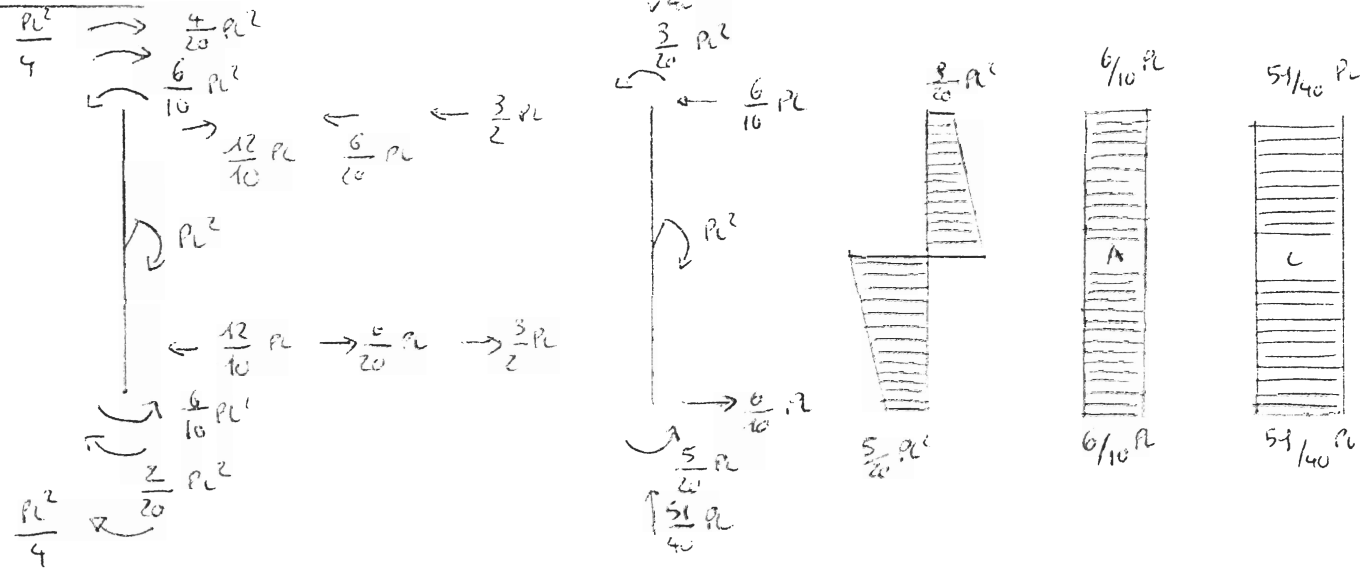




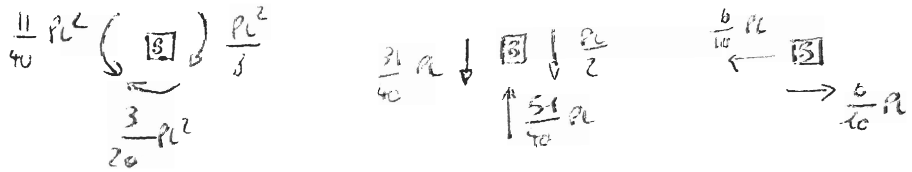
MENSOLA IN B



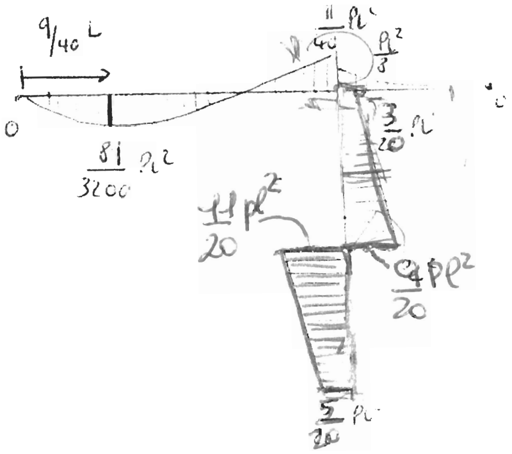
ASTA BC



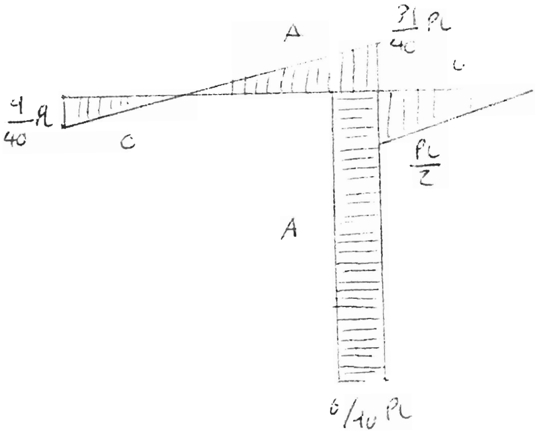
EQUILIBRIO DEL NODO B



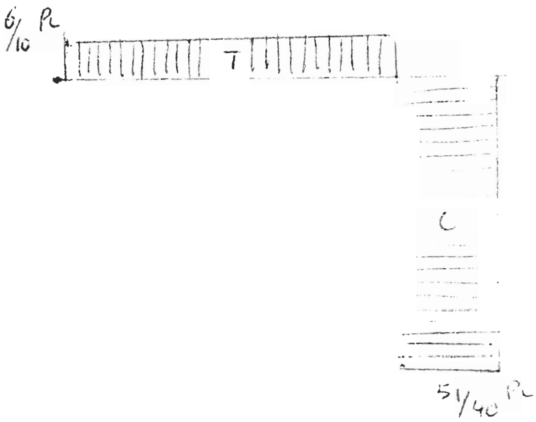
□ DIAGRAMMI DELLE AZIONI INTERNE



MOMENTO FLETTENTE M



AZIONE DI TAGLIO V

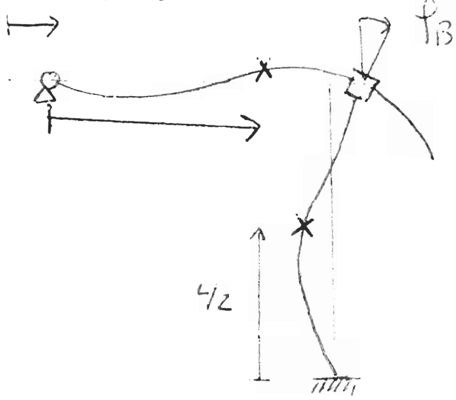


AZIONE ASSIALE N

□ DEFORMATA DELLA STRUTTURA

$\eta_A = \frac{1}{10} \frac{PL^4}{EJ}$

$\varphi_B = \frac{1}{20} \frac{PL^3}{EJ}$



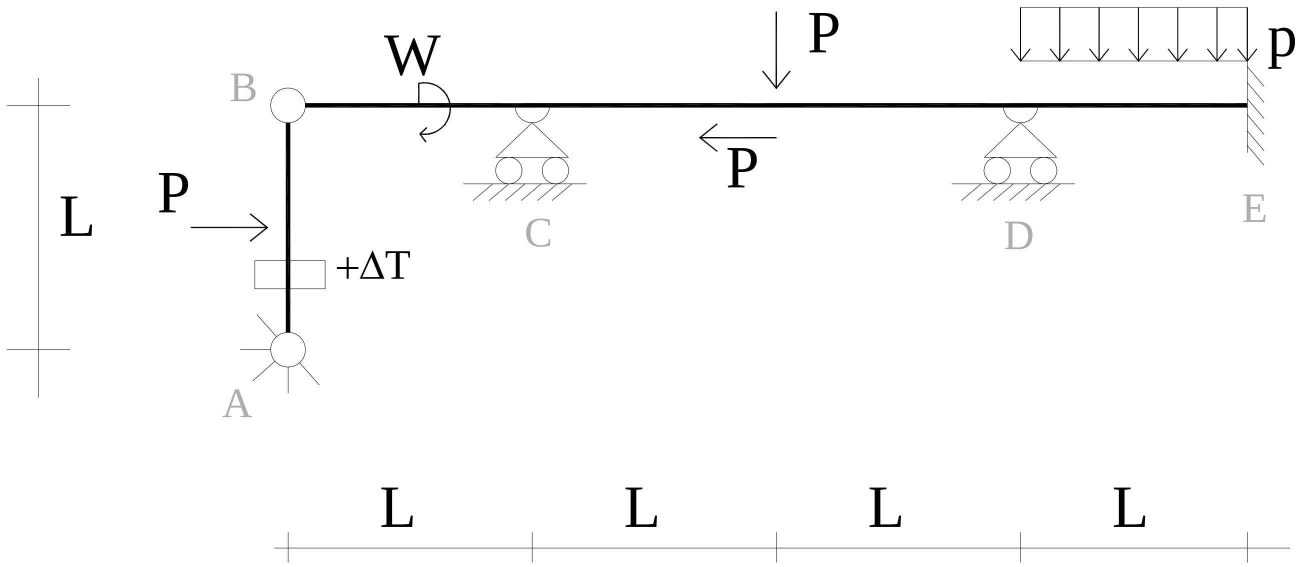
FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 05/07/2010, 3h

ing. F. Minelli; ing. F. Germano

Nome:………………….. ***Cognome:*** ………………………….. ***n.matr.:*** …..……….

Superato ELEMENTI STRUTTURALI A? **Sì** ☐ **NO** ☐

ESERCIZIO 1 (20 punti)



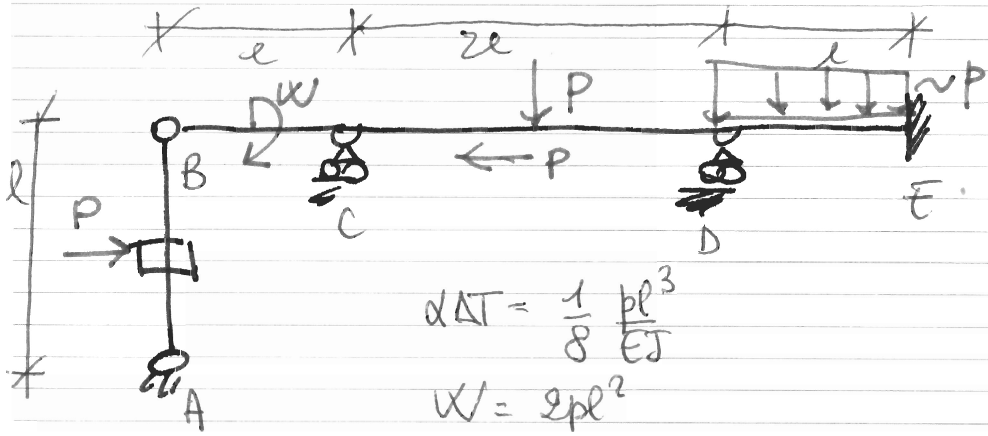
$$P = pL; \qquad \alpha\Delta T = \frac{1}{8}\frac{pL^3}{EJ}; \qquad W = 2\,pL^2; \qquad EA \rightarrow \infty$$

Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi

Tema Esame 05/07/2010

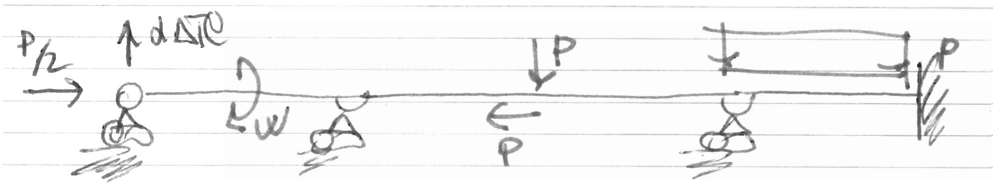


$\alpha \Delta T = \frac{1}{8} \frac{pl^3}{EJ}$

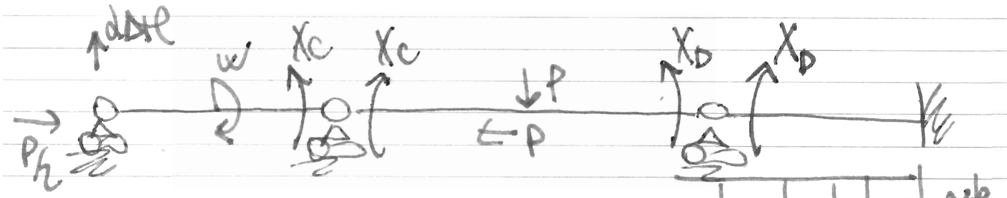
$W = 2pl^2$

$P = pl$

Si osserva la presenza della biella AB.
Si tratta di un'appendice isostatica.
La posso togliere avendo, però, l'accortezza di considerare nella travatura continua BCDE il carico P e il "cedimento" impresso in B dalla deformazione termica.

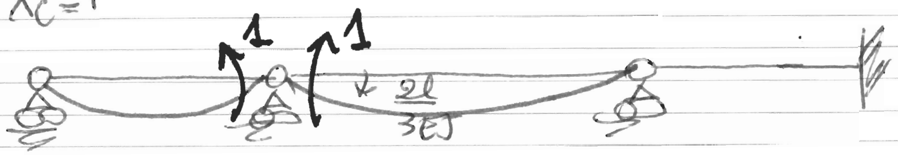


Si tratta di una trave continua. La risolvo con il METODO delle FORZE



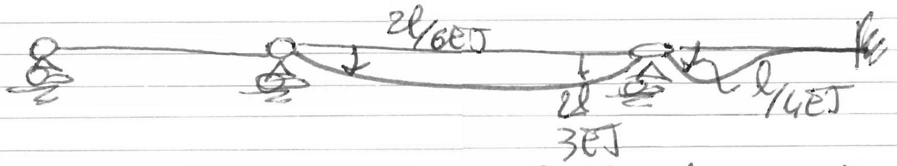
$$\begin{cases} \varphi_{CC} X_C + \varphi_{CD} X_D + \varphi_{C0} = 0 \\ \varphi_{DC} X_C + \varphi_{DD} X_D + \varphi_{D0} = 0 \end{cases}$$

$X_C = 1$



$$\varphi_{CC} = \frac{2l}{3EJ} - \left(-\frac{l}{3EJ}\right) = \frac{l}{EJ} \qquad \varphi_{DC} = 0 - \left(-\frac{2l}{6EJ}\right) = \frac{l}{3EJ}$$

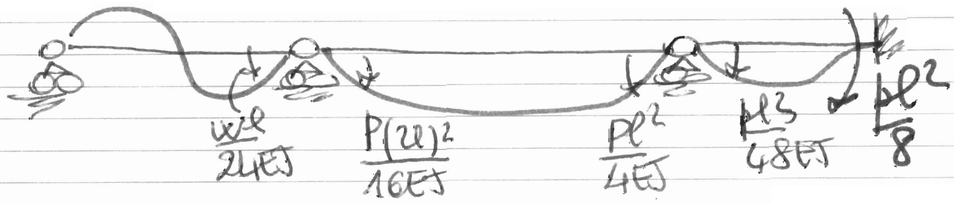
$X_D = 1$



$$\varphi_{DD} = \frac{l}{4EJ} - \left(-\frac{2l}{3EJ}\right) = \frac{3+8}{12}\,\frac{l}{EJ} = \frac{11l}{12EJ}$$

$$\varphi_{CD} = \frac{2l}{6EJ} - 0 = \frac{l}{3EJ}$$

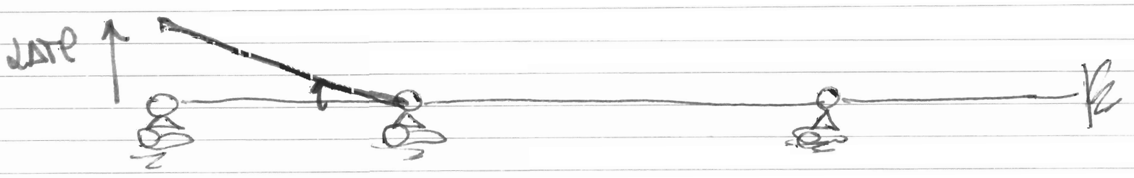
$p \neq 0$



$$\varphi^{p}_{C0} = \frac{pl^2}{4EJ} - \left(-\frac{wl}{24EJ}\right) = \frac{pl^2}{4EJ} + \frac{wl}{4EJ}$$

$$\varphi^{p}_{D0} = \frac{pl^3}{48EJ} - \left(-\frac{wl}{24EJ}\right) = \frac{pl^3}{48EJ} + \frac{pl^2}{4EJ}$$

$\Delta T \neq 0$



$$\varphi^{\Delta T}_{C0} = -\frac{\alpha \Delta T l}{l} = -\alpha \Delta T$$

$$\varphi^{\Delta T}_{D0} = 0$$

$$\varphi_{CO}^{TOT} = \varphi_{CO}^{p} + \varphi_{CO}^{\Delta T} = \frac{pl^2}{4EJ} + \frac{Wl}{24EJ} - \alpha\Delta T$$

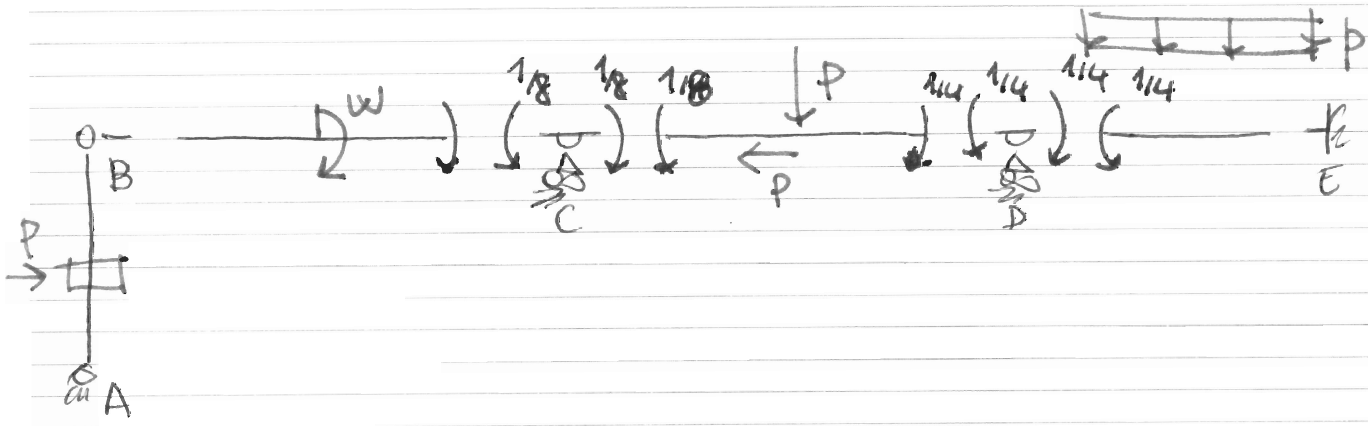
$$\varphi_{DO}^{TOT} = \frac{pl^3}{48EJ} + \frac{pl^2}{4EJ}$$

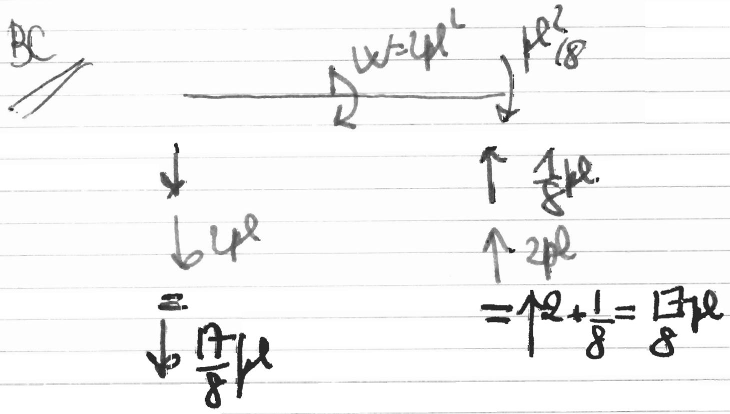
con $\alpha\Delta T = \frac{1}{8}\frac{pl^3}{EJ}$; $W = 2pl^2$

$$\varphi_{CO}^{TOT} = \frac{pl^3}{4EJ} + \frac{2pl^3}{24EJ} - \frac{1}{8}\frac{pl^3}{EJ} = \frac{6+2-3}{24}\frac{pl^3}{EJ} = \frac{5}{24}\frac{pl^3}{EJ}$$

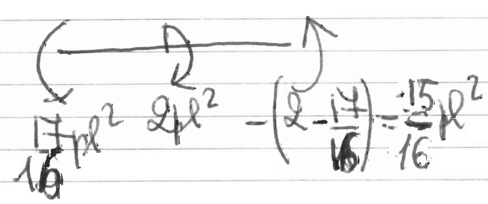
$$\varphi_{DO}^{TOT} = \frac{pl^3}{48EJ} + \frac{pl^3}{4EJ} = \frac{1+12}{48}\frac{pl^3}{EJ} = \frac{13pl^3}{48EJ}$$

$$\begin{cases} \frac{l}{EJ}X_C + \frac{l}{3EJ}X_D + \frac{5}{24}\frac{pl^3}{EJ} = 0 \\ \frac{l}{3EJ}X_C + \frac{11l}{12EJ}X_D + \frac{13}{48}\frac{pl^3}{EJ} = 0 \end{cases} \rightarrow \begin{cases} X_C = -\frac{1}{8}pl^2 \\ X_D = -\frac{1}{4}pl^2 \end{cases}$$





MEZZERIA



CD

$\frac{pl^2}{8}$ $\downarrow pl$ $\frac{pl^2}{4}$

$\uparrow \frac{pl}{2}$ $\quad\quad$ $\uparrow \frac{pl}{2}$

$\downarrow \frac{pl}{8}$ $\quad\quad$ $\uparrow \frac{pl^2}{4}\frac{1}{2l}$

$\uparrow \frac{pl^2}{8}\frac{1}{2l}$ $\quad\quad$ $\downarrow \frac{pl}{16}$

$=$ $\quad\quad$ $=$

$\uparrow \frac{1}{2}-\frac{1}{8}+\frac{1}{16}pl$ $\quad\quad$ $\uparrow \frac{1}{2}+\frac{1}{8}-\frac{1}{16}pl=$

$\frac{8-2+1}{16}pl$ $\quad\quad$ $=\frac{8+2-1}{16}pl$

$=\frac{7}{16}pl$ $\quad\quad$ $=\frac{9}{16}pl$

Mom. mezzeria $\quad$ $\frac{pl^2}{8}$ $\quad \uparrow\frac{7}{16}$ $\quad M$

$M+\frac{1}{8}pl^2-\frac{7}{16}pl^2=0$ $\quad\quad$ $M=\frac{5}{16}pl^2$

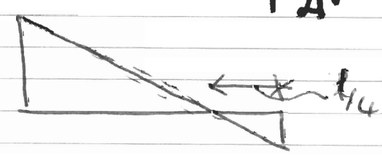
DE

$\frac{pl^2}{4}$ $\uparrow\frac{pl}{2}$ $\uparrow\frac{pl}{4}$ $\quad\quad$ $\frac{pl}{2}\uparrow$ $\frac{pl}{4}\downarrow$ $\quad\quad$ $\frac{pl^2}{8}\ \frac{pl^2}{8}=0$

$=\uparrow\frac{1}{2}+\frac{1}{4}pl$ $\quad\quad$ $=\uparrow\frac{1}{2}-\frac{1}{4}pl$

$\uparrow\frac{3pl}{4}$ $\quad\quad$ $=\uparrow\frac{1}{4}pl$

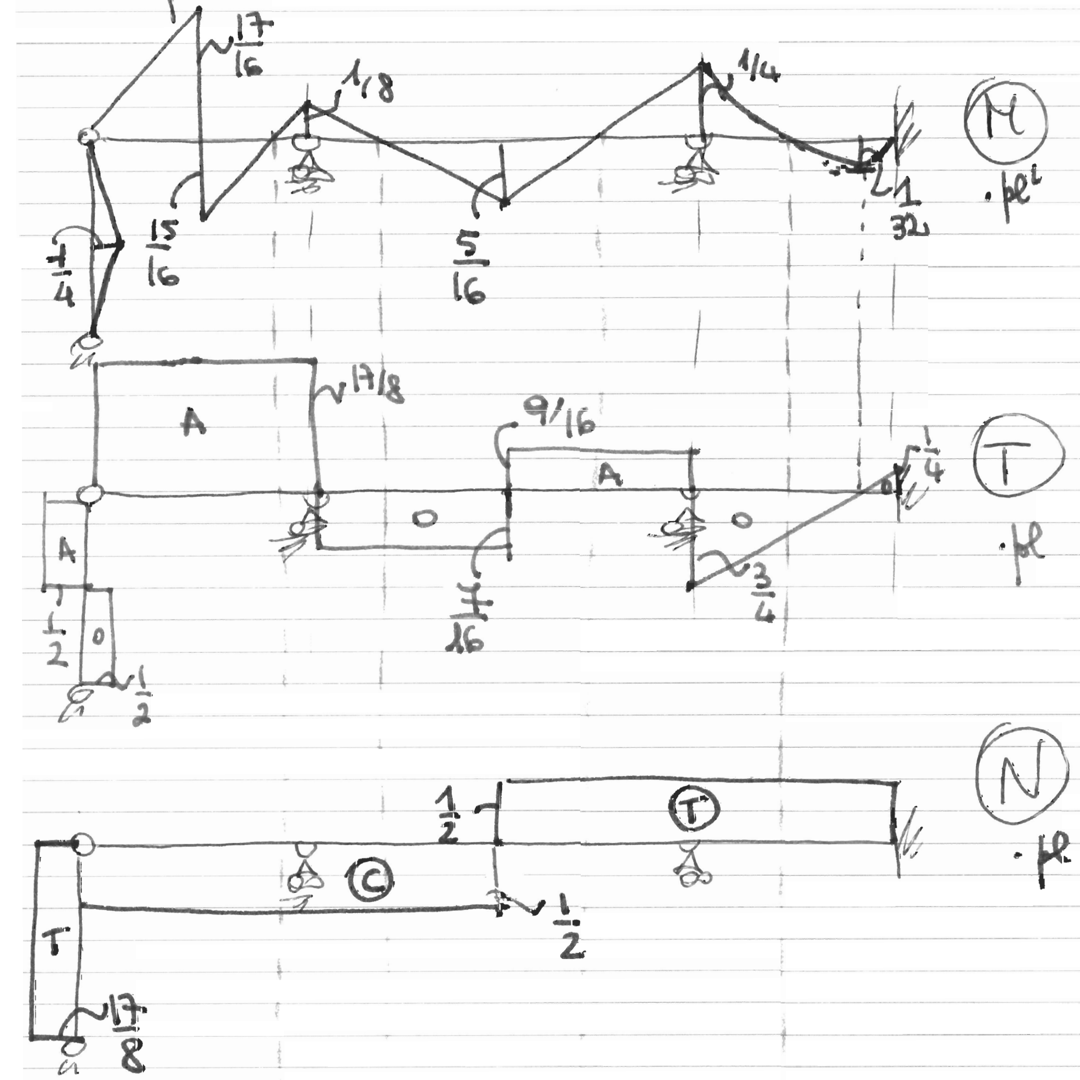
$X=\frac{l}{4}$

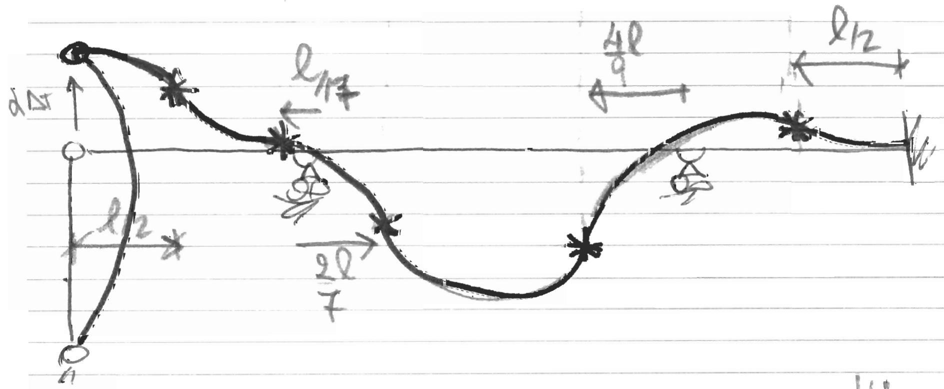


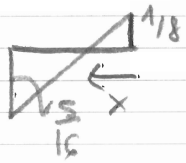
$M_{max}\left(\frac{l}{4}\right)=\frac{pl}{4}\cdot\frac{l}{4}-\frac{pl}{4}\frac{1}{2}\frac{l}{4}$

$=\frac{pl^2}{32}$

Diagramma delle AZIONI INTERNE



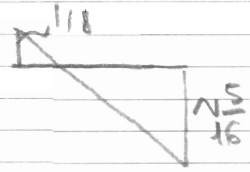




$\frac{1}{8}\cdot x = \frac{15}{16}\left(\frac{l}{2}-x\right)$

$\frac{15}{2}x = \left(\frac{l}{2}-x\right)$

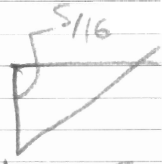
$\left(\frac{15}{2}+1\right)x = \frac{l}{2} \Rightarrow x = \frac{l}{17}$



$\frac{1}{8}\cdot x = \frac{5}{16}\cdot(l-x)$

$\frac{5}{2}x = (l-x)$

$\left(\frac{5}{2}+1\right)x = l \qquad x = \frac{2l}{7}$



$\frac{1}{4}\cdot x = \frac{5}{16}$

$\frac{5}{4}x = (l-$

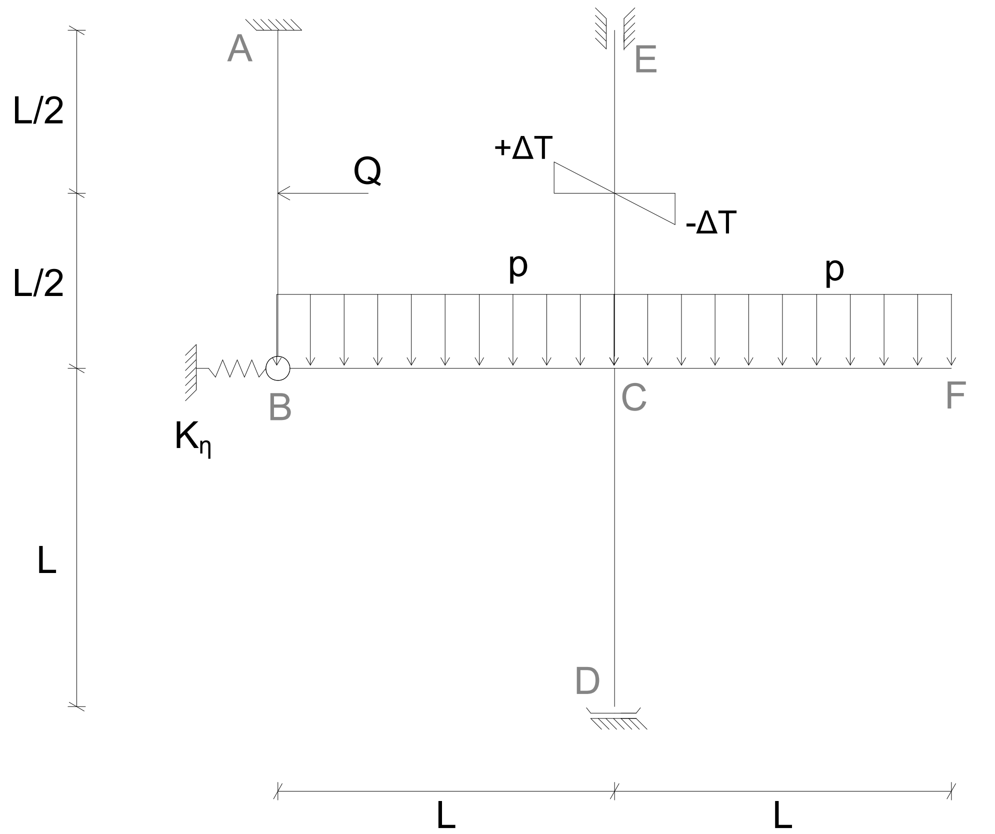
$\left(\frac{5}{4}+1\right)x =$

FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 13/09/2010, 3h

ing. F. Minelli; ing. F. Germano

***Nome:*……………….. *Cognome:* ………………………….. *n.matr.:* …..……….
Superato ELEMENTI STRUTTURALI A? Sì □ NO □

ESERCIZIO 1 (20 punti)

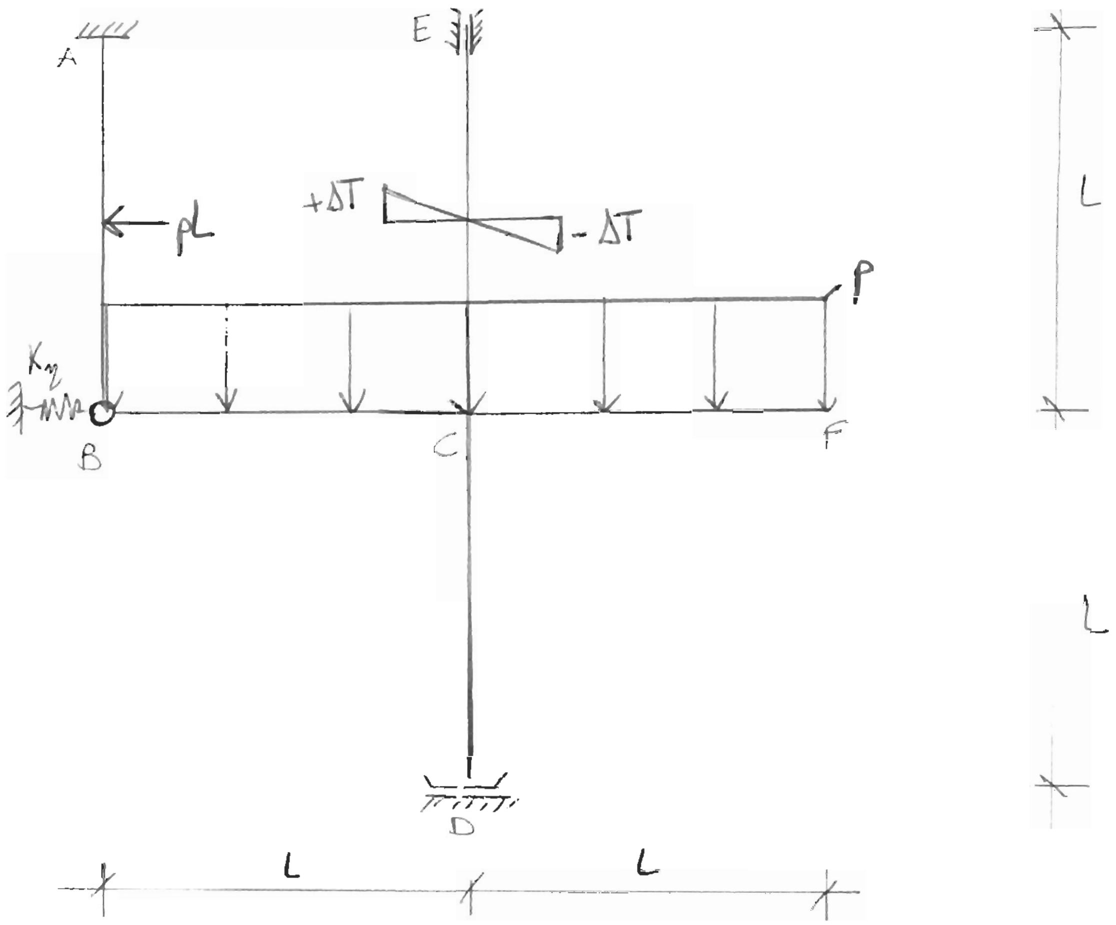


$$Q = pL; \qquad \frac{\alpha \Delta T}{t} = \frac{1}{6}\frac{pL^2}{EJ}; \qquad k_\eta = \frac{9EJ}{L^3}; \qquad EA \to \infty$$

Dato il telaio in figura

Si richiedono:

1- Momento flettente (con il valore e la posizione dei massimi)
2- Taglio
3- Azione assiale
4- Deformata qualitativa con posizione dei flessi



$$K_\eta = \frac{9EJ}{L^3}$$

$$\frac{2\Delta T}{t} = \frac{1}{6}\frac{pL^2}{EJ}$$

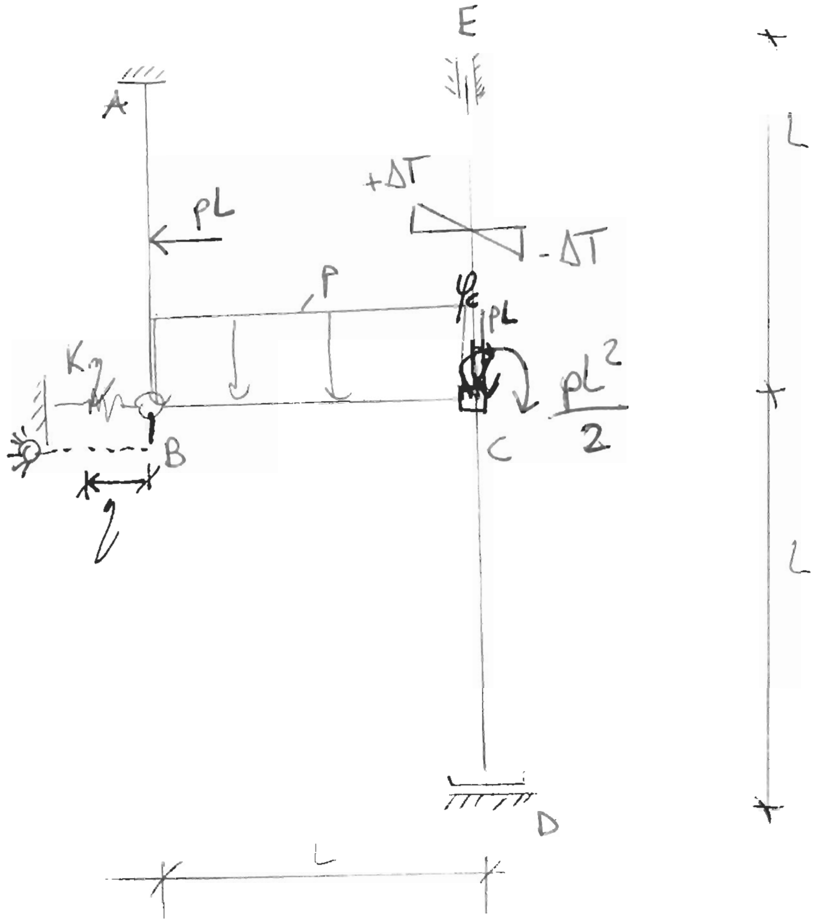
Analisi cinematica

G.L.: 6

G.V.: 9

c'è un'appendice isostatica → (CF)

si può quindi studiare la seguente struttura

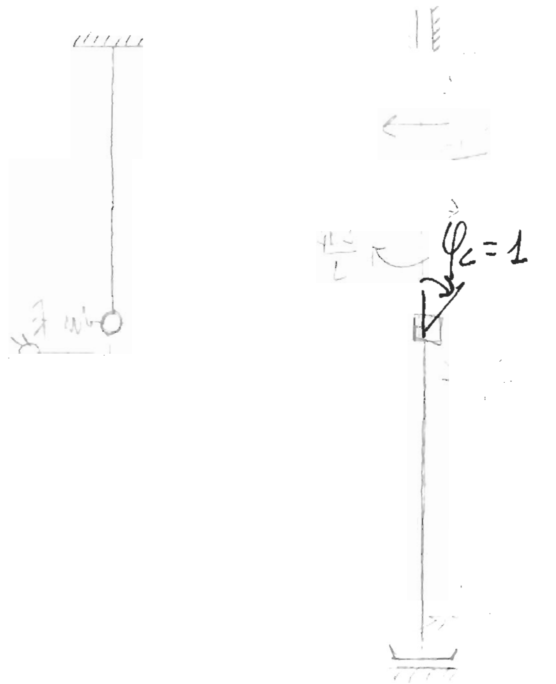


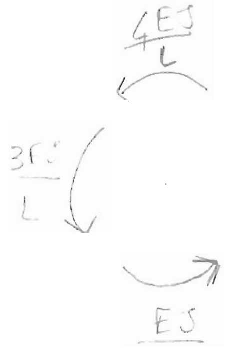
$$\begin{cases} \Sigma m_c = 0 \\ \Sigma F_{b.dd} = 0 \end{cases}$$

$$\begin{cases} m_{cc} \cdot \varphi_c + m_{c\eta} \cdot \eta + m_{c0} = 0 \\ h_{\eta c} \cdot \varphi_c + h_{\eta\eta} \cdot \eta + h_{\eta 0} = 0 \end{cases}$$

NODO +

$\varphi_c = 1 \quad resto = 0$

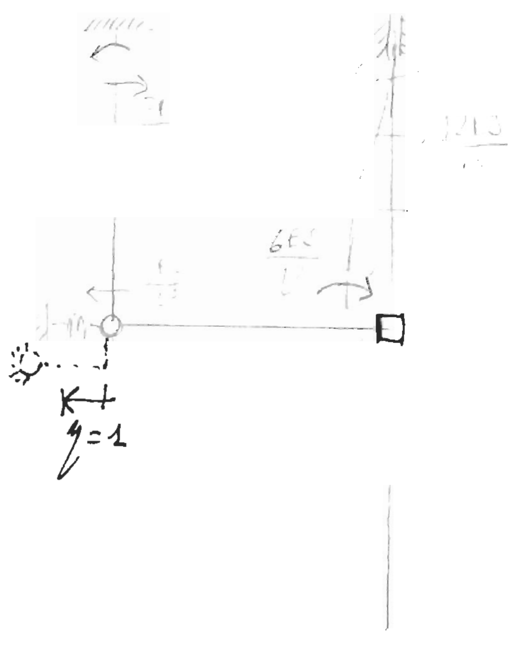




$$m_{cc} = \frac{8EJ}{L}$$

$$k_{\eta c} = \frac{6EJ}{L^2}$$

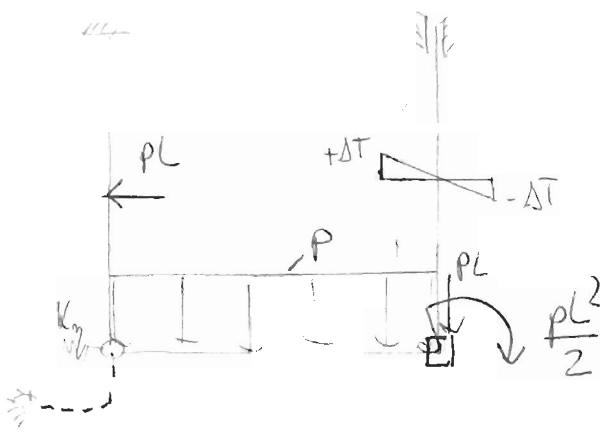
$\eta = 1 \quad resto = 0$

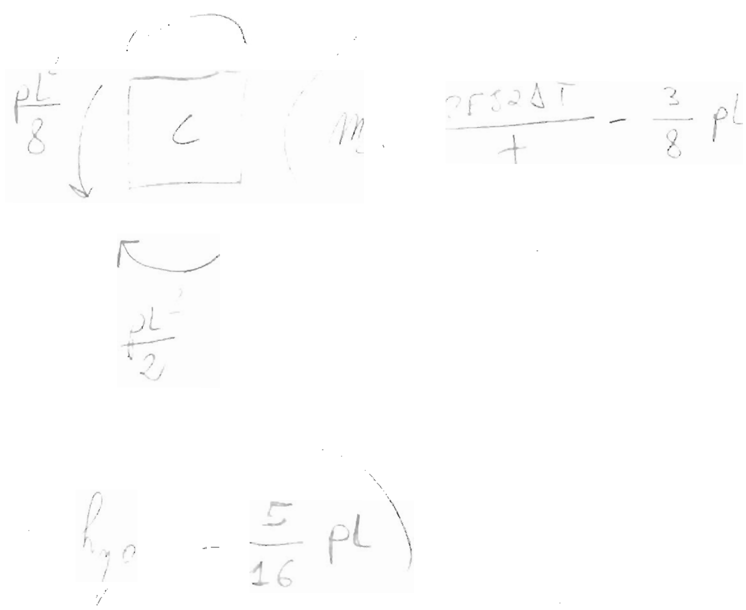


$$m_c = \frac{6EJ}{L}$$

(3) ✓

$\varphi = 0 \quad \eta = 0 \quad$ resto $\neq 0$





$$\begin{cases} \dfrac{8EJ}{L}\cdot\varphi_c - \dfrac{6EJ}{L^2}\eta + \dfrac{2EJ\alpha\Delta T}{t} - \dfrac{3}{8}pL^2 = 0 \\[2ex] -\dfrac{6EJ}{L^2}\cdot\varphi_c + \left(\dfrac{15EJ}{L^2} + K_\eta\right)\eta - \dfrac{5pL}{16} = 0 \end{cases}$$

$$\begin{cases} \dfrac{8EJ}{L}\cdot\varphi_c - \dfrac{6EJ}{L^2}\cdot\eta = \dfrac{3}{8}pL^2 - \dfrac{2EJ\alpha\Delta T}{t} \\[2ex] -\dfrac{6EJ}{L^2}\cdot\varphi_c + \left(\dfrac{15EJ}{L^3} + K_\eta\right)\cdot\eta = \dfrac{pL}{2} \end{cases}$$

$$\begin{bmatrix} \dfrac{8EJ}{L} & -\dfrac{6EJ}{L^2} \\[2ex] -\dfrac{6EJ}{L^2} & \left(\dfrac{15EJ}{L^3} + K_\eta\right) \end{bmatrix} \begin{bmatrix} \varphi_c \\[2ex] \eta \end{bmatrix} = \begin{bmatrix} \dfrac{3}{8}pL^2 - \dfrac{2EJ\alpha\Delta T}{t} \\[2ex] \dfrac{5pL}{16} \end{bmatrix}$$

(4) ✓

considerando

$$K_y = \frac{9EJ}{L^3} \quad \text{ed} \quad \frac{\alpha \Delta T}{t} = \frac{1}{6}\frac{pL^2}{EJ}$$

si ha

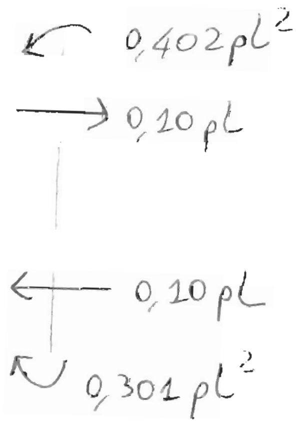
$$\varphi_c = \frac{4}{217}\frac{pL^3}{EJ} \qquad y = \frac{11}{624}\frac{pL^4}{EJ}$$

Diagrammi delle azioni interne

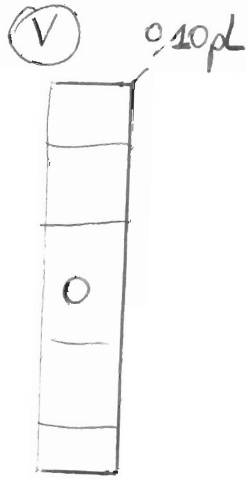
EC

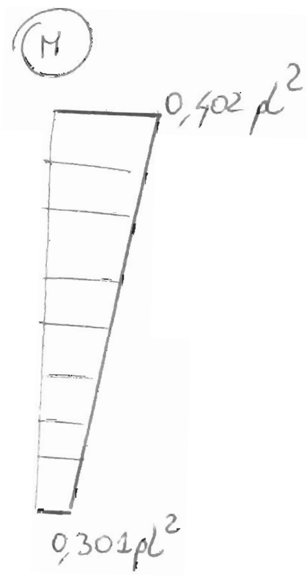
$$\frac{2EJ}{L}\cdot\frac{4}{217}\frac{pL^3}{EJ} - \frac{6EJ}{L^2}\cdot\frac{11}{624}\frac{pL^4}{EJ} - \frac{1}{3}pL^2 = -0{,}402\,pL^2$$

$$\frac{4EJ}{L}\cdot\frac{4}{217}\frac{pL^3}{EJ} - \frac{6EJ}{L^2}\cdot\frac{11}{624}\frac{pL^4}{EJ} + \frac{1}{3}pL^2 = 0{,}301\,pL^2$$



→

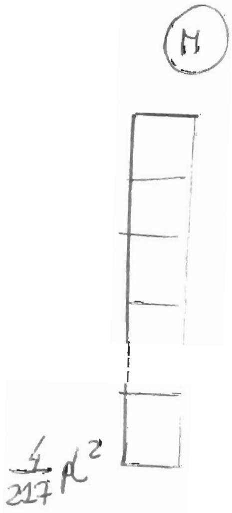




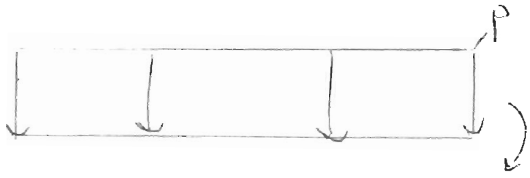
CD

$$\frac{EJ}{L}\cdot\frac{4}{217}\frac{pL^3}{EJ}$$

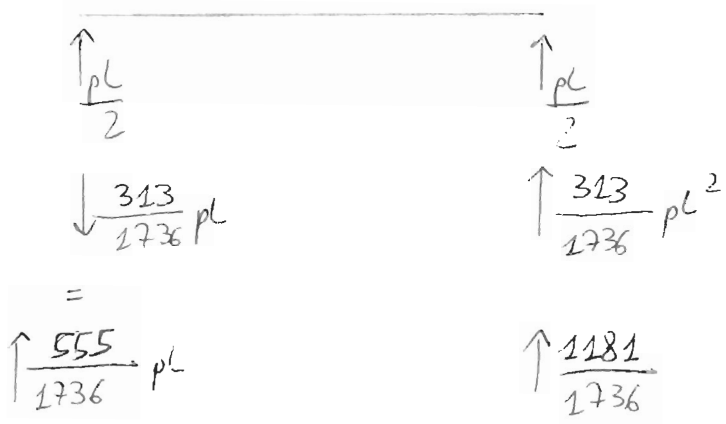
$$\frac{EJ}{L}\;\frac{4}{217}\frac{pL^3}{EJ}$$



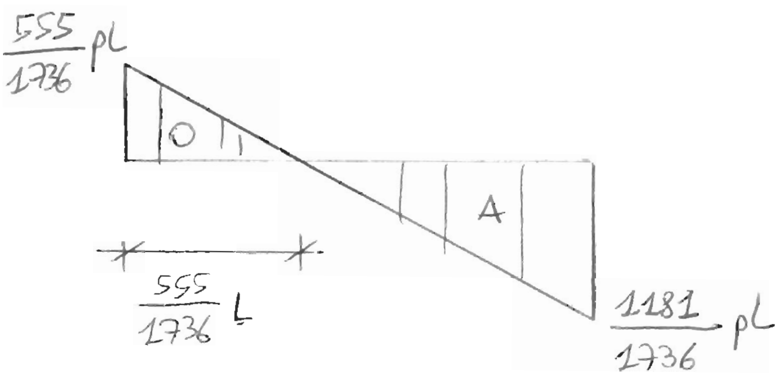
BC



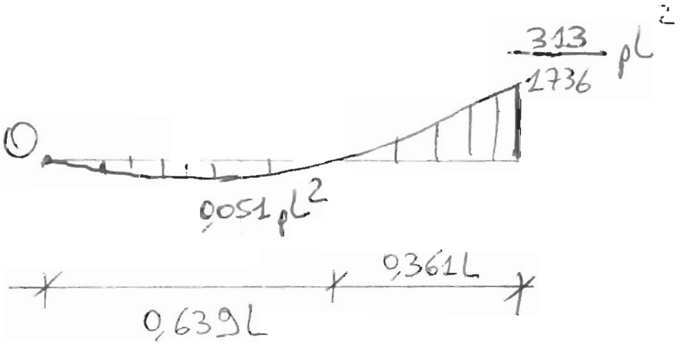
$$\frac{3EJ}{L}\cdot\frac{4}{217}\,\frac{pL^3}{EJ}+\frac{1}{8}pL^2=\frac{313}{1736}\,pL^2=0{,}18\,pL^2$$



(V)



(M)



$$M=\frac{555}{1736}pL\cdot\frac{555}{1736}L-\frac{p\left(\frac{555}{1736}\right)^2L^2}{2}=$$

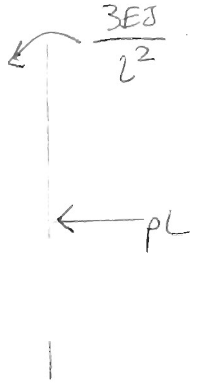
$$=0{,}051\,pL^2$$

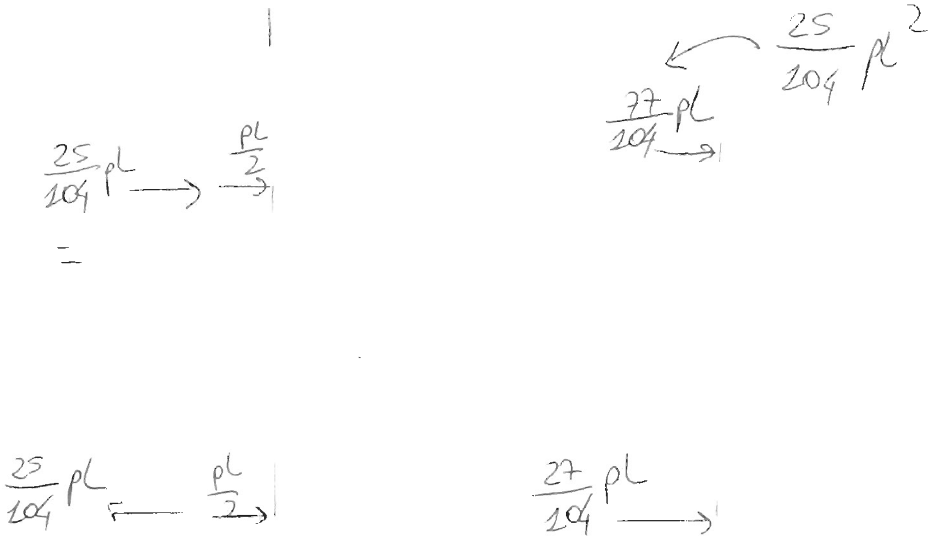
$$\frac{555}{1736}pL\cdot x-p\cdot x\cdot\frac{x}{2}=0$$

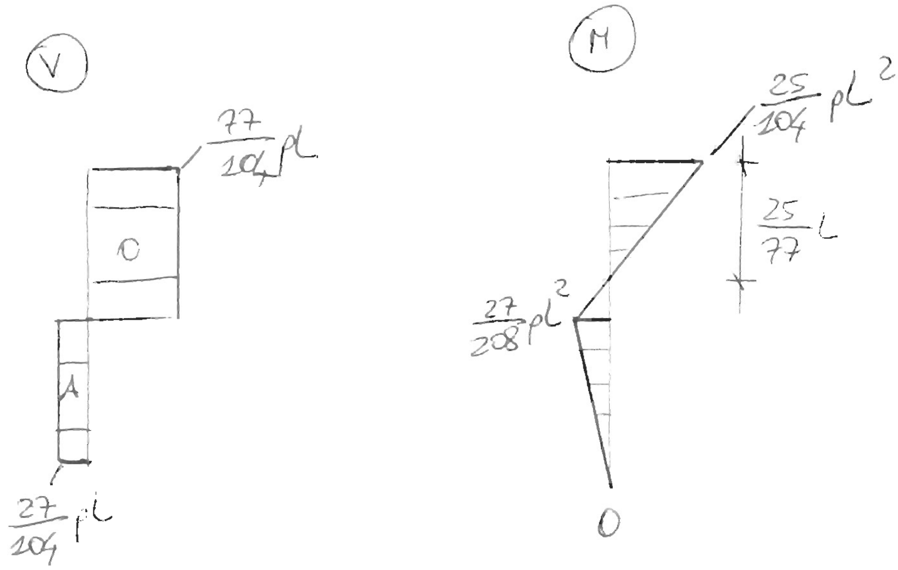
$$-\frac{1}{2}p\,x+\frac{555}{1736}pL\;x=0 \quad \begin{cases} x=0 \\ x=0{,}639 \end{cases}$$

AB

$$\frac{3EJ}{l^2} \cdot \frac{11}{624} \cdot \frac{pL^4}{EJ} + \frac{3}{16} pL^2 = \frac{25}{104} pL^2$$





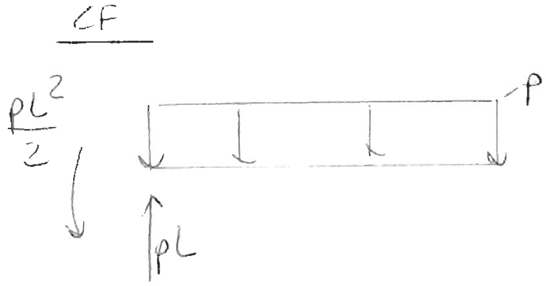


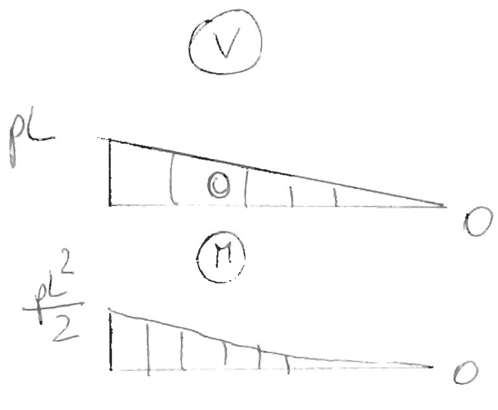
$$\frac{27}{104} pL \cdot \frac{L}{2} = \frac{27}{208} pL^2$$

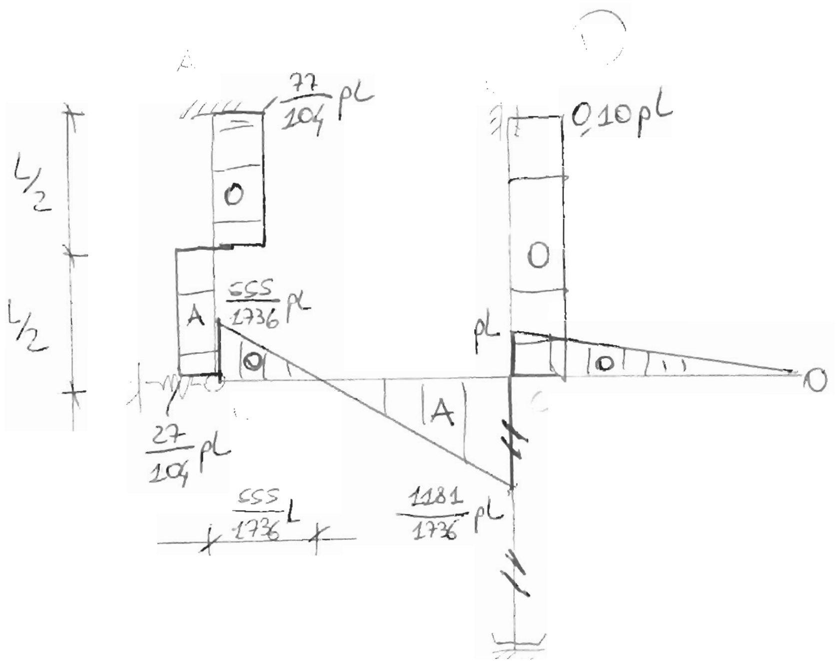
$$\circlearrowleft + \quad \frac{25}{104} pL^2 - \frac{77}{104} pL \cdot x = 0$$

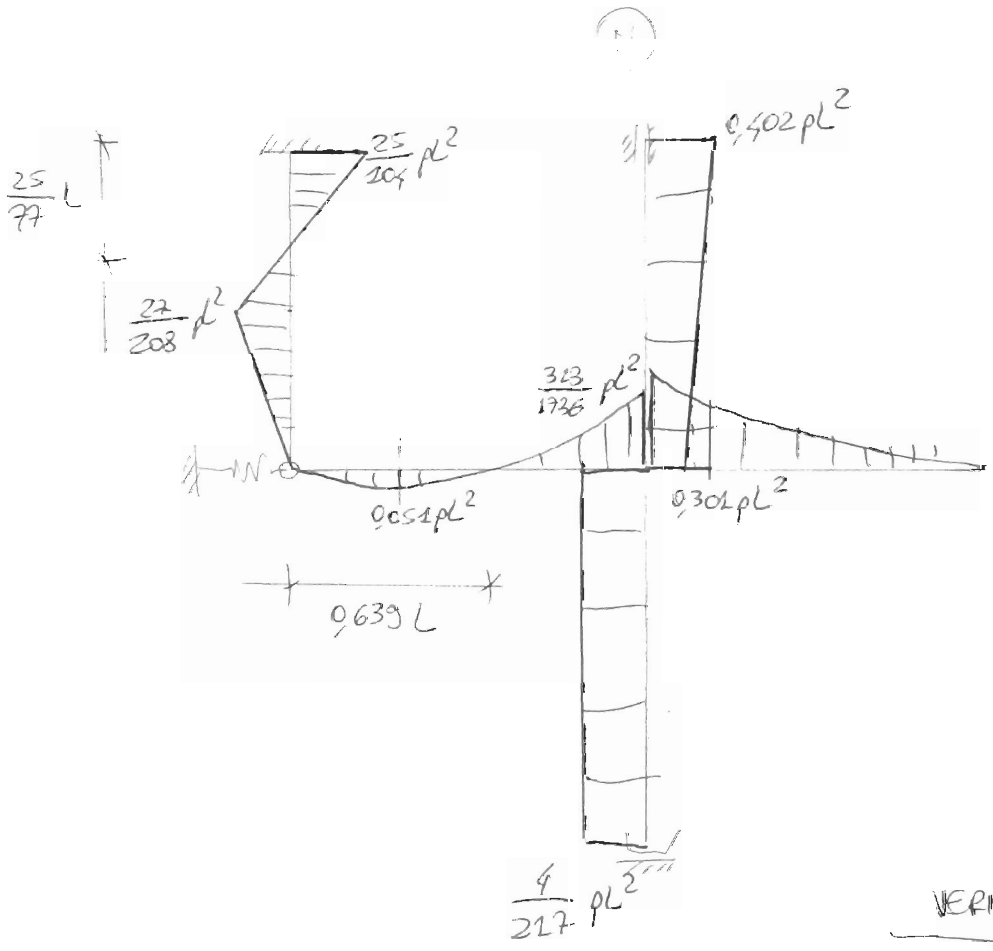
$$x = \frac{25}{77} L$$

CF

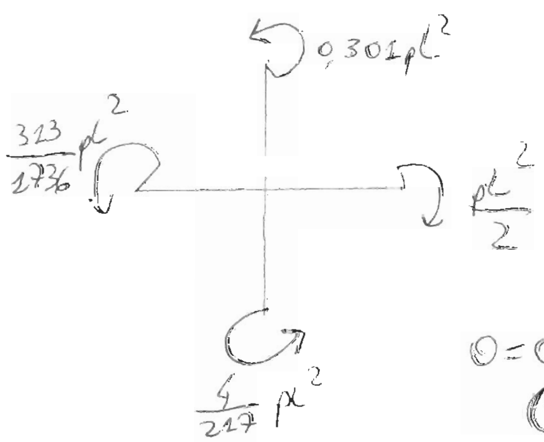






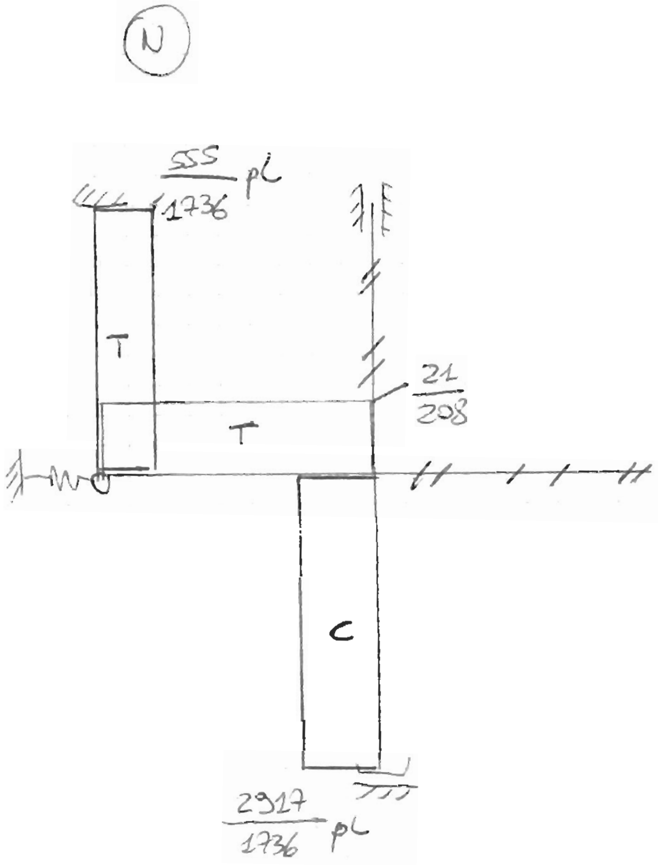


VERIFICA EQUILIBRIO NODI

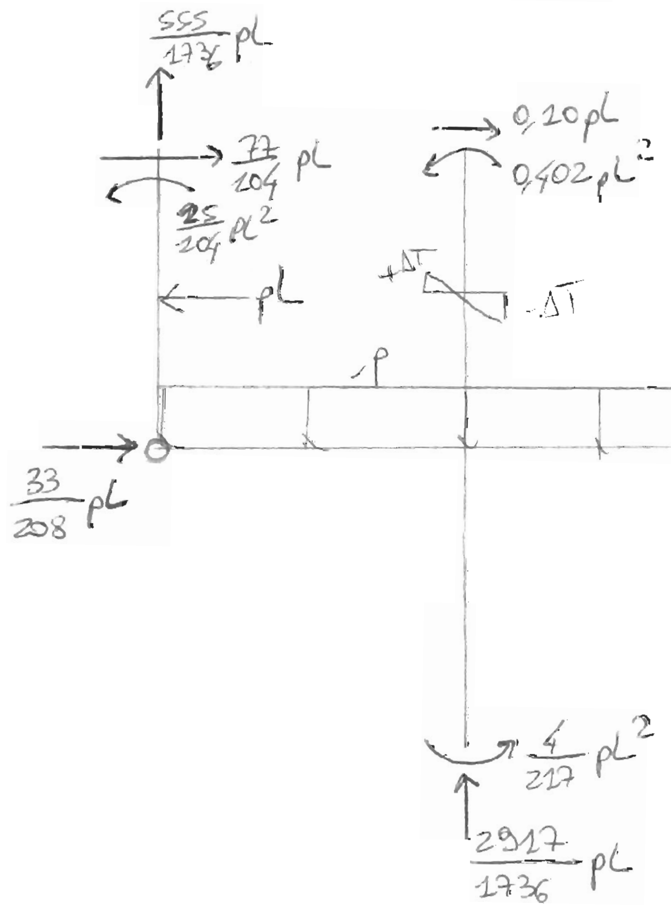


0 = 0 (OK)

(8)



VERIFICA EQUILIBRIO GLOBALE

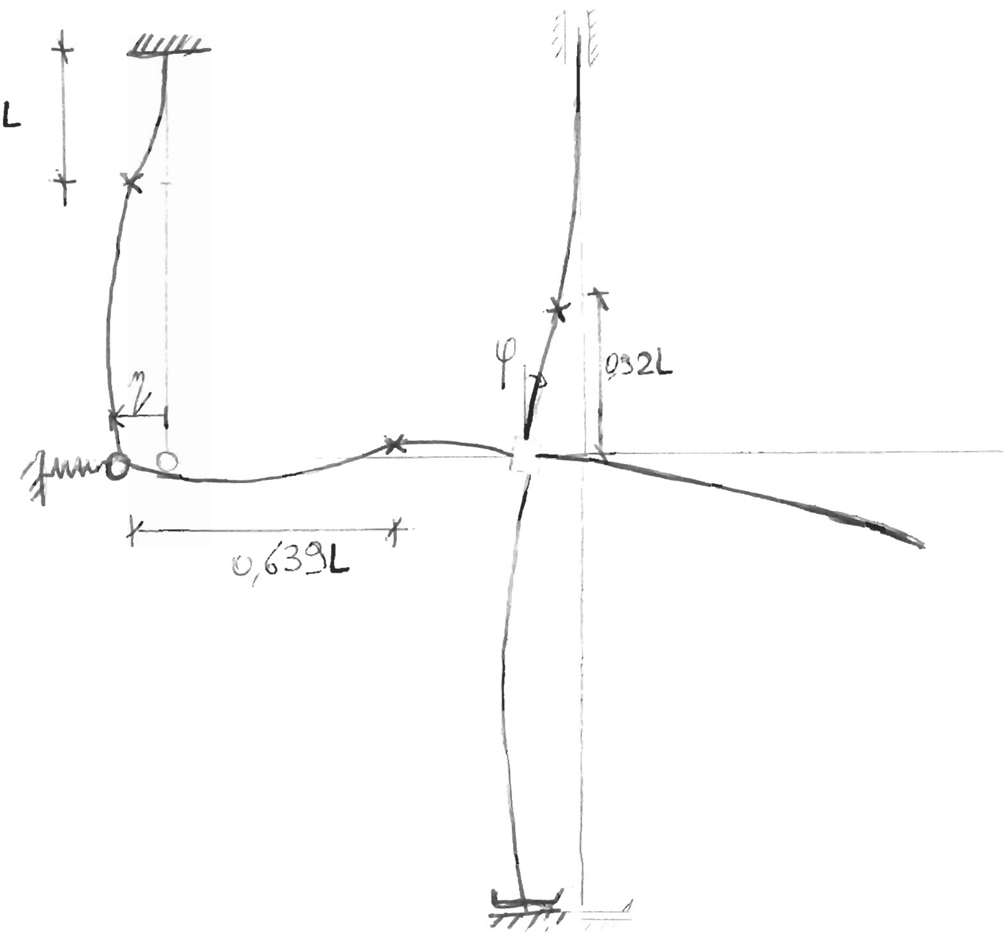


$\xrightarrow{+}$ $\frac{33}{208}pL + \frac{77}{104}pL + 0{,}10\,pL - pL = 0 \qquad 0=0$ ✓

$+\uparrow$ $\frac{555}{1736}pL - 2pL + \frac{2917}{1736}pL = 0 \qquad 0=0$ ✓

$\circlearrowleft$ $\frac{4}{217}pL^2 - pL\frac{L}{2} + pL\frac{L}{2} - \frac{33}{208}pL\cdot L + 0{,}402\,pL^2 - 0{,}10\,pL\cdot 2L + pL\cdot\frac{3}{2}L + \frac{25}{104}pL^2 - \frac{77}{104}pL\cdot 2L - \frac{555}{1736}pL\cdot L$ ...

DEFORMATA QUALITATIVA



$$v''(x) = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{h}$$

$$M(x) = 0{,}301\,pL^2 + 0{,}10\,pL\cdot x$$

$$\frac{2\alpha\Delta T}{h} = 2\cdot\frac{1}{6}\frac{pL^2}{EJ} = \frac{1}{3}\frac{pL^2}{EJ}$$

$$v''(x) = -\frac{0{,}301\,pL^2}{EJ} - \frac{0{,}10\,pL\,x}{EJ} + \frac{1}{3}\frac{pL^2}{EJ} = -0{,}10\frac{pL\,x}{EJ} + 0{,}032\frac{pL^2}{EJ}$$

$$v''(x) = -0{,}10\frac{pL}{EJ}x + 0{,}032\frac{pL^2}{EJ}$$

$v'' > 0 \quad -0{,}10\frac{pL}{EJ}x > -0{,}032\frac{pL^2}{EJ} \rightarrow 0{,}10x < 0{,}032L \quad x < 0{,}32L$

$v'' < 0 \quad x > 0{,}32L$

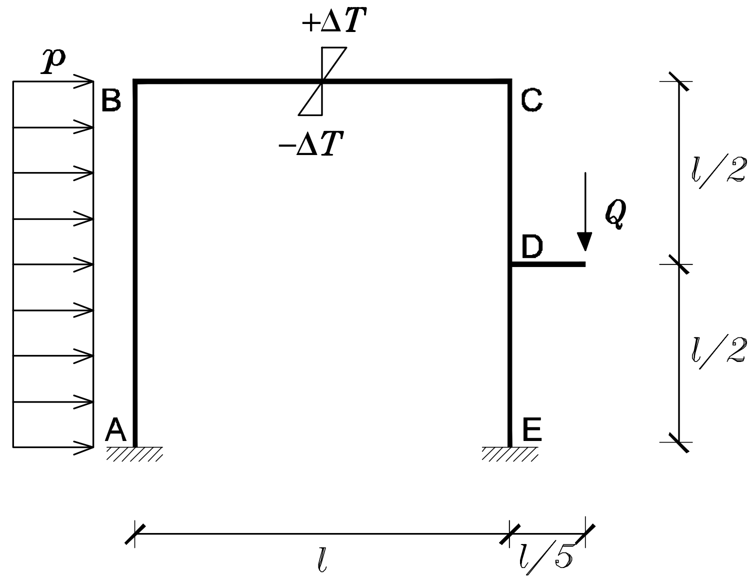
10

TECNICA DELLE COSTRUZIONI

ESAME DEL 24 GENNAIO 2011

DOCENTE: **ING. FAUSTO MINELLI**

ESERCIZIO



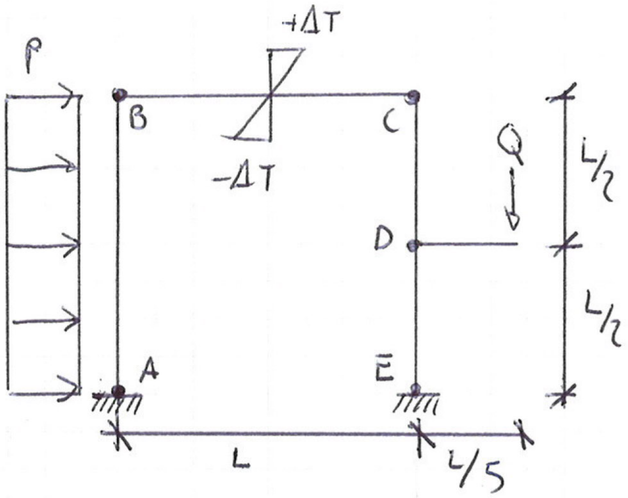
$Q = 3\,pl$

$$\frac{\alpha\,\Delta T}{h} = \frac{1}{6}\frac{pl^2}{EJ}$$

Dato il telaio in figura si richiede il tracciamento:

1. Diagramma del momento flettente (con il valore e la posizione dei massimi);
2. Diagramma del taglio;
3. Diagramma dell'azione assiale;
4. Deformata qualitativa con posizione dei flessi.

PORTALE IPERSTATICO ①



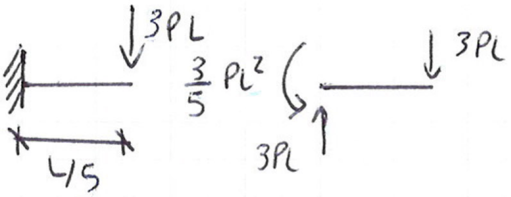
$$\begin{cases} \dfrac{\alpha \Delta T}{t} = \dfrac{1}{6}\dfrac{pL^2}{EJ} \\ Q = 3pL \end{cases}$$

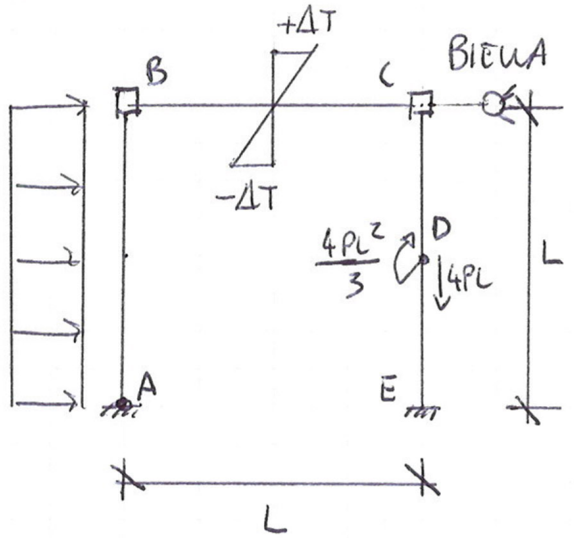
Ⓐ ANALISI CINEMATICA

GdiL = 3

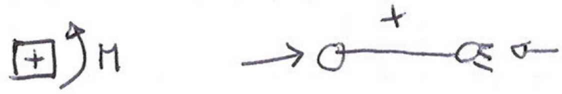
GdiV = 6 → STRUTTURA 3 VOLTE IPERSTATICA A NODI SPOSTABILI

APPENDICE ISOSTATICA: MENSOLA IN D

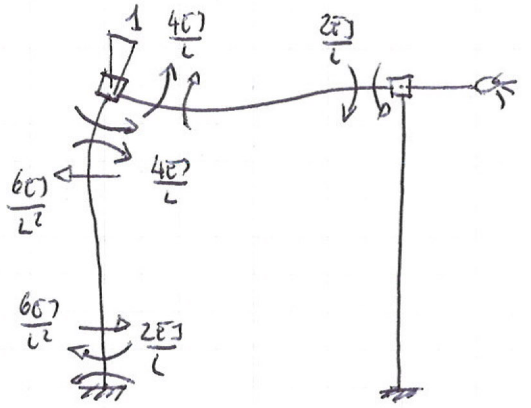




$$\begin{cases} m_{BB}\cdot\varphi_B + m_{BC}\varphi_C + m_{B\eta}\eta + m_{B0} = 0 \\ m_{CB}\varphi_B + m_{CC}\varphi_C + m_{C\eta}\eta + m_{C0} = 0 \\ h_{\eta B}\varphi_B + h_{\eta C}\varphi_C + h_{\eta\eta}\eta + h_{\eta 0} = 0 \end{cases}$$



⊛ $\varphi_B = 1$

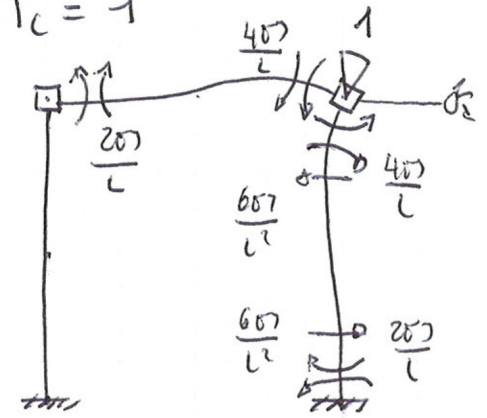


$$m_{BB} = +\frac{4EJ}{L} + \frac{4EJ}{L} = \frac{8EJ}{L}$$

$$m_{CB} = +\frac{2EJ}{L}$$

$$h_{\eta B} = +\frac{6EJ}{L^2}$$

(*) $\varphi_C = 1$

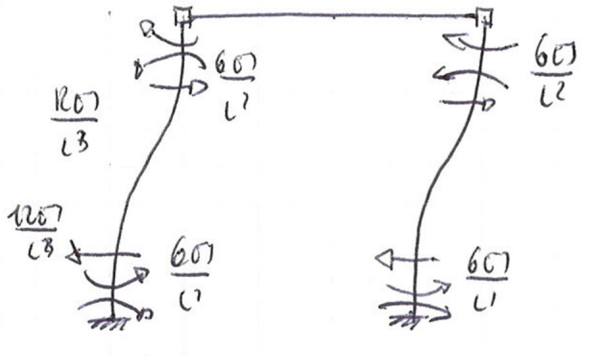


$$m_{BC} = +\frac{2EJ}{L}$$

$$m_{CC} = +\frac{4EJ}{L} + \frac{4EJ}{L} = \frac{8EJ}{L}$$

$$h_{\eta C} = +\frac{6EJ}{L^2}$$

(*) $\eta = 1$

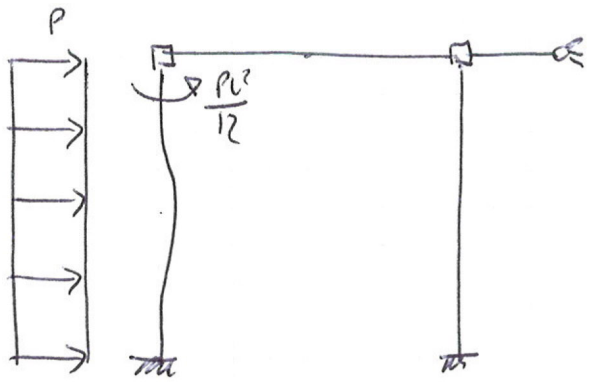


$$m_{B\eta} = -\frac{6EJ}{L^2}$$

$$m_{C\eta} = -\frac{6EJ}{L^2}$$

$$h_{\eta\eta} = -\frac{24EJ}{L^3}$$

(*) $P \neq 0$ (4)

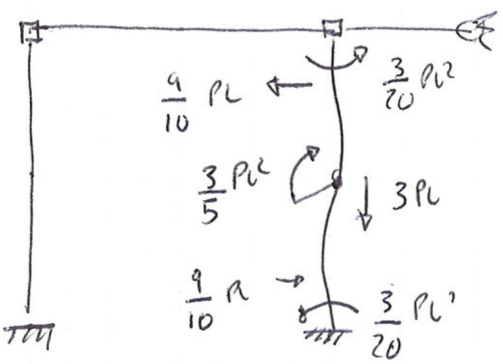


$$m_{BC_1, I} = \frac{PL^2}{12}$$

$$m_{CD_1, I} = 0$$

$$h_{CD_1, I} = \frac{PL}{2}$$

(*) $Q \neq 0$



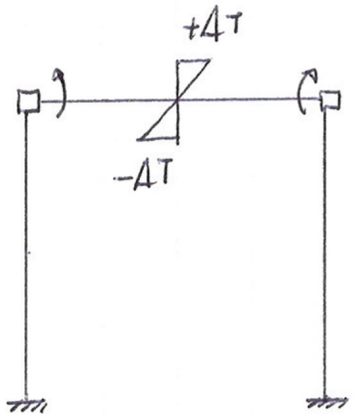
$$m_{BC_1, II} = 0$$

$$m_{CD_1, II} = \frac{3}{20} PL^2$$

$$h_{HD_1, II} = \frac{9}{10} PL$$

⊛ $\Delta T \neq 0$

(5)



$$m_{B0,III} = \frac{2EJ\alpha\Delta T}{t} = \frac{1}{3}pL^2$$

$$m_{C0,III} = \frac{2EJ\alpha\Delta T}{t} = -\frac{1}{3}pL^2$$

$$h_{\eta 0,III} = \phi$$

⊛ SISTEMA RISOLVENTE

$$\begin{cases} 1) \quad \left(8\frac{EJ}{L}\right)\varphi_B + \left(2\frac{EJ}{L}\right)\varphi_C + \left(-\frac{6EJ}{L^2}\right)\eta = -\frac{5}{12}pL^2 \\ 2) \quad \left(2\frac{EJ}{L}\right)\varphi_B + \left(8\frac{EJ}{L}\right)\varphi_C + \left(-\frac{6EJ}{L^2}\right)\eta = +\frac{11}{60}pL^2 \\ 3) \quad \left(6\frac{EJ}{L^2}\right)\varphi_B + \left(6\frac{EJ}{L^2}\right)\varphi_C + \left(-24\frac{EJ}{L^3}\right)\eta = -\frac{7}{5}pL \end{cases}$$

$$\varphi_B = -\frac{1}{60}\frac{pL^3}{EJ} \qquad \varphi_C = +\frac{1}{12}\frac{pL^3}{EJ} \qquad \eta = +\frac{3}{40}\frac{pL^4}{EJ}$$

$$\cdot 1 \begin{cases} 8\frac{EJ}{L}\varphi_B + 2\frac{EJ}{L}\varphi_C - \frac{6EJ}{L^2}\eta = -\frac{5}{12}\,pL^2 \\ 2\frac{EJ}{L}\varphi_B + 8\frac{EJ}{L}\varphi_C - \frac{6EJ}{L^2}\eta = +\frac{11}{60}\,pL^2 \\ 6\frac{EJ}{L^2}\varphi_B + \frac{6EJ}{L^2}\varphi_C - 24\frac{EJ}{L^3}\eta = -\frac{7}{5}\,pL^2 \end{cases}$$

$$\cdot 4 \begin{cases} 8\varphi_B + 2\varphi_C - 6\frac{\eta}{L} = -\frac{5}{12}\frac{pL^3}{EJ} \\ 2\varphi_B + 8\varphi_C - 6\frac{\eta}{L} = +\frac{11}{60}\frac{pL^3}{EJ} \\ 6\varphi_B + 6\varphi_C - 24\frac{\eta}{L} = -\frac{7}{5}\frac{pL^3}{EJ} \end{cases}$$

$$3 \begin{cases} 6\varphi_B - 6\varphi_C = \left(-\frac{5}{12} - \frac{11}{60}\right)\frac{pL^3}{EJ} = \left(-\frac{25+11}{60}\right)\frac{pL^3}{EJ} = -\frac{3}{5}\frac{pL^3}{EJ} \\ -2\varphi_B - 16\varphi_C = \left(-\frac{11}{15} - \frac{7}{5}\right)\frac{pL^3}{EJ} = -\left(\frac{11-21}{15}\right)\frac{pL^3}{EJ} = -\frac{32}{15}\frac{pL^3}{EJ} \\ -\quad 84\varphi_C = \left(-\frac{3}{5} - \frac{32}{5}\right)\frac{pL^3}{EJ} \end{cases}$$

$$84\varphi_C = \frac{35}{5}7\,\frac{pL^3}{EJ}$$

$$\boxed{\varphi_C = \frac{1}{12}\frac{pL^3}{EJ}}$$

(7)

$$6\varphi_B - 6\left(\frac{1}{12}\frac{Pl^3}{EJ}\right) = -\frac{3}{5}\frac{Pl^3}{EJ}$$

$$6\varphi_B = \left(\frac{1}{2} - \frac{3}{5}\right)\frac{Pl^3}{EJ} \qquad 6\varphi_B = \frac{5-6}{10}\frac{Pl^3}{EJ}$$

$$\boxed{\varphi_B = -\frac{1}{60}\frac{Pl^3}{EJ}}$$

$$6\cdot\left(-\frac{1}{60}\frac{Pl^3}{EJ}\right) + 6\left(\frac{1}{12}\frac{Pl^3}{EJ}\right) - 24\left(\frac{\eta}{l}\right) = -\frac{7}{5}\frac{Pl^3}{EJ}$$

$$-\frac{1}{10}\frac{Pl^4}{EJ} + \frac{1}{2}\frac{Pl^4}{EJ} - 24\eta = -\frac{7}{5}\frac{Pl^4}{EJ}$$

$$+24\eta = -\left(\frac{1}{10} - \frac{1}{2} - \frac{7}{5}\right)\frac{Pl^4}{EJ}$$

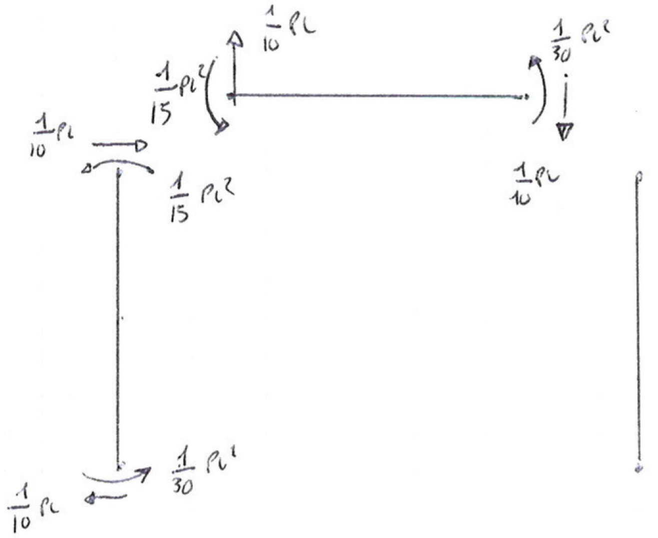
$$\eta = \frac{1}{24}\cdot\left[-\left(\frac{1-5-14}{10}\right)\frac{Pl^4}{EJ}\right] = \frac{1}{24}\cdot\frac{18}{10}\frac{Pl^4}{EJ}$$

$$\boxed{\eta = \frac{3}{40}\frac{Pl^4}{EJ}}$$

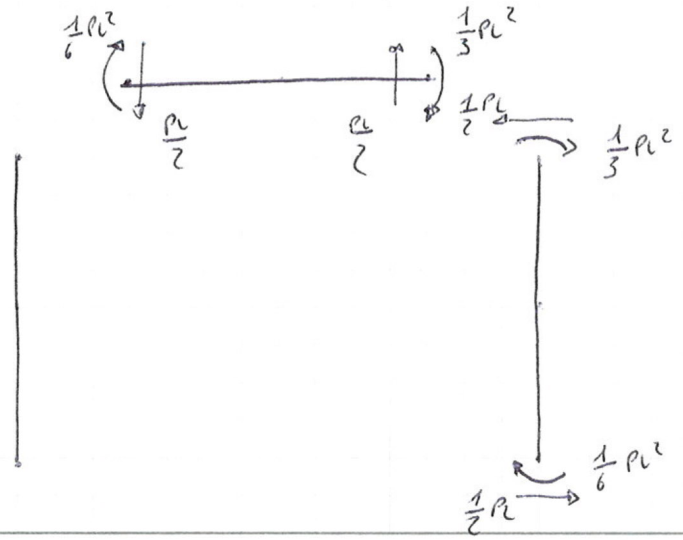
APPLICANDO IL PRINCIPIO DI SOVR. EFFETTI

(8)

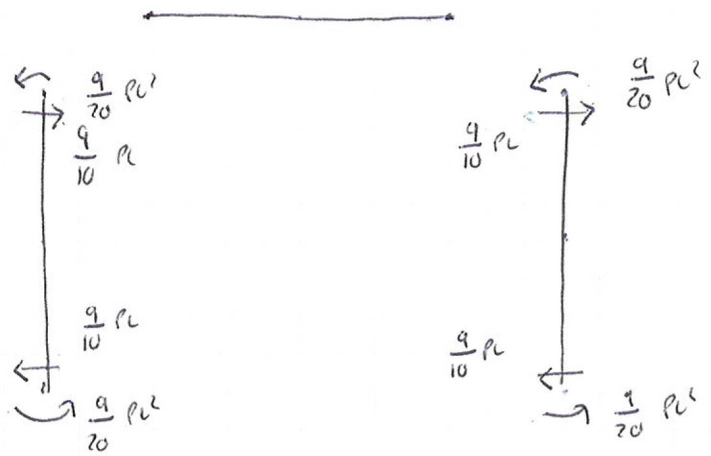
(*) $f_B = -\frac{1}{60} \frac{PL^3}{EJ}$



(*) $f_C = \frac{1}{12} \frac{PL^3}{EJ}$

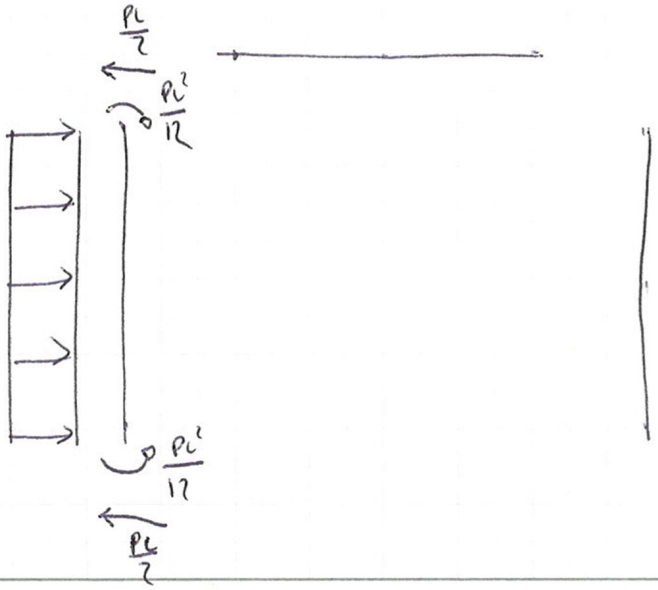


(*) $\eta = + \frac{3}{40} \frac{PL^4}{EJ}$ (9)

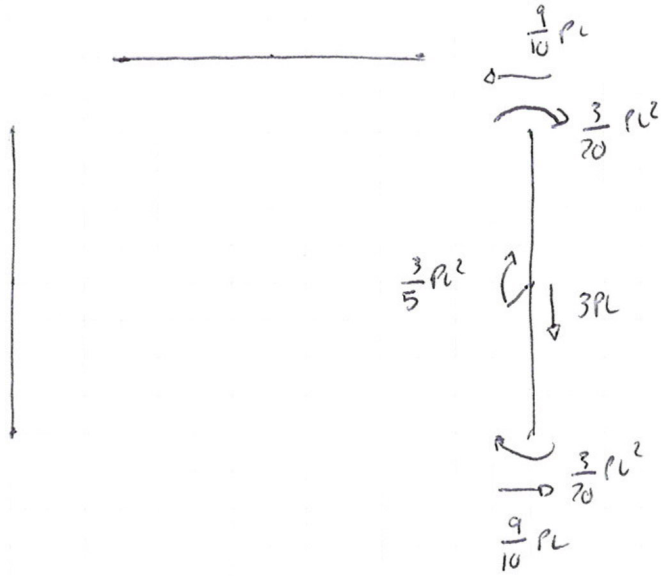


$\frac{3}{40}$, $\frac{63}{20}$

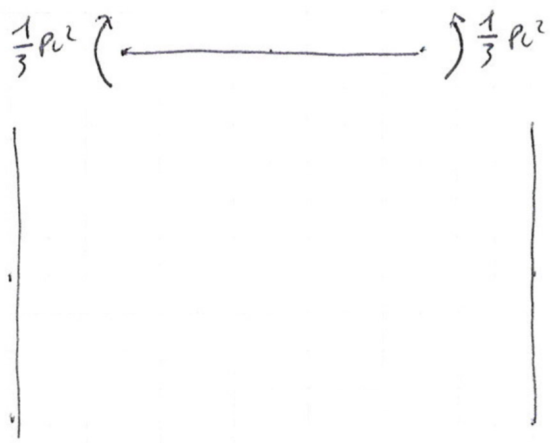
(*) $p \neq 0$



⊛ $Q \neq 0$

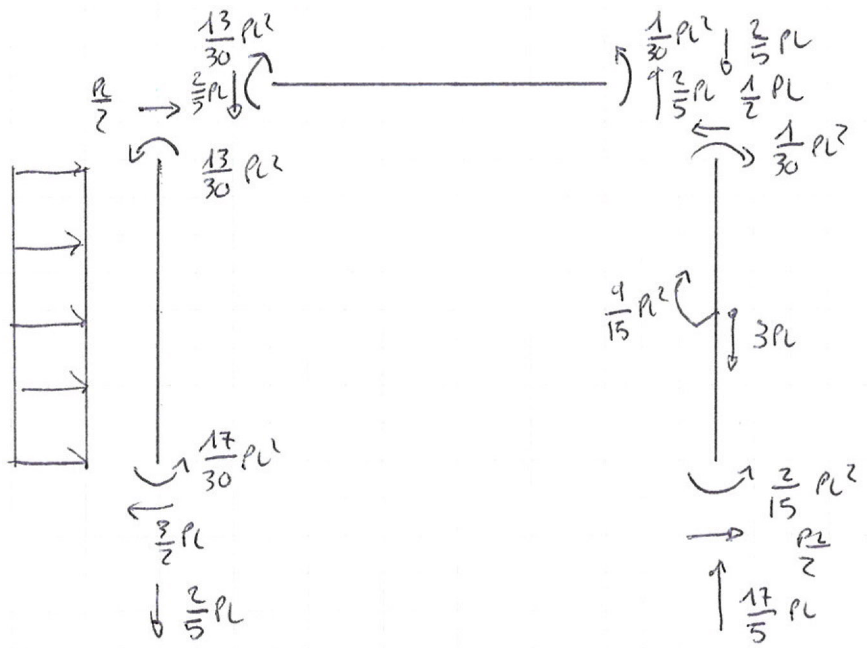


⊛ $\Delta T \neq 0$



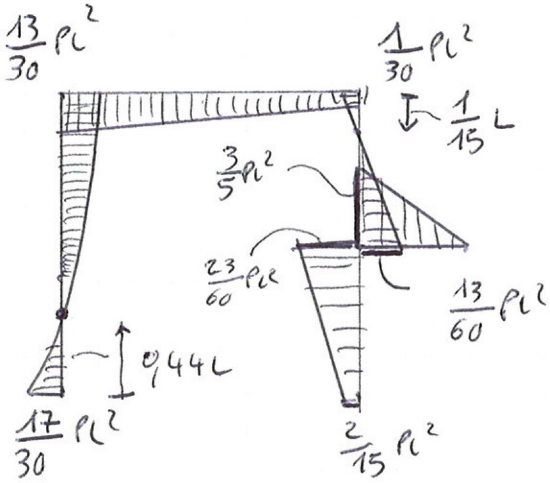
TOTALE

(11)



13/30 pl²
pl/2
2/5 pl
13/30 pl²
17/30 pl²
3/2 pl
2/5 pl
1/30 pl²
2/5 pl
2/5 pl
1/2 pl
1/30 pl²
4/15 pl²
3pl
2/15 pl²
pl/2
17/5 pl

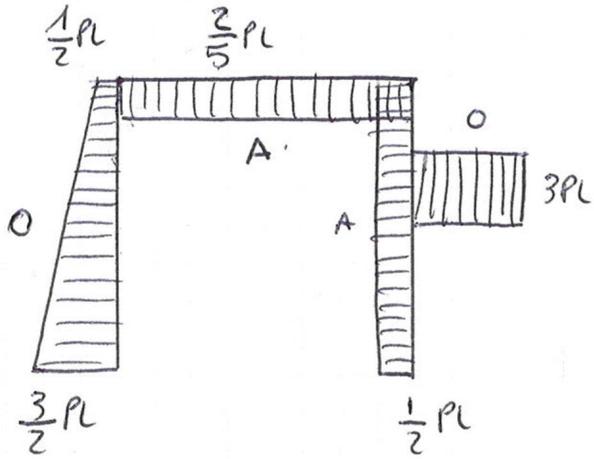
M



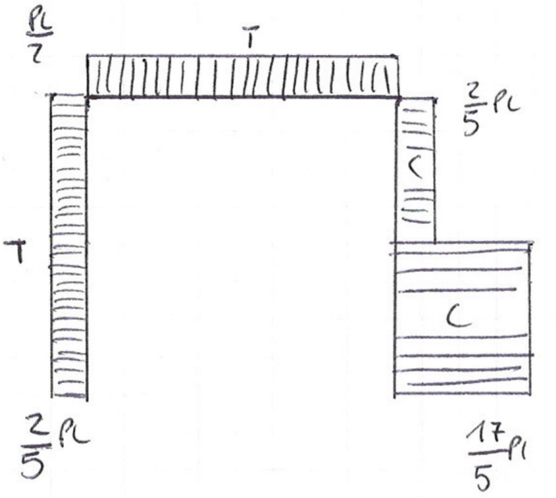
$$M_{AB}(x) = \frac{17}{30}pL^2 - \frac{3}{2}pLx - \frac{px^2}{2} = 0 \rightarrow x^2 - 3xL + \frac{17}{15}L^2 = 0$$

$$x_{1,2} = \frac{+3L \pm \sqrt{9L^2 - \frac{68}{15}L^2}}{2} = \begin{cases} 3{,}17\,L \\ 0{,}44\,L \end{cases}$$

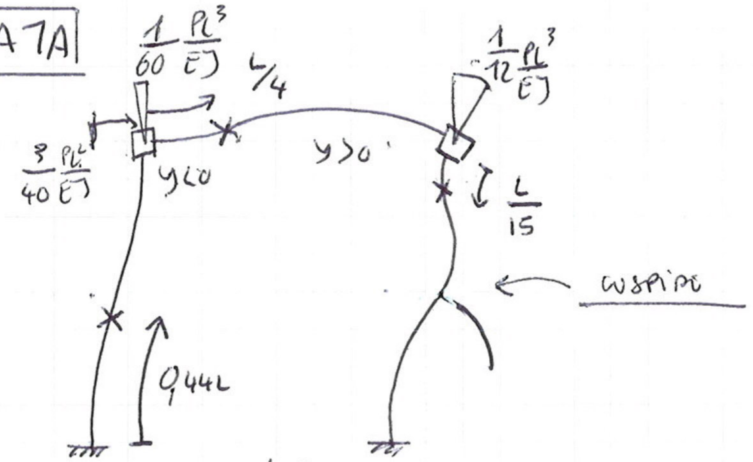
V



(N) (13)



DEFORMATA

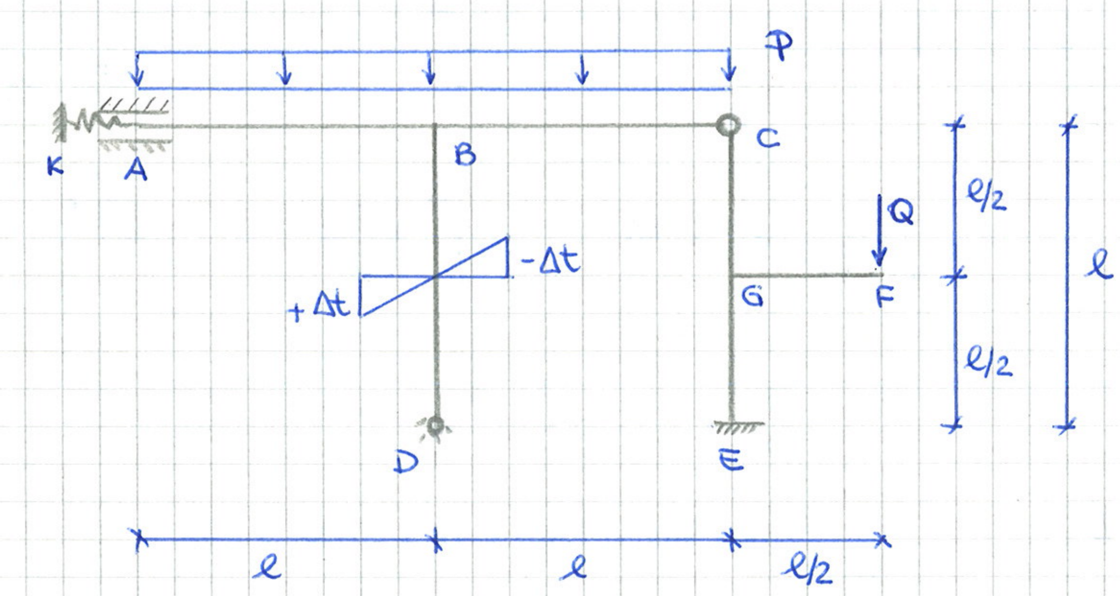


$$y''_{BC}(x) = -\frac{M}{EJ} + \frac{2\alpha\Delta T}{t}$$

$$M(x) = \frac{13}{30}PL^2 - \frac{2}{5}PLx \qquad \frac{2\alpha\Delta T}{t} = \frac{1}{3}\frac{PL^2}{EJ}$$

$$y''(x) = \frac{2}{5}\frac{PL}{EJ}x - \frac{13}{30}\frac{PL^2}{EJ} + \frac{1}{3}\frac{PL^2}{EJ} \qquad y''>0 \text{ PER } x > \frac{L}{4}$$

TEMA D'ESAME 18/04/2011



$K = 3\frac{EJ}{\ell^3}$ $EJ = \text{cost.}$

$Q = 10\,p\ell$ $EA \to \infty$

$\frac{\alpha \Delta t}{t} = \frac{13}{24}\frac{p\ell^2}{EJ}$

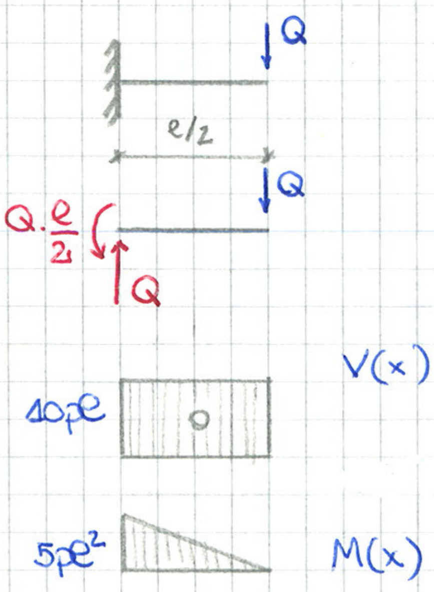
- Analisi cinematica

GDL $3+3=6$

GDV $2+4+2+2+2=12$

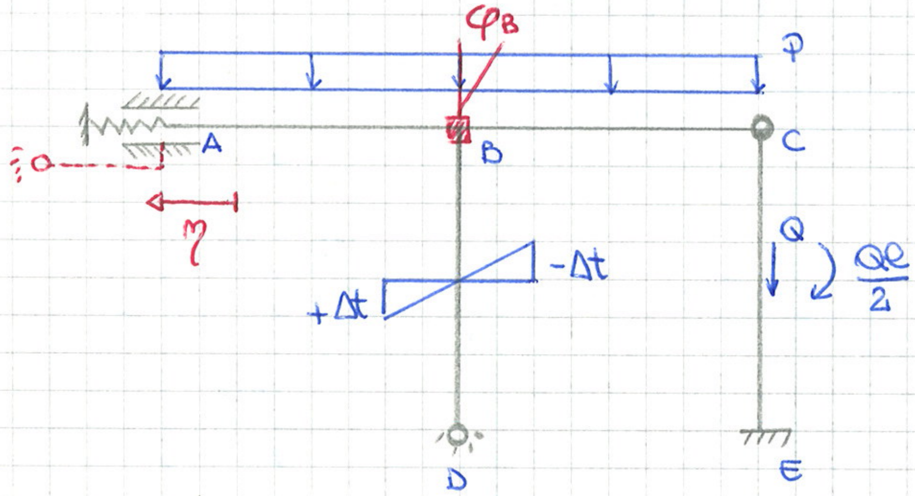
Struttura a nodi spostabili.

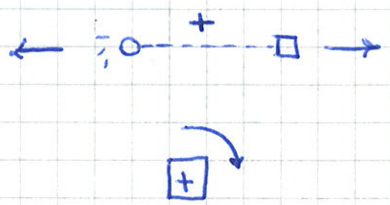
- Appendice isostatica – asta FG



$Q = 10\,p\ell$

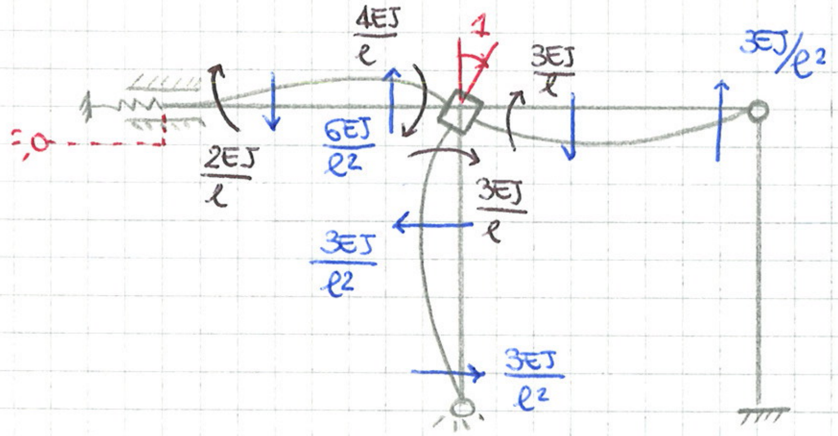
- Sistema risolvente / convenzione di segno

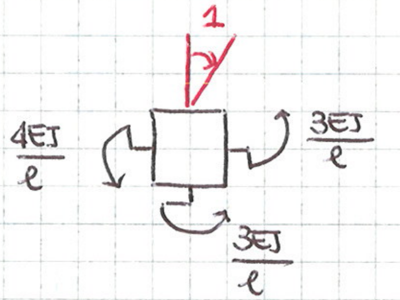


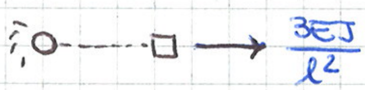


$$\begin{cases} \Sigma m_B = \emptyset \\ \Sigma F\ biella = \emptyset \end{cases} \qquad \begin{cases} m_{BB}\cdot\varphi_B + m_{B\eta}\cdot\eta + m_{B0} = \emptyset \\ r_{AB}\cdot\varphi_B + r_{A\eta}\cdot\eta + r_{A0} = \emptyset \end{cases}$$

① $\varphi_B = 1 \qquad \eta = p = \Delta t = \emptyset$



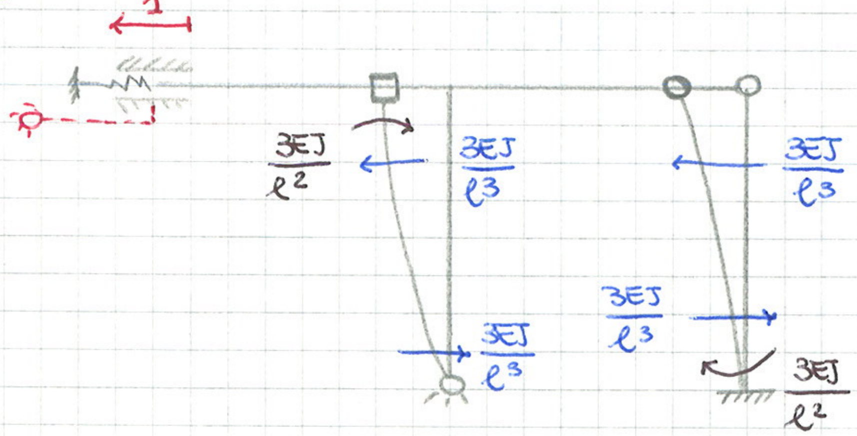


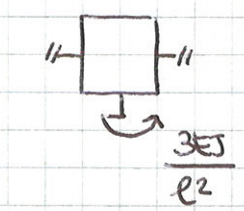


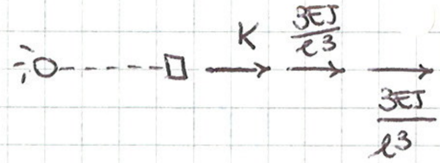
- $m_{BB} = -\frac{3EJ}{l} - \frac{3EJ}{l} - \frac{4EJ}{l} = -10\frac{EJ}{l}$

- $r_{AB} = \frac{3EJ}{l^2}$

② $\eta = 1 \qquad \varphi_B = p = \Delta t = \emptyset$



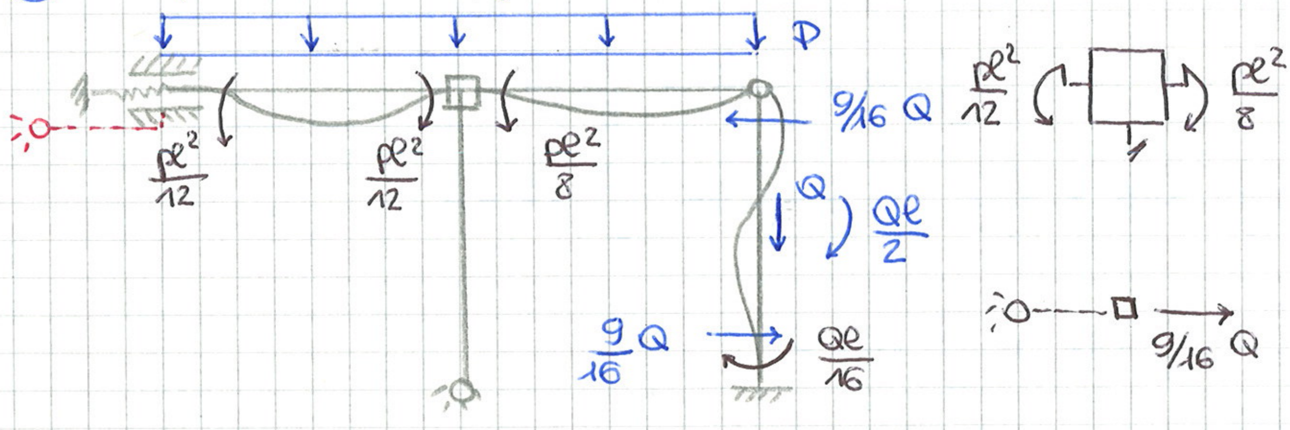




$$m_{B\eta} = -\frac{3EJ}{\ell^2}$$

$$R_{A\eta} = K + \frac{3EJ}{\ell^3} + \frac{3EJ}{\ell^3} = \frac{9EJ}{\ell^3}$$

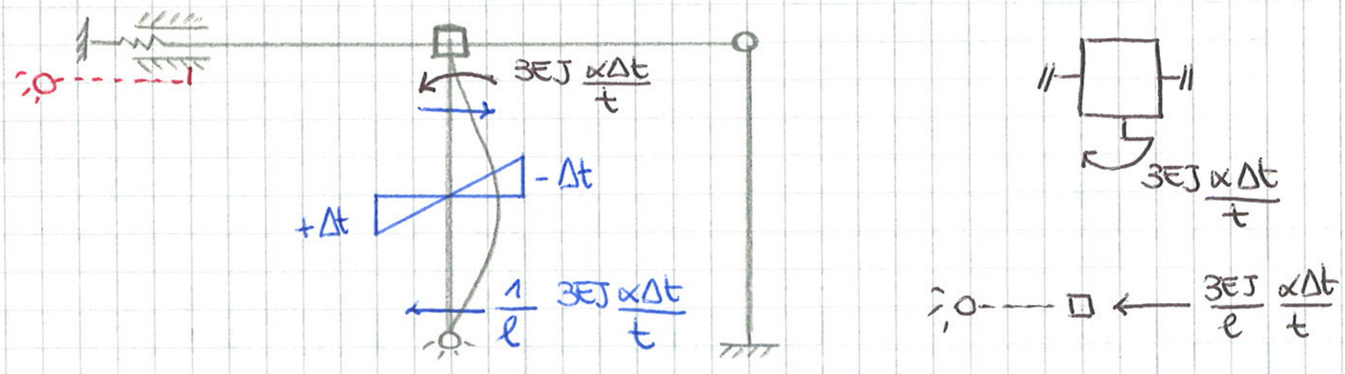
③ $p \neq 0$ $\varphi_B = \eta = \Delta t = 0$



$$m_{B0} = \frac{p\ell^2}{8} - \frac{p\ell^2}{12} = \frac{p\ell^2}{24}$$

$$R_{A0} = \frac{9}{16}Q = \frac{9}{16}(10p\ell) = \frac{45}{8}p\ell$$

④ $\Delta t \neq 0$ $\varphi_B = \eta = p = 0$



$$m_{B0} = 3EJ\,\frac{\alpha\Delta t}{t} = 3EJ\cdot\frac{13}{24}\frac{p\ell^2}{EJ} = \frac{13}{8}p\ell^2$$

$$R_{A0} = -\frac{3EJ}{\ell}\,\frac{\alpha\Delta t}{t} = -\frac{3EJ}{\ell}\cdot\frac{13}{24}\frac{p\ell^2}{EJ} = -\frac{13}{8}p\ell$$

• Sistema risolvente

$$\begin{cases} \varphi_B \left(-10\frac{EJ}{\ell}\right) - \frac{3EJ}{\ell^2}\eta + \frac{1}{24}p\ell^2 + \frac{13}{8}p\ell^2 = 0 \quad \left(\cdot \frac{3}{\ell}\right) \\ \varphi_B \cdot \frac{3EJ}{\ell^2} + \frac{9EJ}{\ell^3}\eta + \frac{45}{8}p\ell - \frac{13}{8}p\ell = 0 \end{cases}$$

$$\begin{cases} -30\frac{EJ}{\ell^2}\varphi_B - \frac{9EJ}{\ell^3}\eta + \frac{1}{8}p\ell + \frac{39}{8}p\ell = 0 \\ \frac{3EJ}{\ell^2}\varphi_B + \frac{9EJ}{\ell^3}\eta + \frac{45}{8}p\ell - \frac{13}{8}p\ell = 0 \end{cases}$$

$$-27\frac{EJ}{\ell^2}\varphi_B + \frac{72}{8}p\ell = 0$$

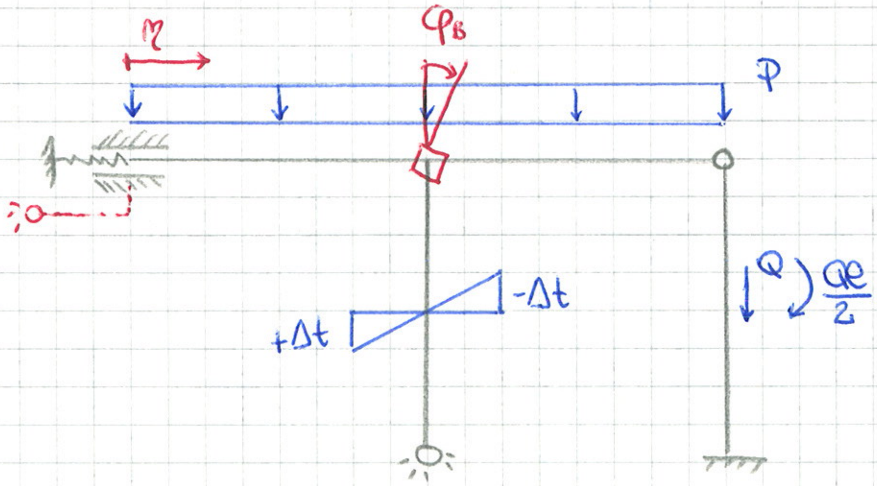
$$\boxed{\varphi_B = \frac{72}{8}\frac{p\ell^3}{EJ}\cdot\frac{1}{27} = \frac{1}{3}\frac{p\ell^3}{EJ}}$$

$$-10\frac{EJ}{\ell}\left(\frac{1}{3}\frac{p\ell^3}{EJ}\right) - \frac{3EJ}{\ell^2}\eta + \frac{40}{24}p\ell^2 = 0$$

$$\frac{3EJ}{\ell^2}\eta = -\frac{40}{24}p\ell^2$$

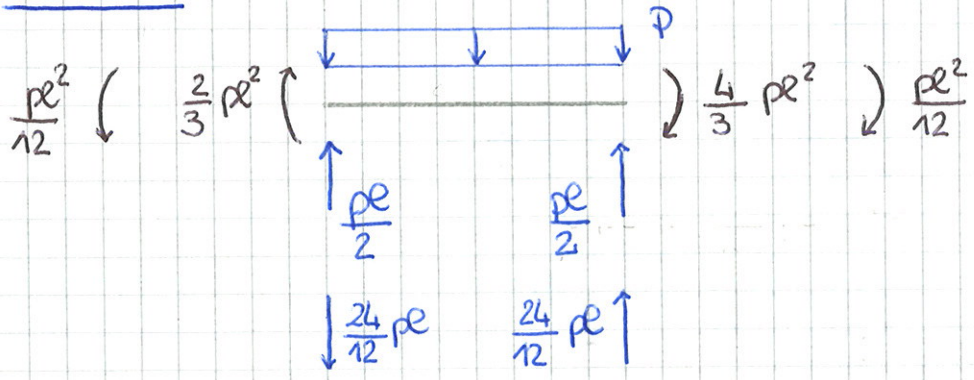
$$\boxed{\eta = -\frac{5}{3}p\ell^2\cdot\frac{1}{3}\frac{\ell^2}{EJ} = -\frac{5}{9}\frac{p\ell^4}{EJ}}$$

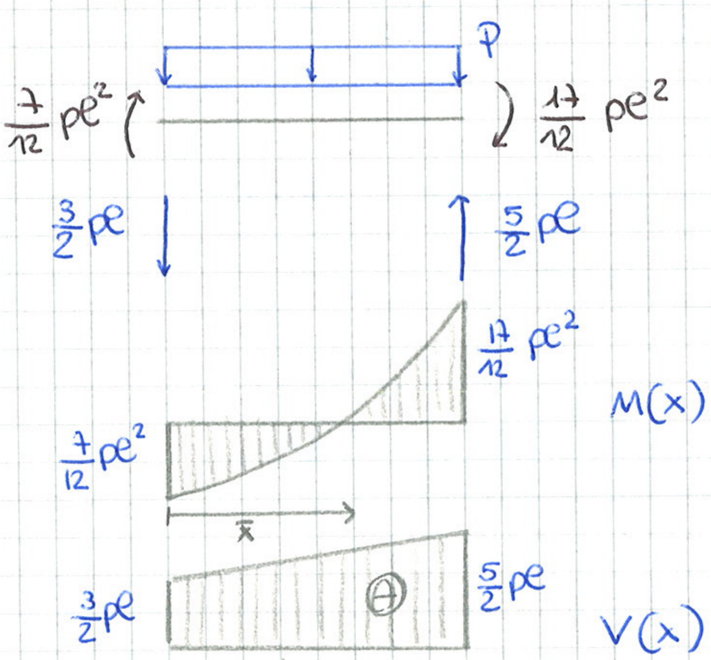
$$\begin{cases} \varphi_B = \frac{1}{3}\frac{p\ell^3}{EJ} \\ \eta = -\frac{5}{9}\frac{p\ell^4}{EJ} \end{cases}$$



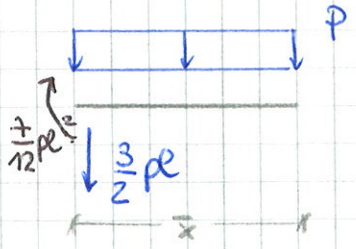
. Determinazione delle azioni interne

Asta AB





$\bar{x} = ?$



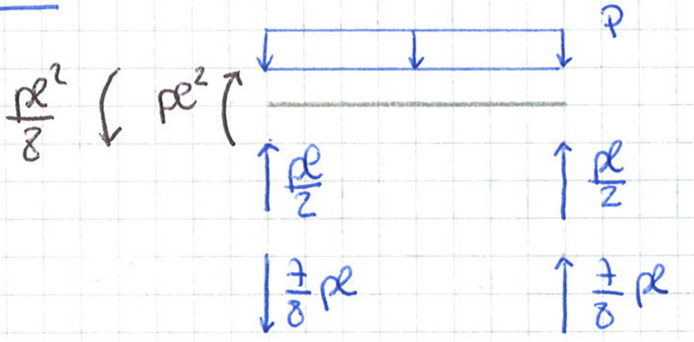
$$\frac{7}{12} p\ell^2 - \frac{3}{2} p\ell \bar{x} - \frac{p\bar{x}^2}{2} = 0$$

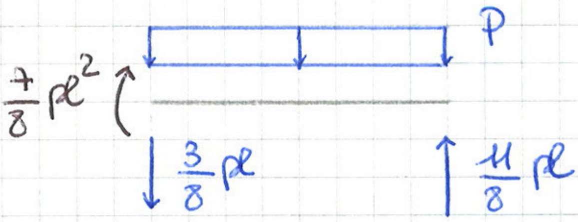
$$x_{1,2} = \frac{3/2 \pm \sqrt{9/4 + 7/6}}{(-1)}$$

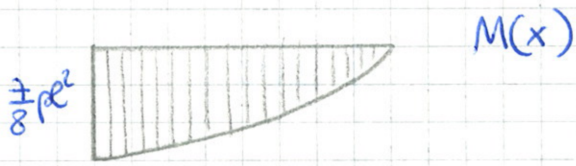
$$x_{1,2} = \begin{cases} -3{,}34 & \text{non accettabile} \\ 0{,}348 & \end{cases}$$

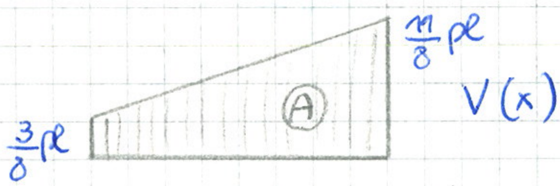
$\bar{x} = 0{,}348\,\ell$

Asta BC

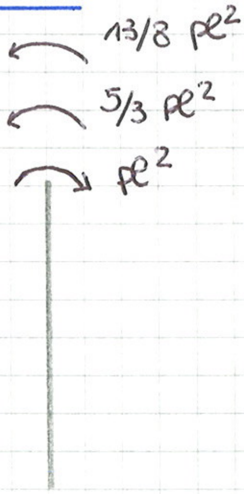


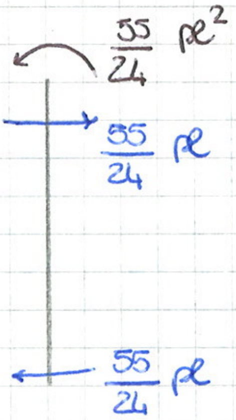


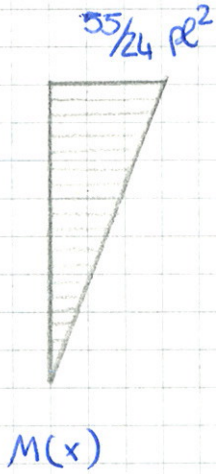


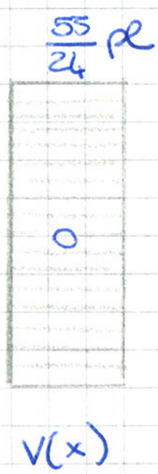


Asta BD

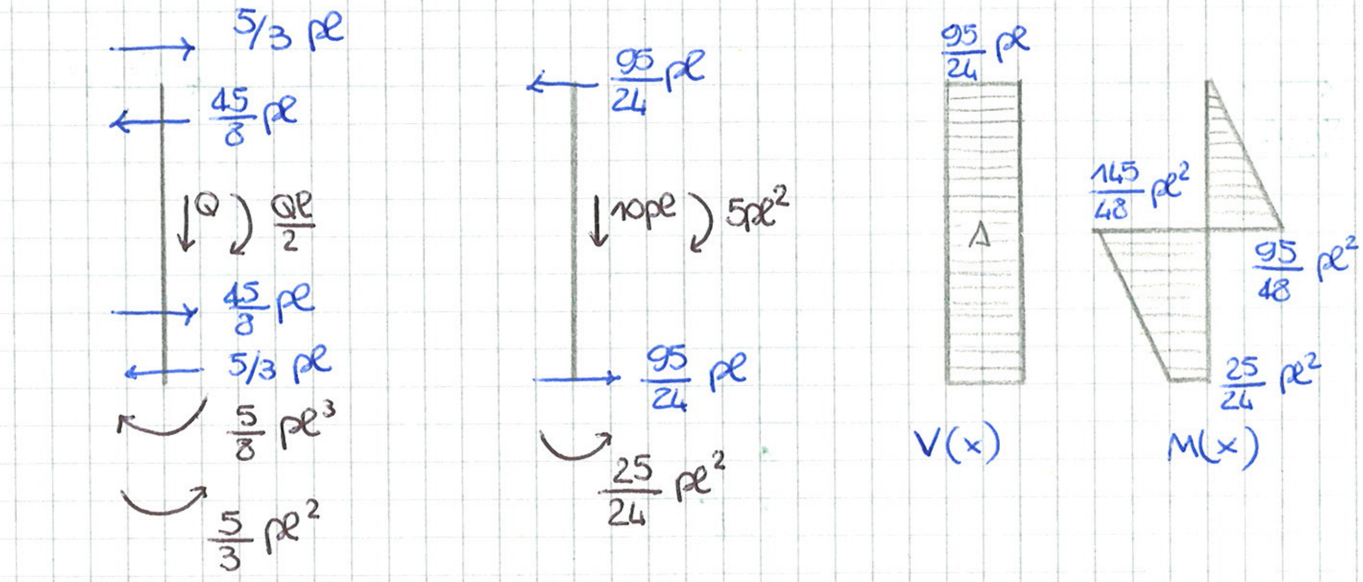




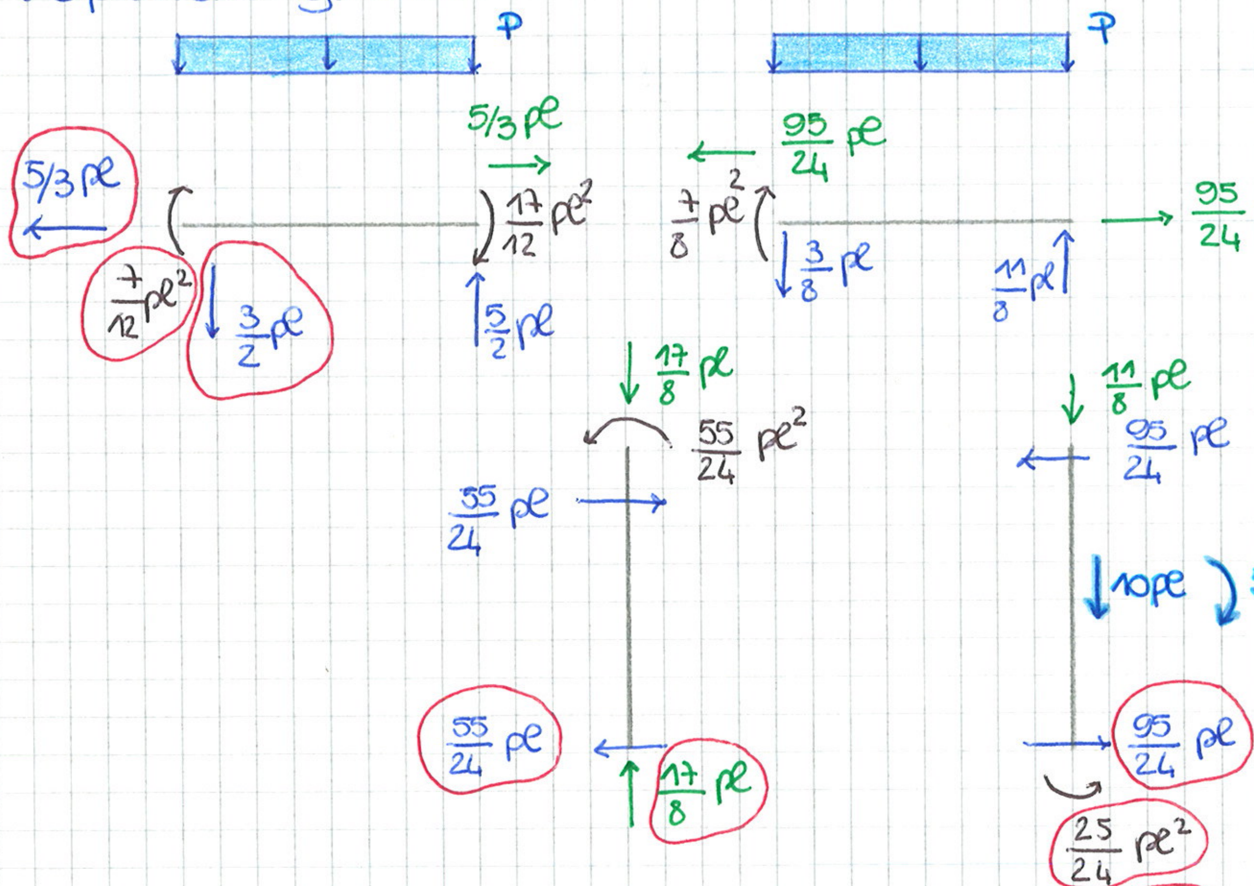




Asta CE



. Equilibrio globale



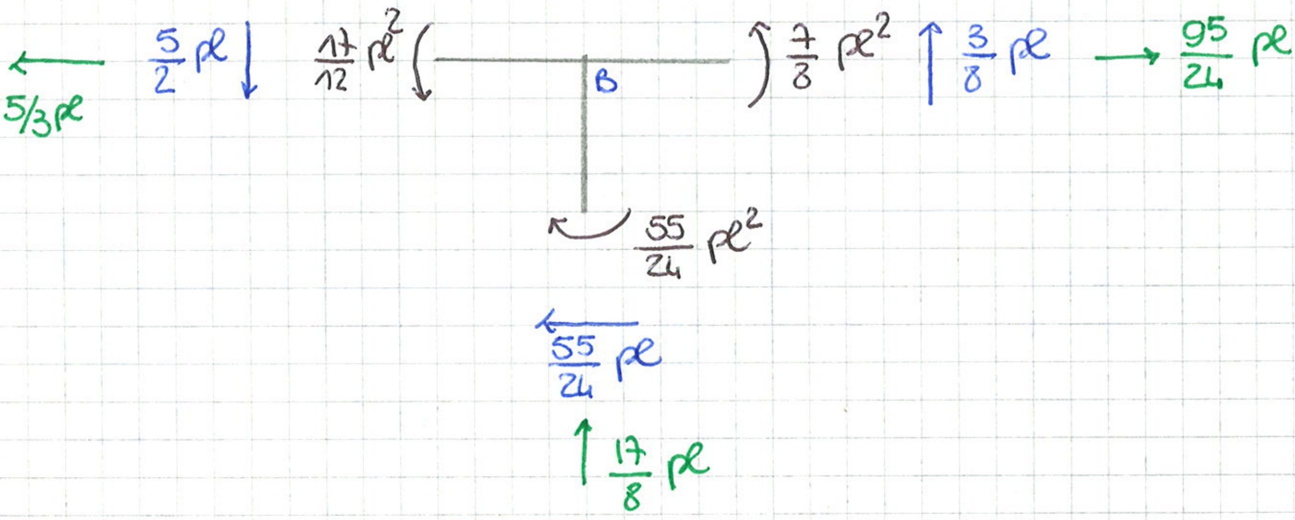
$\Sigma V = 0 \rightarrow -\frac{3}{2}p\ell + \frac{17}{8}p\ell + \frac{91}{8}p\ell = 2p\ell + 10p\ell$ ok

$\Sigma H = 0 \rightarrow -\frac{5}{3}p\ell - \frac{55}{24}p\ell + \frac{95}{24}p\ell = 0$ ok

$\Sigma M(D) = 0 \rightarrow 10p\ell \cdot \ell + 5p\ell^2 - \frac{25}{24}p\ell^2 - \frac{91}{8}p\ell^2 - \frac{5}{3}p\ell^2 + \frac{7}{12}p\ell^2 - \frac{3}{2}p\ell^2 = 0$

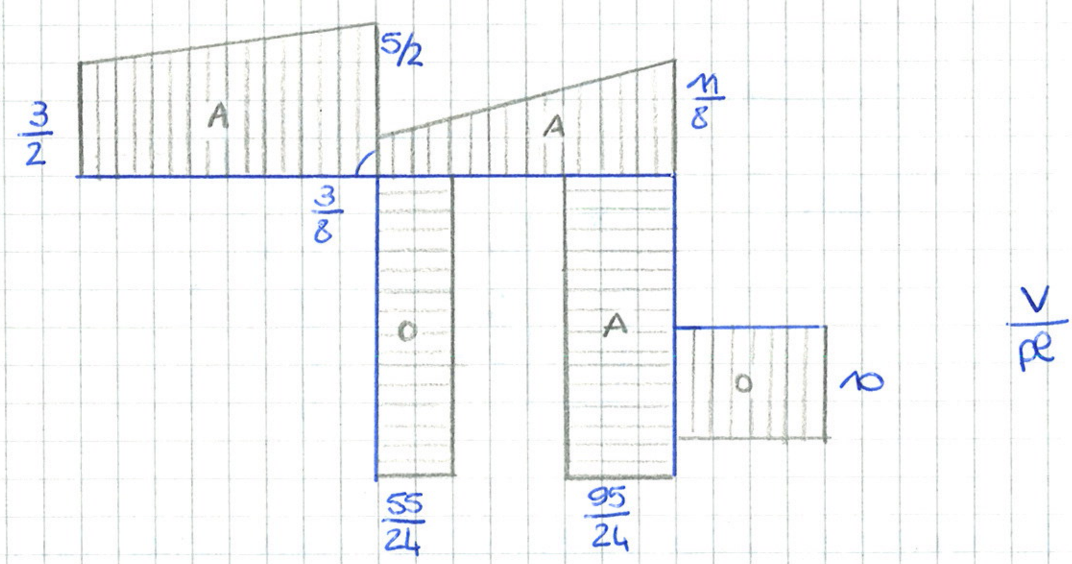
ok

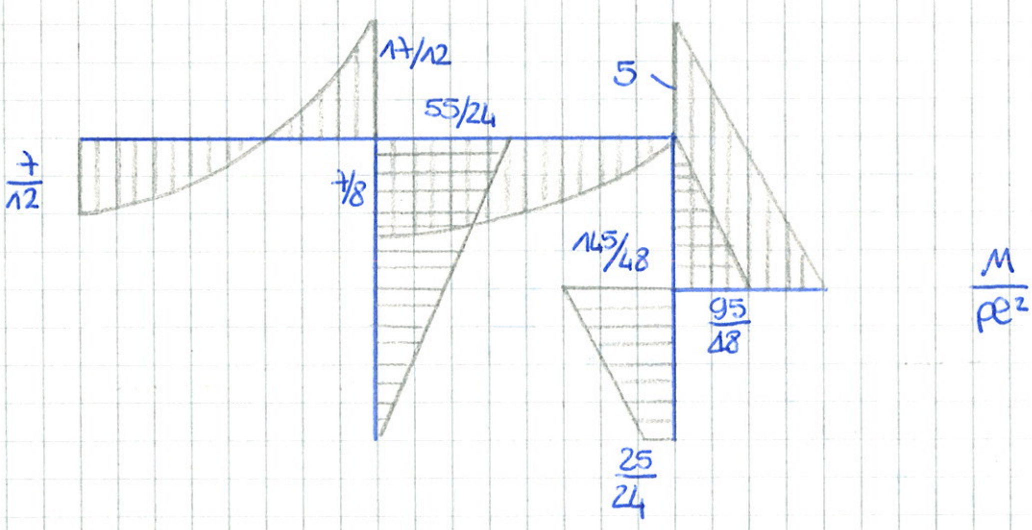
- Equilibrio al nodo B

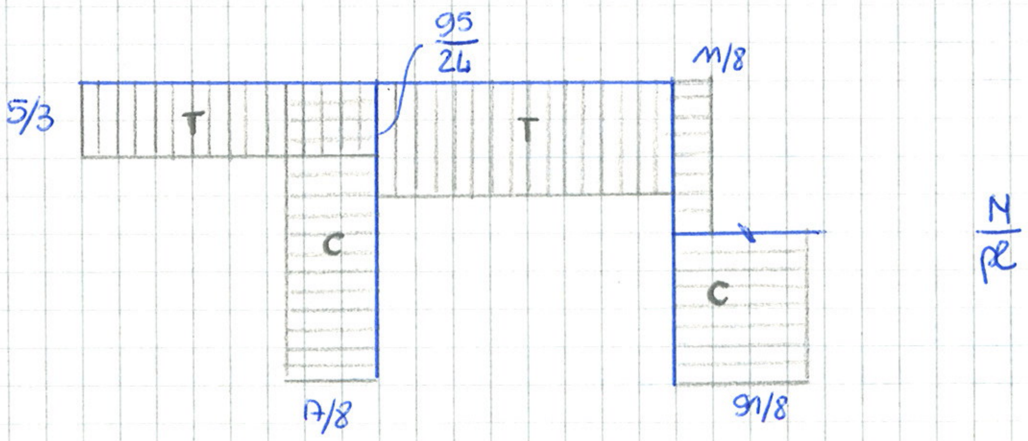


$$\begin{cases} \Sigma H = 0 \rightarrow -\frac{5}{3}p\ell + \frac{95}{24}p\ell - \frac{55}{24}p\ell = \frac{-40+95-55}{24}p\ell = \emptyset \\ \Sigma V = 0 \rightarrow -\frac{5}{2}p\ell + \frac{17}{8}p\ell + \frac{3}{8}p\ell = \frac{-20+17+3}{8}p\ell = \emptyset \\ \Sigma M(B) = 0 \rightarrow \frac{17}{12}p\ell^2 - \frac{55}{24}p\ell^2 + \frac{7}{8}p\ell^2 = \frac{34-55+21}{24}p\ell^2 = \emptyset \end{cases}$$

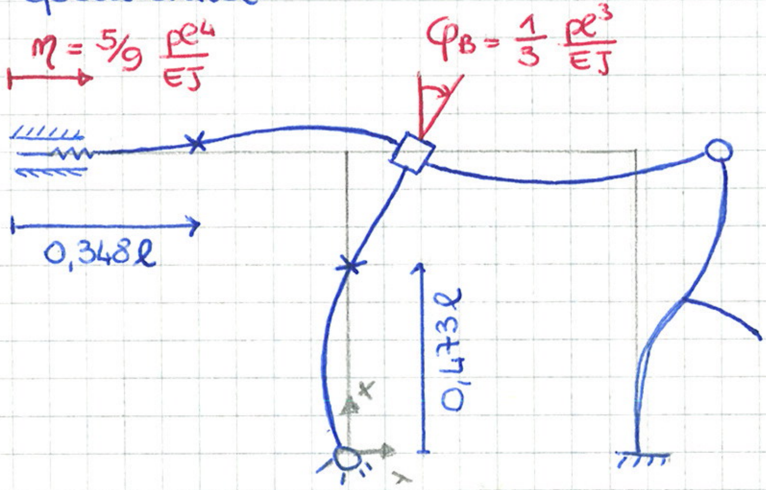
. Tracciamento delle azioni interne



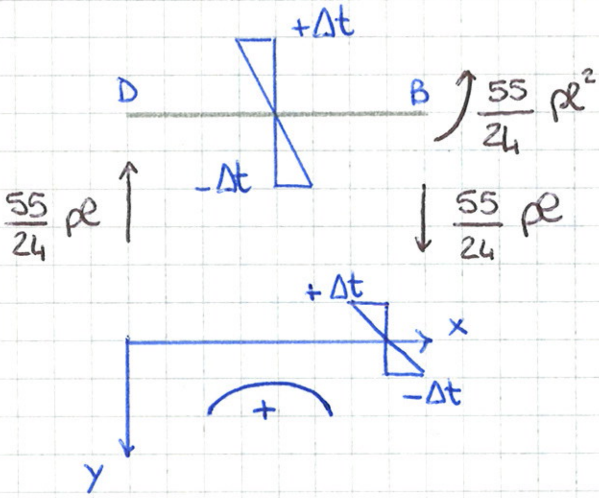




• Deformata qualitativa



- Determinazione della deformata sull'asta BD:



$$\frac{\alpha \Delta t}{t} = \frac{13}{24} \frac{p\ell^2}{EJ}$$

$$M(x) = \frac{55}{24} p\ell \cdot x$$

$$\boxed{y''(x) = -\frac{M(x)}{EJ} + \frac{2\alpha \Delta t}{t}}$$

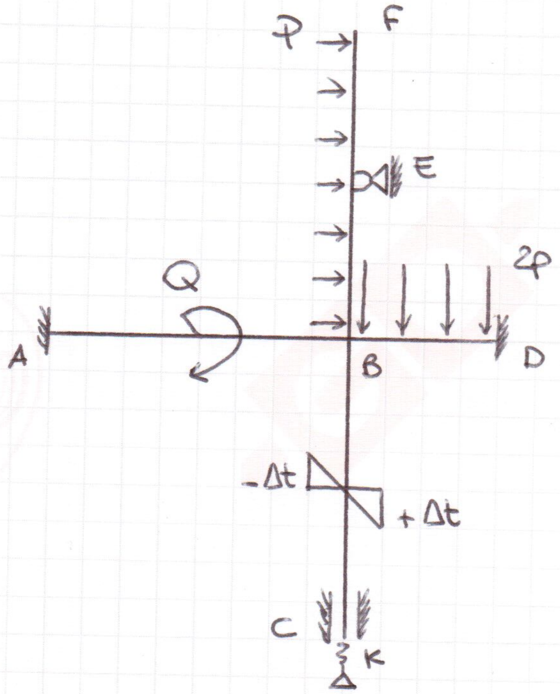
$$y''(x) = -\frac{55}{24} p\ell \frac{x}{EJ} + \frac{13}{12} \frac{p\ell^2}{EJ}$$

$$y''(x) > 0 \longrightarrow x < \frac{26}{55} \ell = 0,473\, \ell$$

se $x < 0,473\,\ell$ concavità verso il basso

se $x > 0,473\,\ell$ concavità verso l'alto

T.E. 4/07/2011

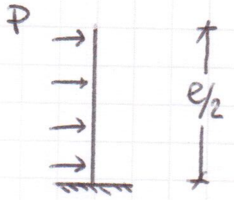


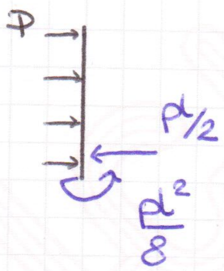
$$\frac{\alpha \Delta t}{h} = \frac{35}{64} \frac{pl^2}{EJ}$$

$$Q = pl^2$$

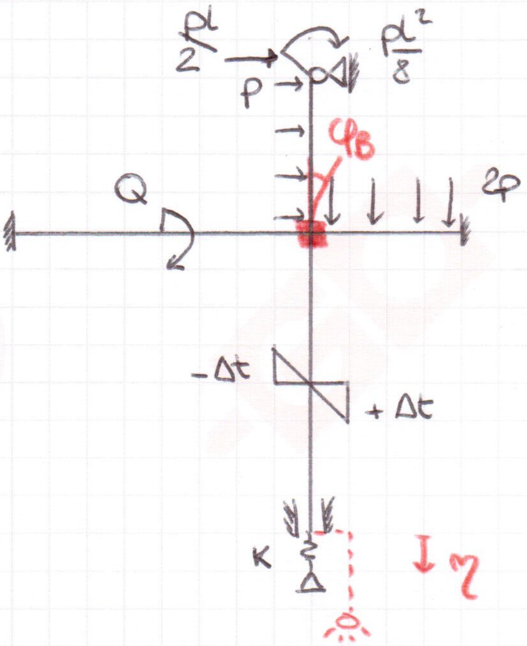
$$K = 18 \frac{EJ}{l^3}$$

- Appendice isostatica EF:



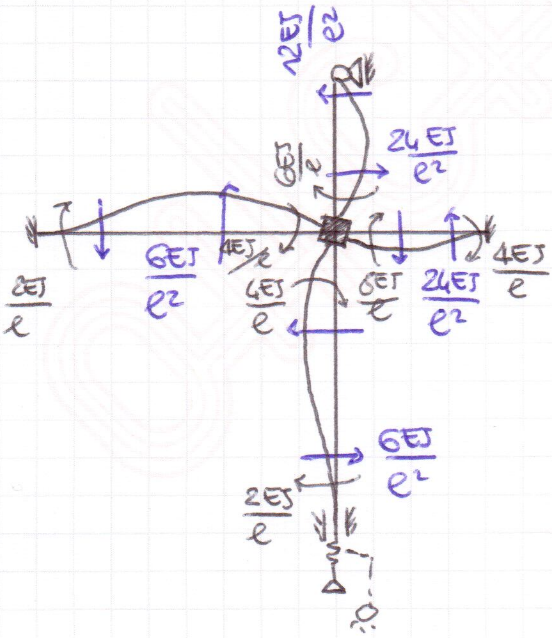


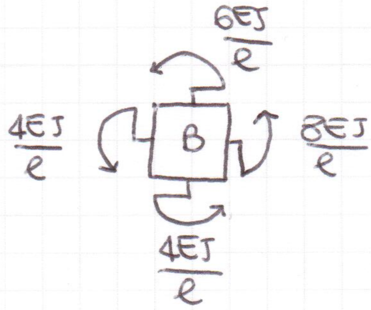
la struttura diventa:



$$\begin{cases} \varphi_B \cdot m_{BB} + \eta \cdot m_{B\eta} + m_{B0} = 0 \\ k_{CB} \cdot \varphi_B + k_{CC} \cdot \eta + k_{C0} = 0 \end{cases}$$

①

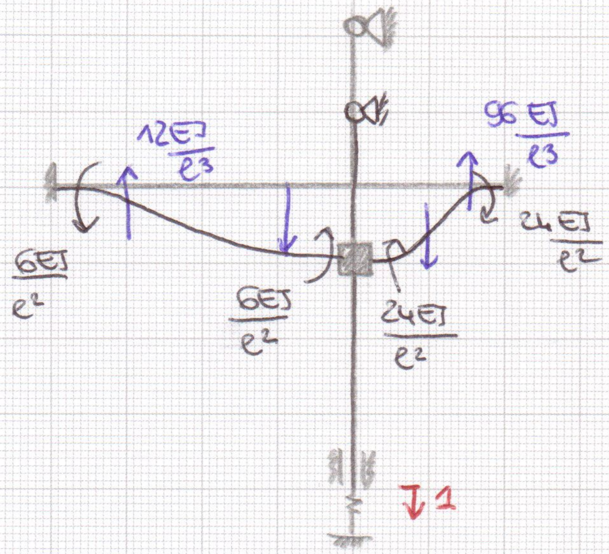


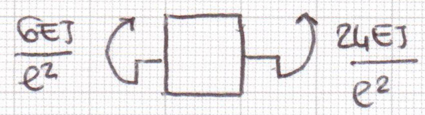


$m_{BB} = -22 \dfrac{EJ}{l}$

$k_{CB} = 18 \dfrac{EJ}{l^2}$

② $\eta \neq 0$

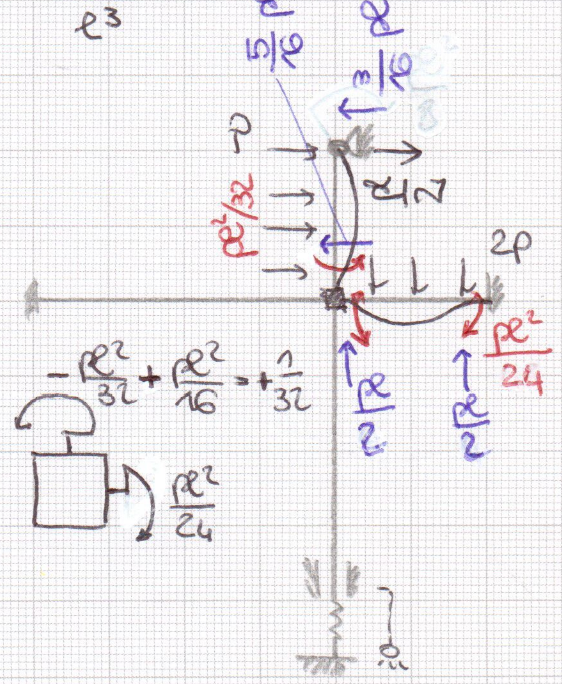


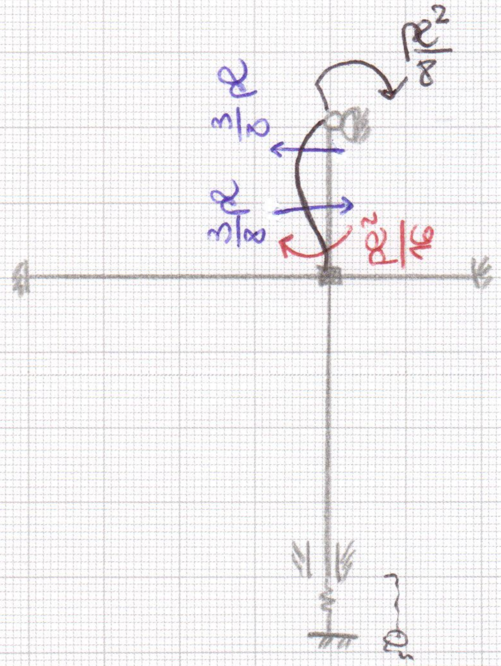


$$m_{B\eta} = -\frac{18EJ}{\ell^2}$$

$$r_{cc} = K + 108\frac{EJ}{\ell^3}$$

③ $p \neq 0$

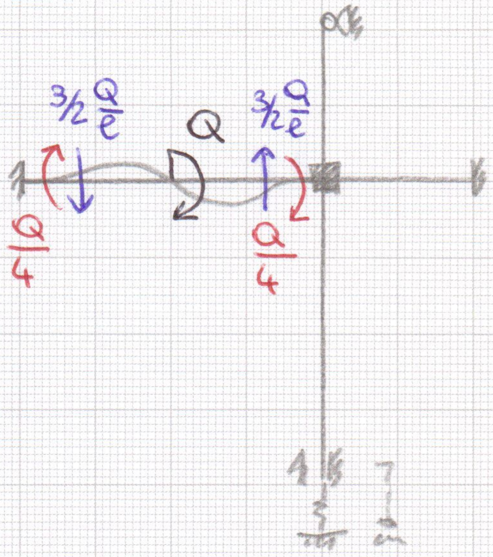


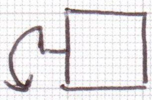


$$m_{B0} = +\frac{1}{96}\, p\ell^2$$

$$r_{c0} = -p\ell$$

④ $Q \neq 0$





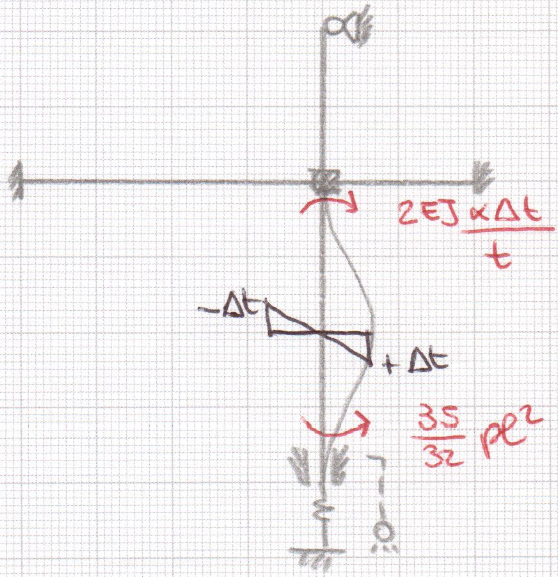
$$\frac{Q}{4} = \frac{p\ell^2}{4}$$

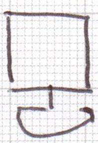
$$\frac{3}{2}\frac{Q}{\ell} = \frac{3}{2}\,p\ell$$

$$m_{BO} = -\frac{p\ell^2}{4}$$

$$h_{CO} = -\frac{3}{2}\,p\ell$$

⑤ $\Delta t \neq 0$





$$m_{BO} = -\frac{35}{32}\,p\ell^2$$

$$h_{CO} = \emptyset$$

Risoluzione del sistema:

$$\begin{cases} -22\dfrac{EJ}{\ell}\varphi_B - 18\dfrac{EJ}{\ell^2}\eta + \dfrac{1}{96}p\ell^2 - \dfrac{p\ell^2}{4} - \dfrac{35}{32}p\ell^2 = 0 \\ 18\dfrac{EJ}{\ell^2}\varphi_B + \left(k + 108\dfrac{EJ}{\ell^3}\right)\eta - \dfrac{p\ell}{2} - \dfrac{3}{2}p\ell = 0 \end{cases}$$

$$\begin{cases} -22\dfrac{EJ}{\ell}\varphi_B - 18\dfrac{EJ}{\ell^2}\eta - \dfrac{4}{3}p\ell^2 = 0 \\ 18\dfrac{EJ}{\ell^2}\varphi_B + 126\dfrac{EJ}{\ell^3}\eta - 2p\ell = 0 \end{cases}$$

$$126\frac{EJ}{\ell^3}\eta = \left(2p\ell - 18\frac{EJ}{\ell^2}\varphi_B\right)$$

$$\eta = \frac{1}{63}\frac{p\ell^4}{EJ} - \frac{9}{63}\ell\varphi_B$$

$$-22\frac{EJ}{\ell}\varphi_B - 18\frac{EJ}{\ell^2}\left(\frac{1}{63}\frac{p\ell^4}{EJ} - \frac{9}{63}\ell\varphi_B\right) - \frac{4}{3}p\ell^2 = 0$$

$$-22\frac{EJ}{\ell}\varphi_B - \frac{18}{63}p\ell^2 + \frac{162}{63}\varphi_B\frac{EJ}{\ell} - \frac{4}{3}p\ell^2 = 0$$

$$-22\frac{EJ}{\ell}\varphi_B + \frac{162}{63}\varphi_B\frac{EJ}{\ell} = \frac{4}{3}p\ell^2 + \frac{18}{63}p\ell^2$$

$$\frac{-1386+162}{63}\varphi_B\frac{EJ}{\ell} = \frac{84+18}{63}p\ell^2 \qquad \varphi_B = -\frac{102}{1224}\frac{p\ell^3}{EJ}$$

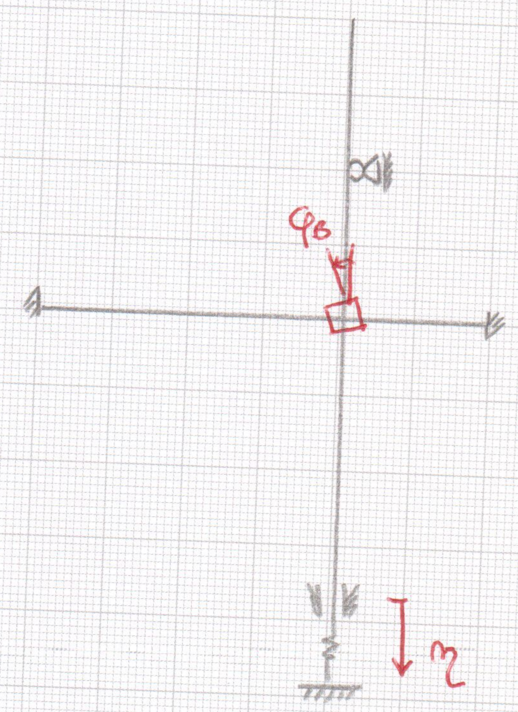
$$\boxed{\varphi_B = -\,{}^{1}\!/_{12}\; p\ell^3/EJ}$$

$$18\,\frac{EJ}{\ell^2}\left(-\frac{1}{12}\,\frac{p\ell^3}{EJ}\right)+126\,\frac{EJ}{\ell^3}\,\eta-2p\ell=0$$

$$-\frac{3}{2}\,p\ell+126\,\frac{EJ}{\ell^3}\,\eta-2p\ell=0$$

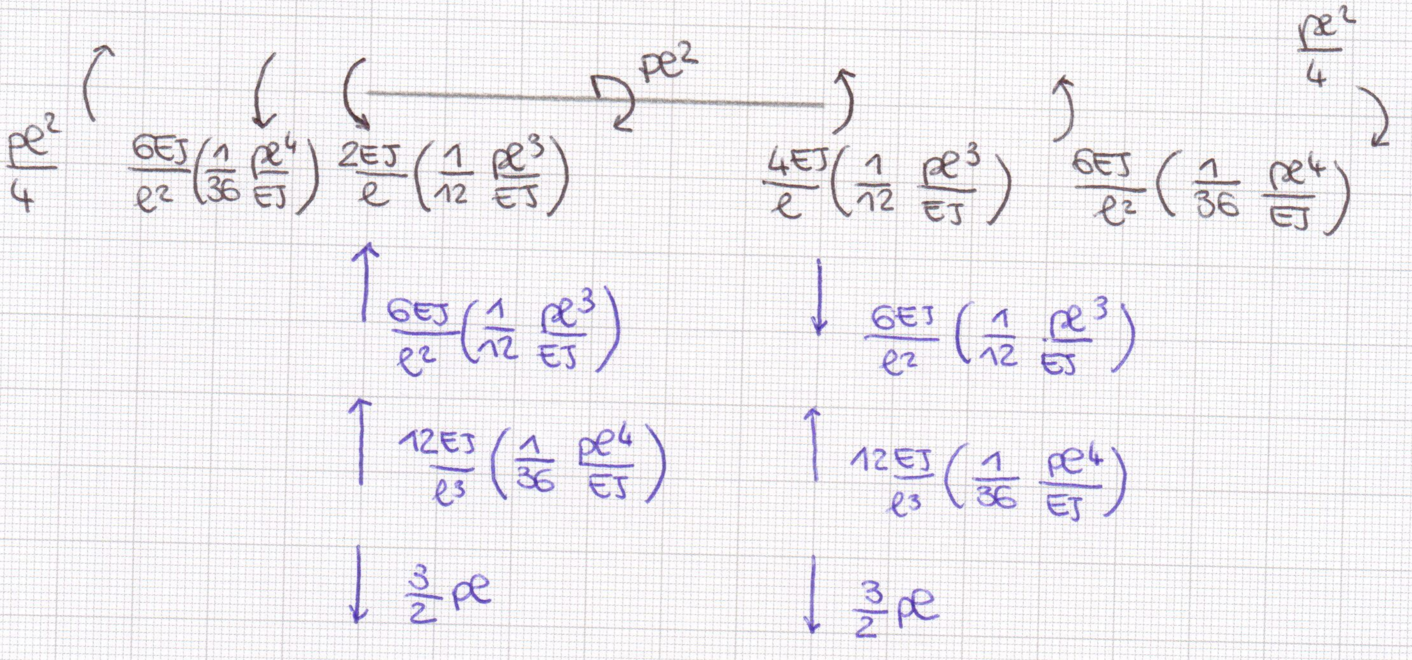
$$126\,\frac{EJ}{\ell^3}\,\eta=\frac{7}{2}\,p\ell$$

$$\boxed{\eta=\frac{1}{36}\,\frac{p\ell^4}{EJ}}$$

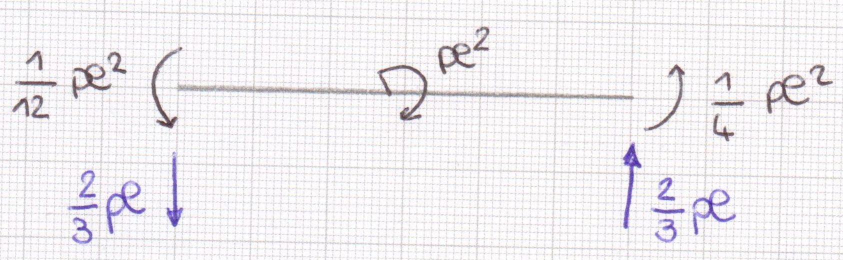


$$\begin{cases}\varphi_B=\dfrac{1}{12}\,\dfrac{p\ell^3}{EJ}\\[2ex]\eta=\dfrac{1}{36}\,\dfrac{p\ell^4}{EJ}\end{cases}$$

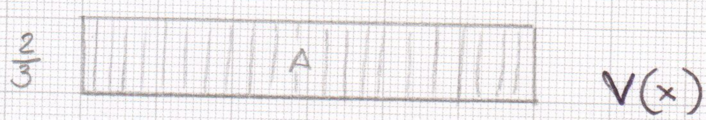
Asta AB



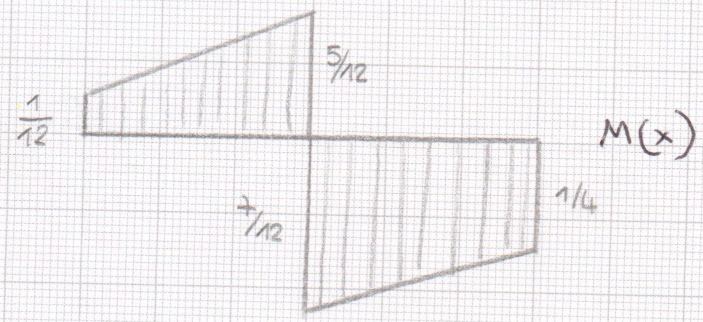
$\frac{p\ell^2}{4}$
$\frac{6EJ}{\ell^2}\left(\frac{1}{36}\frac{p\ell^4}{EJ}\right)$
$\frac{2EJ}{\ell}\left(\frac{1}{12}\frac{p\ell^3}{EJ}\right)$
$p\ell^2$
$\frac{4EJ}{\ell}\left(\frac{1}{12}\frac{p\ell^3}{EJ}\right)$
$\frac{6EJ}{\ell^2}\left(\frac{1}{36}\frac{p\ell^4}{EJ}\right)$
$\frac{p\ell^2}{4}$
$\frac{6EJ}{\ell^2}\left(\frac{1}{12}\frac{p\ell^3}{EJ}\right)$
$\frac{6EJ}{\ell^2}\left(\frac{1}{12}\frac{p\ell^3}{EJ}\right)$
$\frac{12EJ}{\ell^3}\left(\frac{1}{36}\frac{p\ell^4}{EJ}\right)$
$\frac{12EJ}{\ell^3}\left(\frac{1}{36}\frac{p\ell^4}{EJ}\right)$
$\frac{3}{2}p\ell$
$\frac{3}{2}p\ell$



$\frac{1}{12}p\ell^2$
$p\ell^2$
$\frac{1}{4}p\ell^2$
$\frac{2}{3}p\ell$
$\frac{2}{3}p\ell$

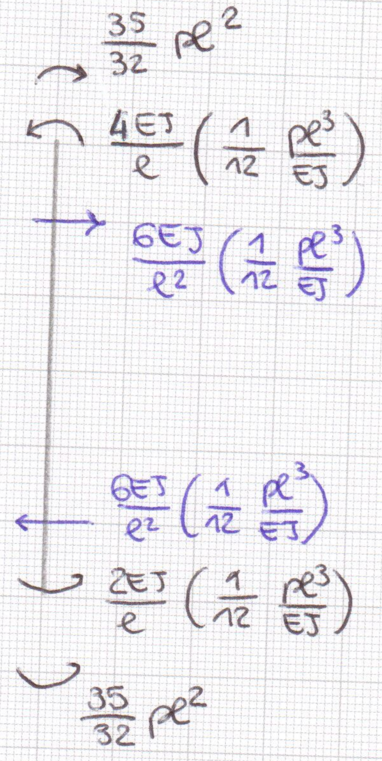


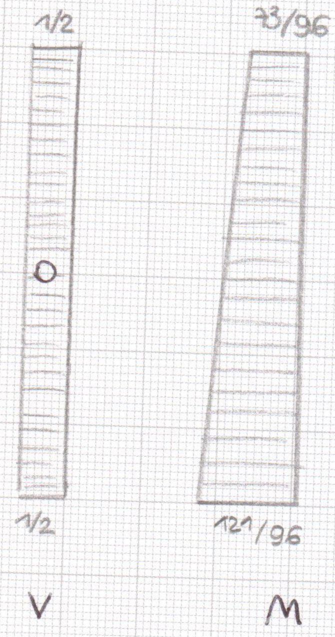
2/3
A
V(x)



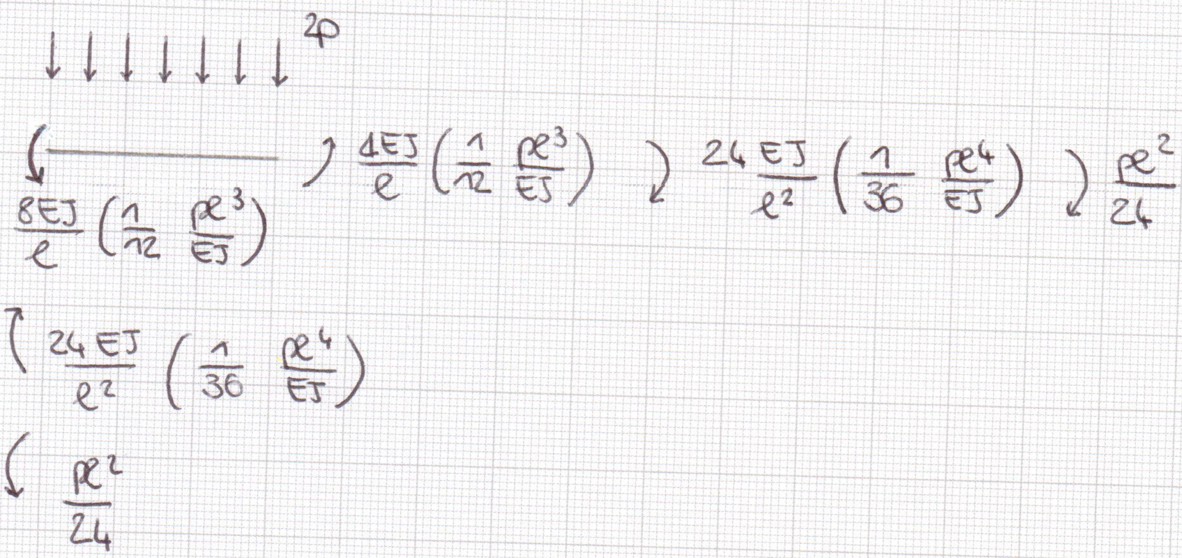
1/12
5/12
M(x)
7/12
1/4

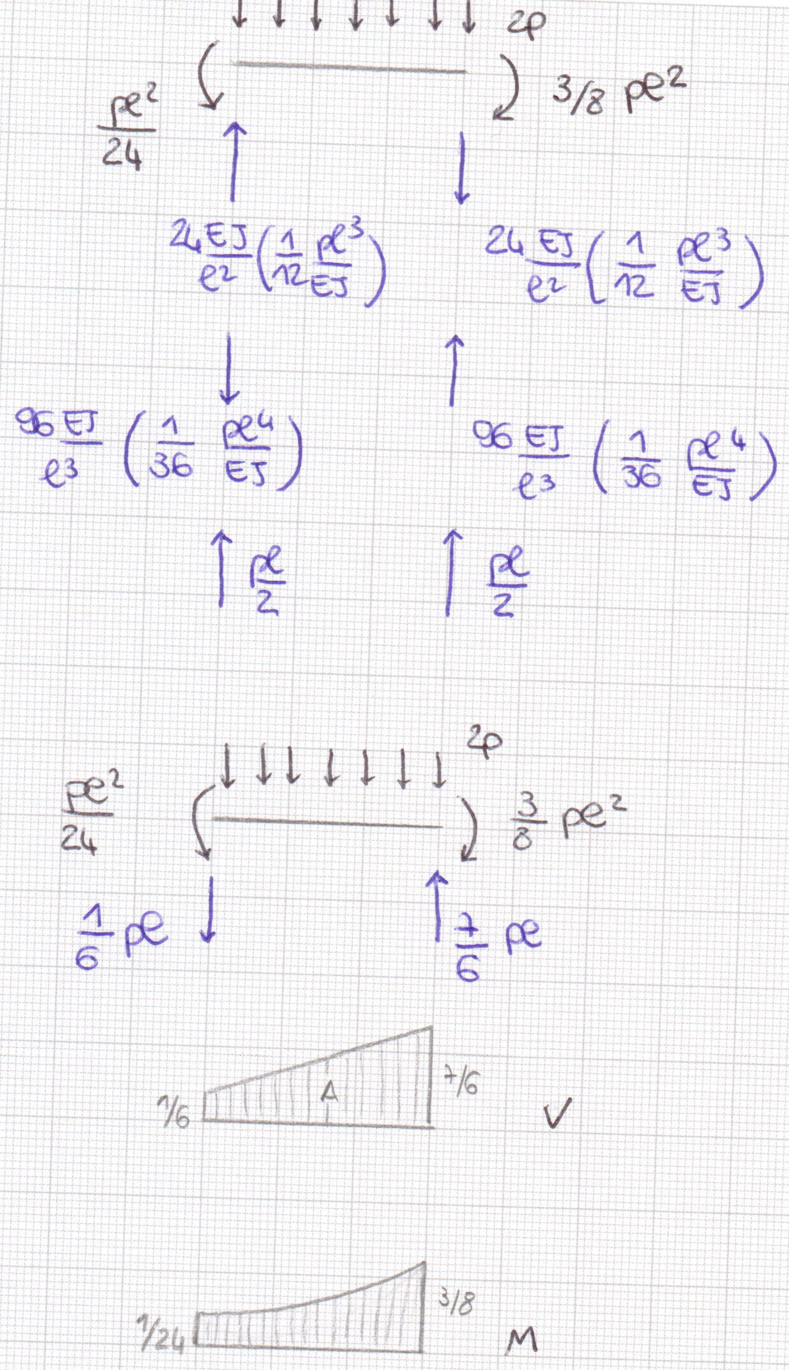
Asta BC





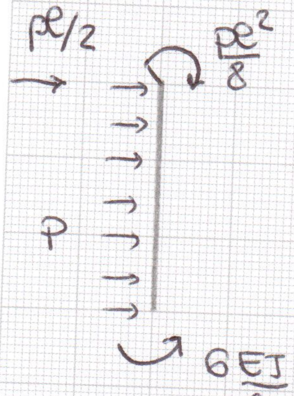
Asta BD





2p
pl²/24
3/8 pl²
24 EJ/l² (1/12 pl³/EJ)
24 EJ/l² (1/12 pl³/EJ)
96 EJ/l³ (1/36 pl⁴/EJ)
96 EJ/l³ (1/36 pl⁴/EJ)
pl/2
pl/2
2p
pl²/24
3/8 pl²
1/6 pl
7/6 pl
1/6
7/6
A
V
1/24
3/8
M

Asta BE



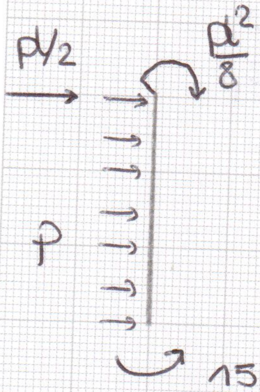
$\rightarrow \frac{12EJ}{l^2}\left(\frac{1}{12}\frac{pl^3}{EJ}\right) \quad \leftarrow \frac{3}{16}pl \quad \leftarrow \frac{3}{8}pl \quad \leftarrow \frac{pl}{2}$

$\leftarrow \frac{12EJ}{l^2}\left(\frac{1}{12}\frac{pl^3}{EJ}\right) \quad \leftarrow \frac{5}{16}pl \quad \rightarrow \frac{3}{8}pl$

$\frac{6EJ}{l}\left(\frac{1}{12}\frac{pl^3}{EJ}\right)$

$\frac{1}{32}pl^2$

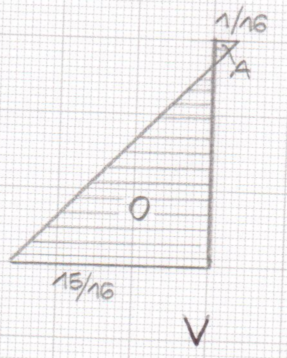
$\frac{pl^2}{16}$

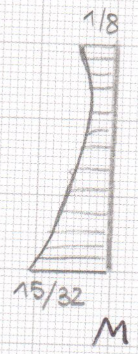


$\leftarrow \frac{1}{16}pl$

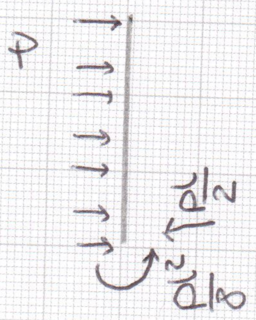
$\leftarrow \frac{15}{16}pl$

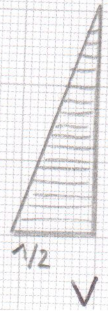
$\frac{15}{32}pl^2$

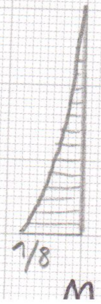




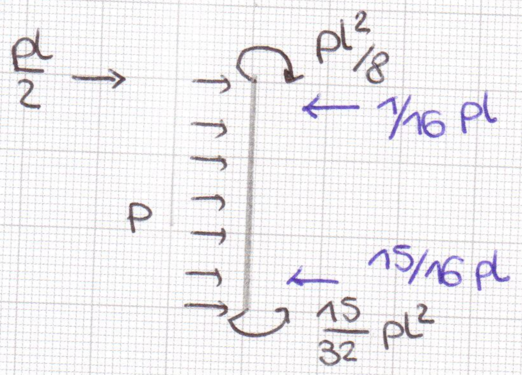
Appendice isostatica

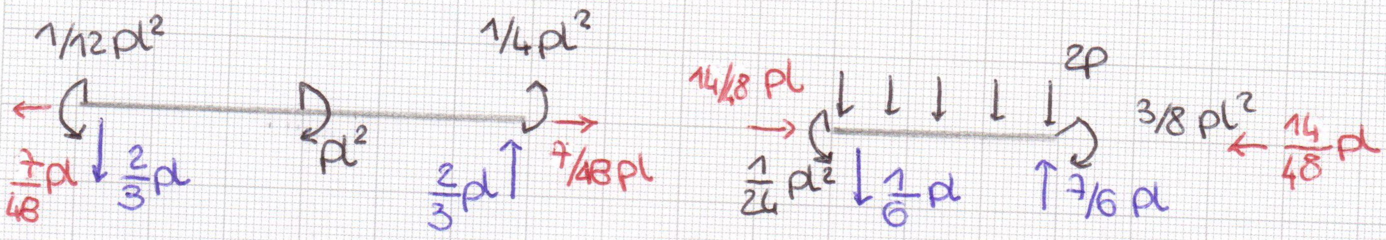


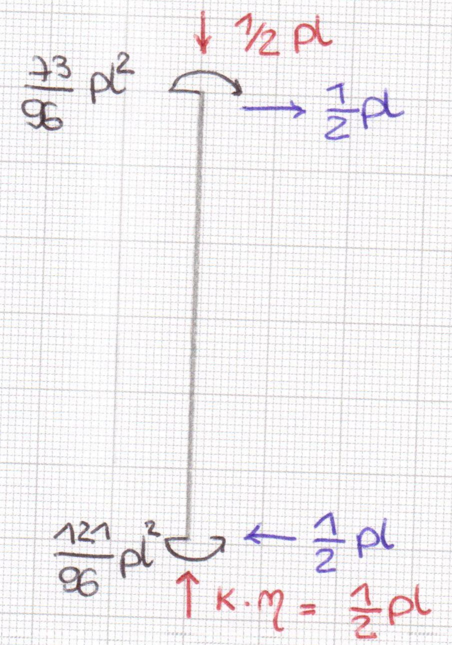




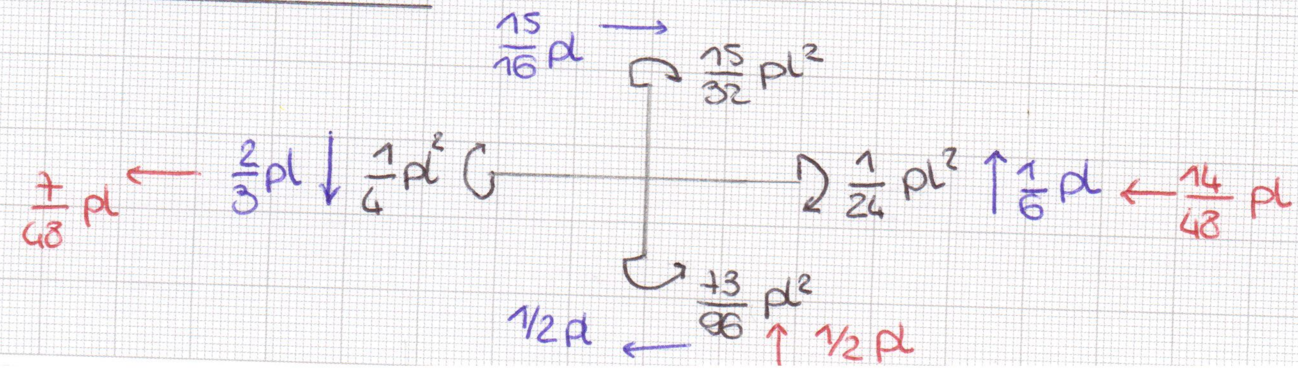
Equilibrio globale







Equilibrio al nodo



$\cdot \sum V = 0 \rightarrow \frac{2}{3} pl - \frac{1}{2} pl - \frac{1}{6} pl = 0$

$$\frac{4-3-1}{6} pl = 0 \qquad \text{ok}$$

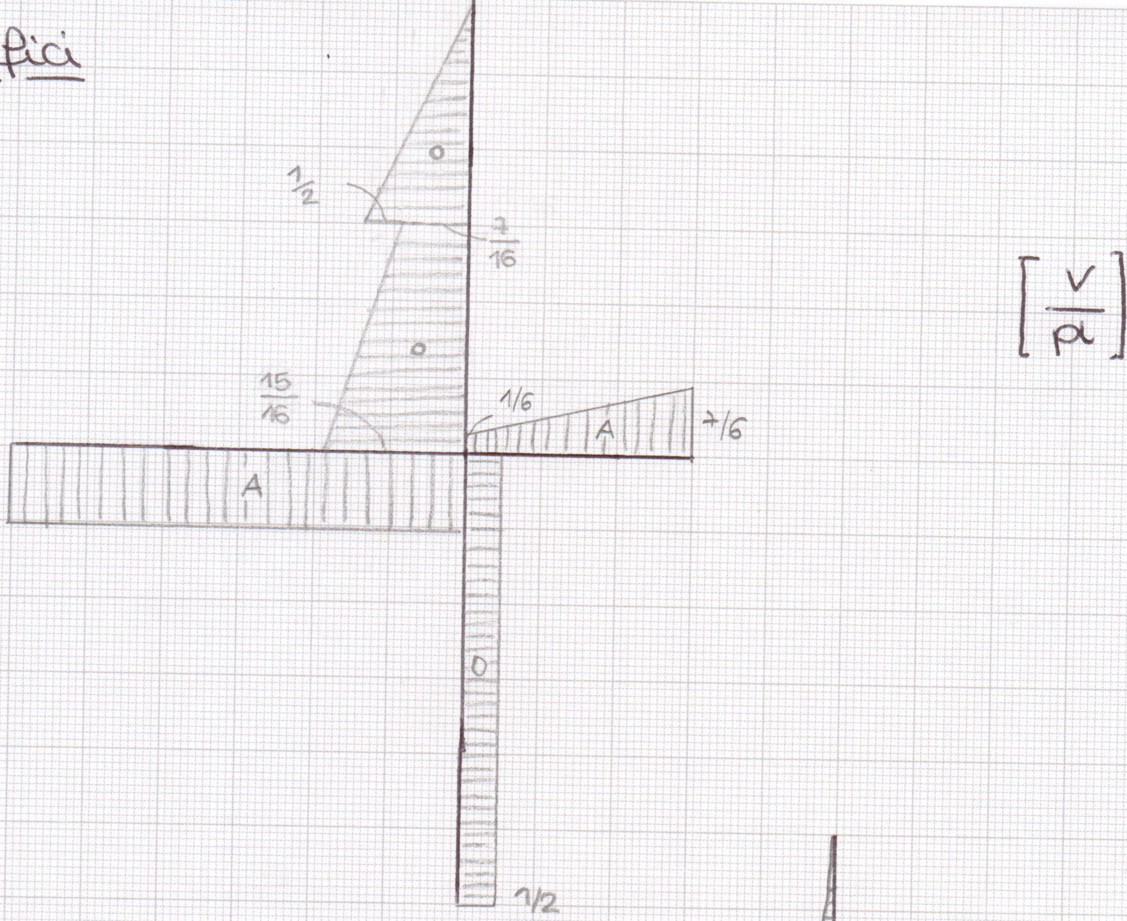
$\cdot \sum H = 0 \rightarrow \frac{7}{48} pl - \frac{15}{16} pl + \frac{1}{2} pl + \frac{14}{48} pl = 0$

$$\frac{7-45+24+14}{48} pl = 0 \qquad \text{ok}$$

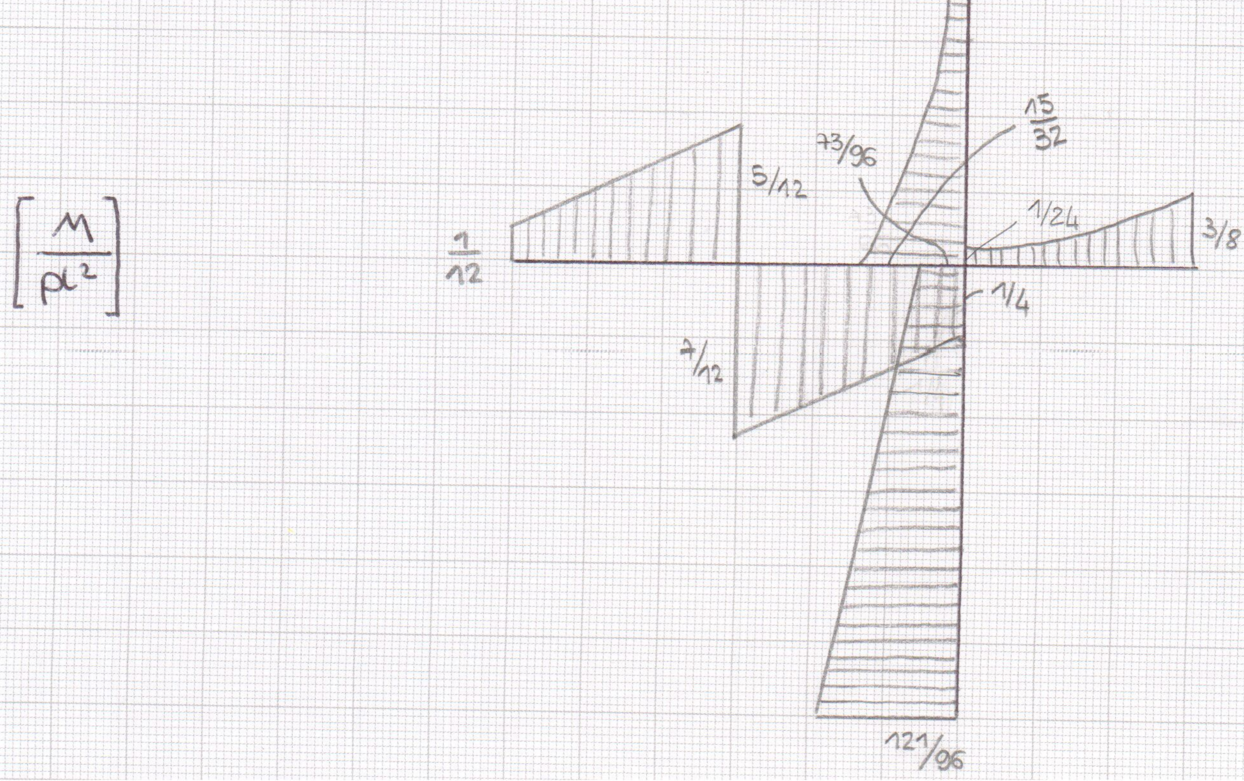
$\cdot \sum M = 0 \rightarrow \frac{1}{4} pl^2 + \frac{15}{32} pl^2 + \frac{1}{24} pl^2 - \frac{73}{96} pl^2 = 0$

$$\frac{24+45+4-73}{96} pl^2 = 0 \qquad \text{ok}$$

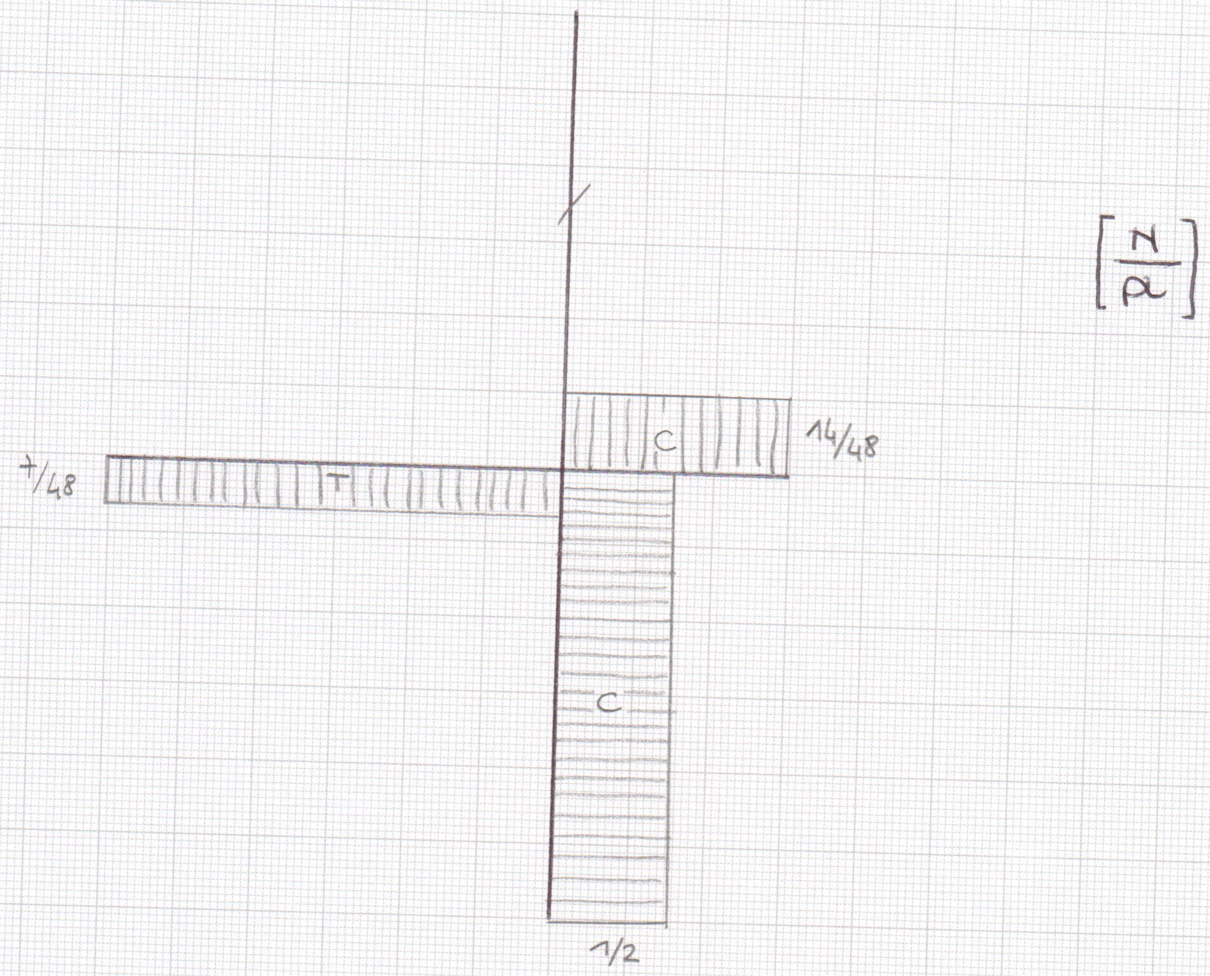
Grafici



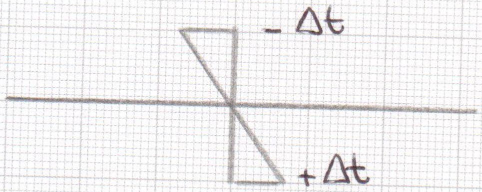
1/2
7/16
15/16
1/6
A
7/6
2/3
A
0
1/2
[V/pl]

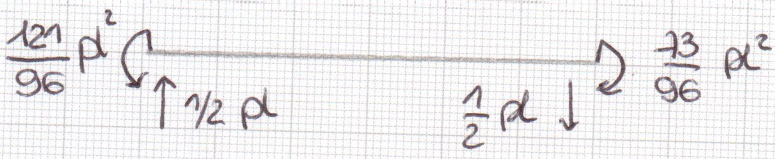


[M/pl²]
5/12
73/96
15/32
1/24
3/8
1/12
1/4
7/12
121/96



Calcolo deformata sull'asta BC





$$y''(x) = -\frac{M(x)}{EJ} \pm \frac{2\alpha \Delta t}{t}$$

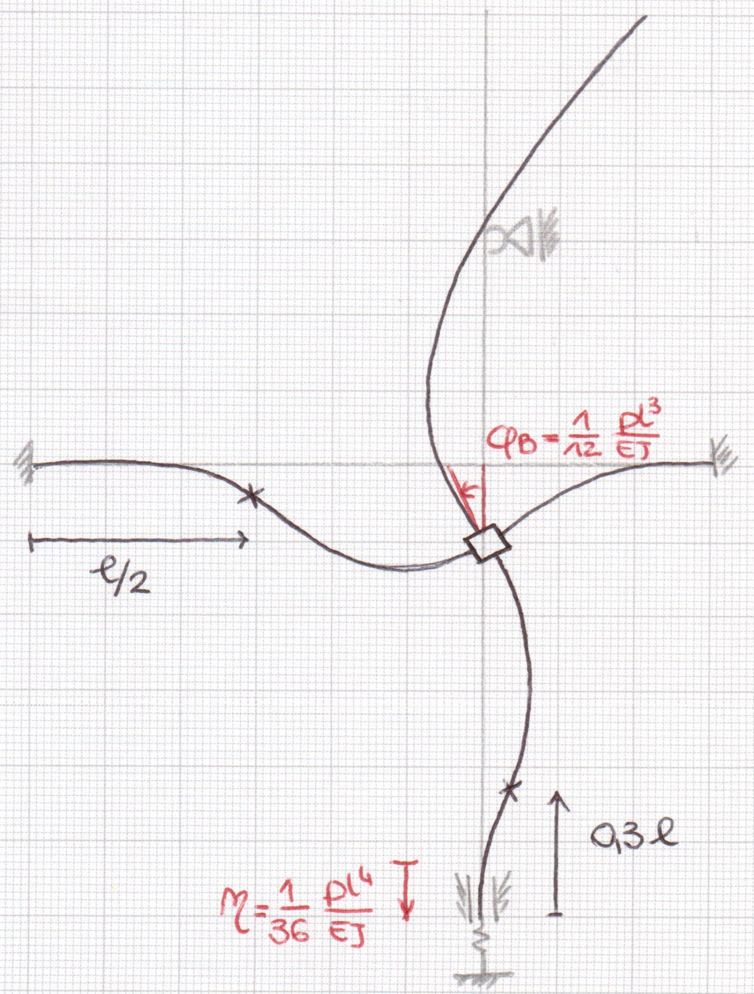
$$y''(x) = -\frac{1}{EJ}\left(-\frac{121}{96}\,pl^2 + \frac{1}{2}\,pl\,x\right) - \frac{35}{32}\,\frac{pl^2}{EJ}$$

$$\frac{121}{96}\frac{pl^2}{EJ} - \frac{1}{2}\,pl\frac{x}{EJ} - \frac{35}{32}\frac{pl^2}{EJ} > 0$$

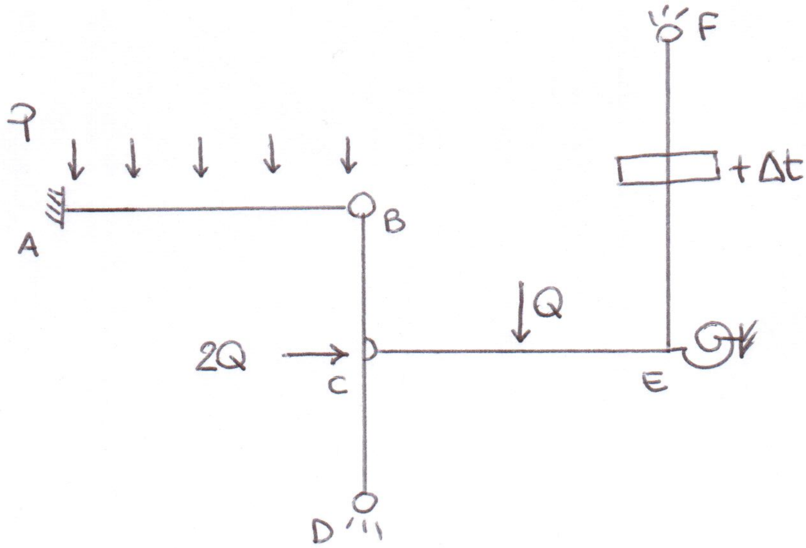
$$-\frac{1}{2}x > \frac{105-121}{96}\,l$$

$$-\frac{1}{2}x > -\frac{16}{96}\,l$$

$$x < \frac{16}{48}\,l \qquad\qquad x < 0{,}3\,l$$



• T.E. 18/07/2011



$$K = \frac{3EJ}{\ell}$$

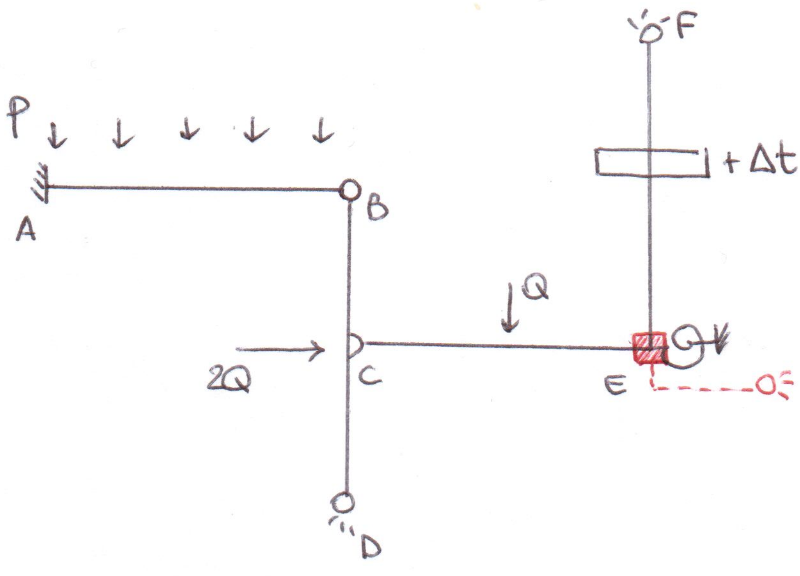
$$\alpha \Delta t = \frac{49}{48} \frac{Q\ell^2}{EJ}$$

$$p = \frac{Q}{\ell}$$

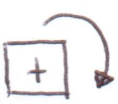
- Telaio a nodi spostabili.

- Sistema risolvente:

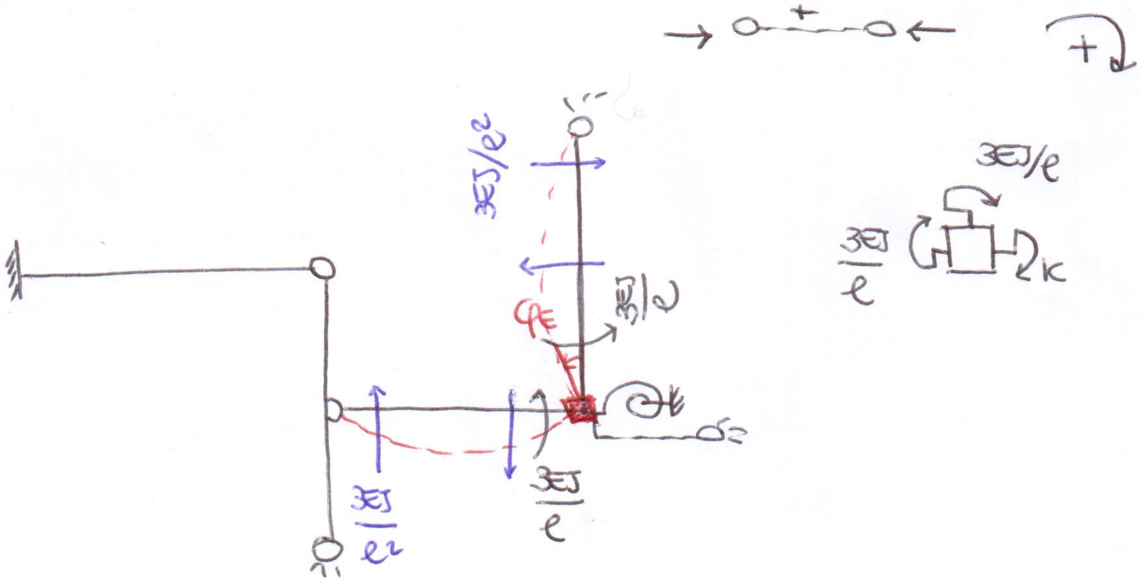
$$\begin{cases} m_{EE} \cdot \varphi_E + m_{E\eta} \cdot \eta + m_{E0} = 0 \\ h_{\eta E} \cdot \varphi_E + h_{\eta\eta} \cdot \eta + h_{\eta 0} = 0 \end{cases}$$



- Convenzioni di segno:

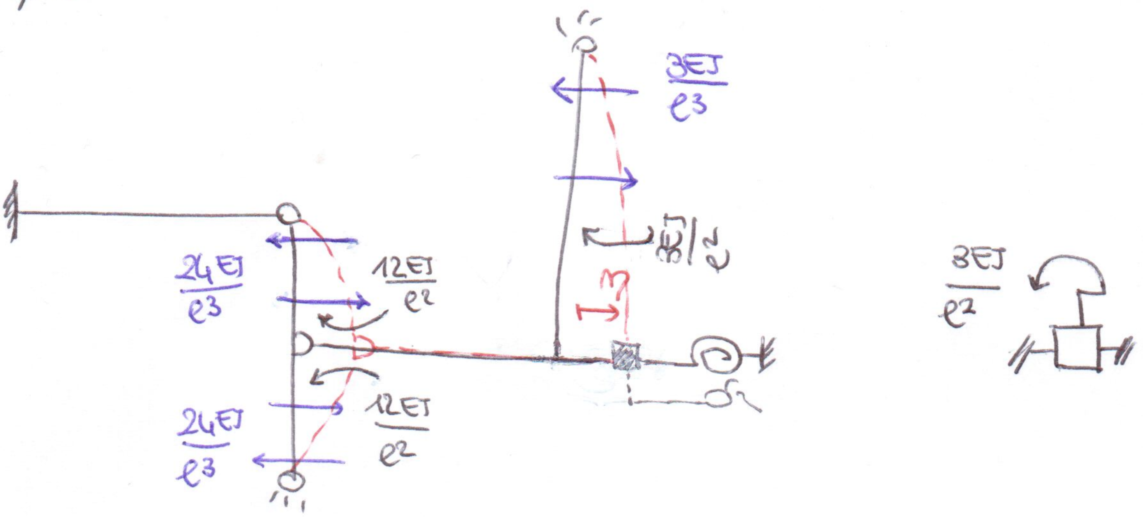


① $\varphi_E \neq 0$



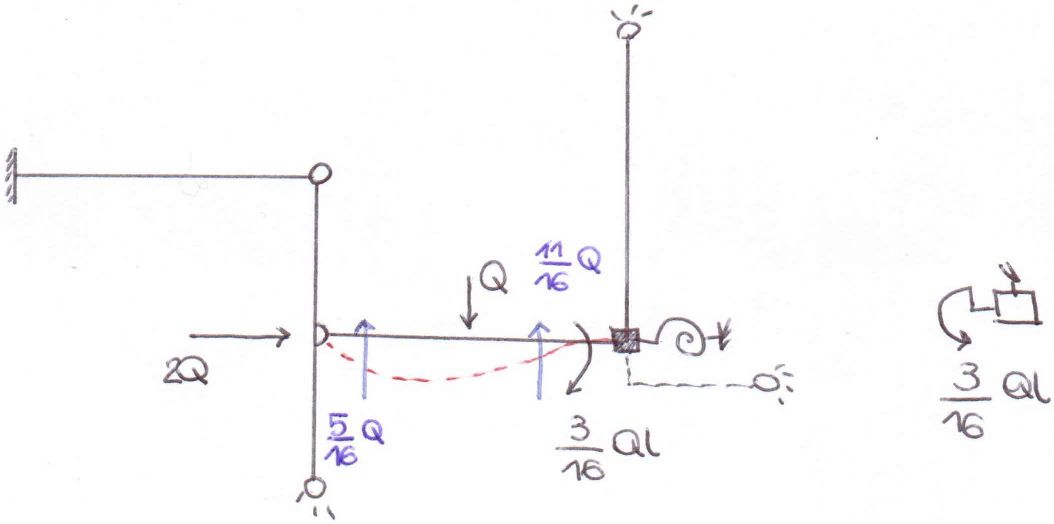
$$\begin{cases} m_E = \dfrac{3EJ}{\ell} + \dfrac{3EJ}{\ell} + k = \dfrac{6EJ}{\ell} + k = \dfrac{6EJ}{\ell} + \dfrac{3EJ}{\ell} = \dfrac{9EJ}{\ell} \\ h_E = \dfrac{3EJ}{\ell^2} \end{cases}$$

② $\eta_E \neq 0$



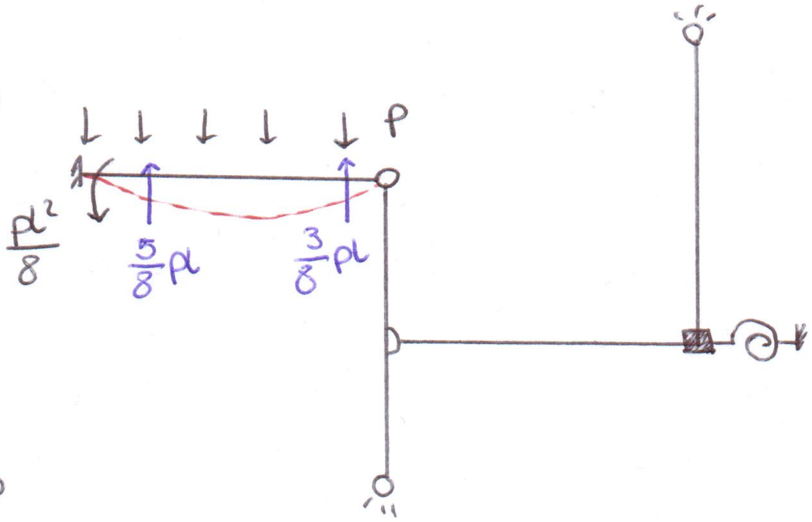
$$\begin{cases} m_E = -\dfrac{3EJ}{\ell^2} \\ h_E = -\dfrac{48EJ}{\ell^3} - \dfrac{3EJ}{\ell^3} = -\dfrac{51EJ}{\ell^3} \end{cases}$$

③ $Q \neq 0$



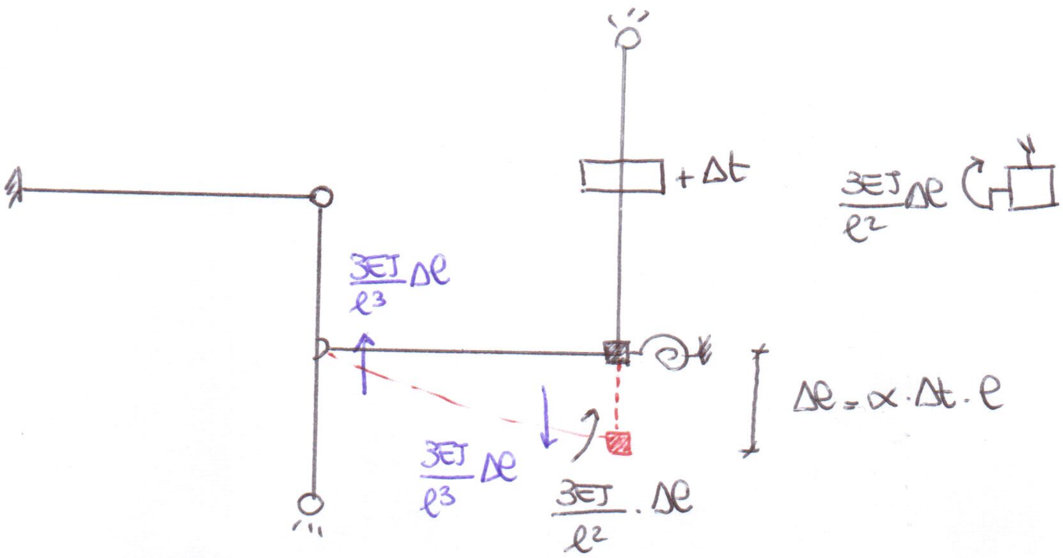
$$\begin{cases} m_{E0} = -\frac{3}{16} Ql \\ r_{E0} = 2Q \end{cases}$$

④ $p \neq 0$



$$\begin{cases} m_{E0} = 0 \\ r_{E0} = 0 \end{cases}$$

⑤ $\Delta t \neq 0$



$$\begin{cases} m_{E0} = \frac{3EJ}{l^2} \Delta l \\ r_{E0} = 0 \end{cases}$$

$$\begin{cases} \left(\frac{6EJ}{\ell}+k\right)\varphi - \frac{3EJ}{\ell^2}\cdot\eta - \frac{3}{16}Q\ell + \frac{3EJ}{\ell^2}\cdot\alpha\Delta t\cdot\ell = 0 \\ \frac{3EJ}{\ell^2}\cdot\varphi - 51\frac{EJ}{\ell^3}\cdot\eta + 2Q = 0 \end{cases}$$

$$-\frac{3}{16}+\frac{49}{16}$$

$$\begin{cases} \left(\frac{6EJ}{\ell}+\frac{3EJ}{\ell}\right)\varphi - \frac{3EJ}{\ell^2}\eta - \frac{3}{16}Q\ell + \frac{3EJ}{\ell^2}\,\frac{49}{48}\,\frac{QL^3}{EJ} = 0 \\ \frac{3EJ}{\ell^2}\cdot\varphi - 51\frac{EJ}{\ell^3}\cdot\eta + 2Q = 0 \end{cases}$$

$$\begin{cases} \frac{9EJ}{\ell}\varphi - \frac{3EJ}{\ell^2}\eta + \frac{46}{16}Q\ell = 0 \\ \frac{3EJ}{\ell^2}\cdot\varphi - 51\frac{EJ}{\ell^3}\eta + 2Q = 0 \quad (-3\ell) \end{cases}$$

$$\frac{46}{16}-6=\frac{46-96}{16}$$

$$\begin{cases} \frac{9EJ}{\ell}\varphi - \frac{3EJ}{\ell^2}\eta + \frac{46}{16}Q\ell = 0 \\ -\frac{9EJ}{\ell}\cdot\varphi + \frac{153EJ}{\ell^2}\eta - 6Q\ell = 0 \end{cases}$$

$$//\quad 150\frac{EJ}{\ell^2}\eta - \frac{50}{16}Q\ell = 0$$

$$150\frac{EJ}{\ell^2}\eta = \frac{25}{8}Q\ell$$

$$\eta = \frac{25}{8}\cdot\frac{1}{150}\,\frac{\ell^2}{EJ}Q\ell$$

$$\boxed{\eta = \frac{1}{48}\frac{Q\ell^3}{EJ}}$$

$$\frac{288-153}{48}=\frac{135}{48}$$

$$-\frac{9EJ}{\ell}\cdot\varphi + \frac{153EJ}{\ell^2}\left(\frac{1}{48}\frac{QL^3}{EJ}\right) - 6QL = 0$$

$$-\frac{9EJ}{\ell}\cdot\varphi = 6QL - \frac{153}{48}QL$$

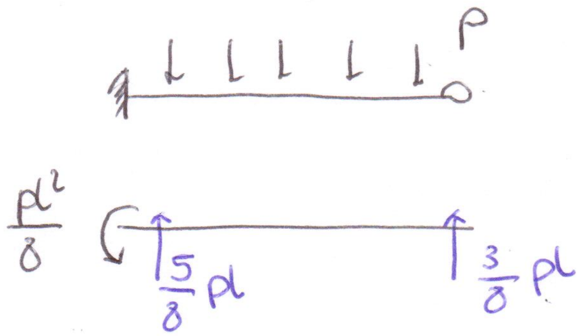
$$-\frac{9EJ}{\ell}\varphi = \frac{135}{48}Q\ell$$

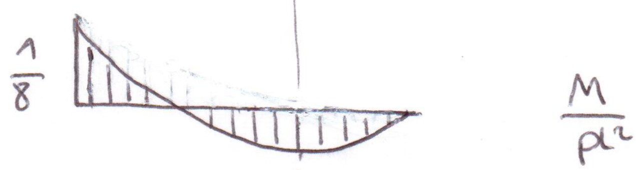
$$\varphi = \frac{\cancel{135}^{\,15}}{48}\cdot\frac{1}{\cancel{9}}\frac{Q\ell^2}{EJ}$$

$$\boxed{\eta = \frac{1}{48}\frac{Ql^3}{EJ}}$$

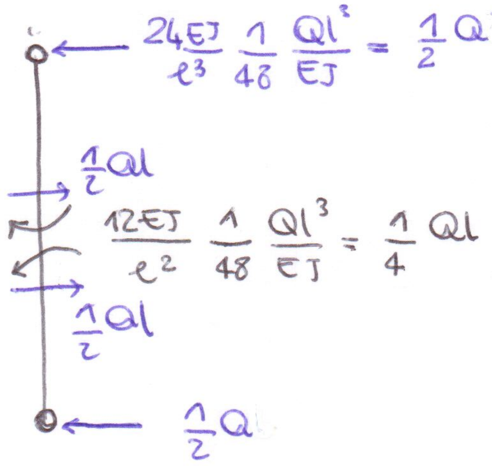
$$\varphi = -\frac{15}{48}\frac{Ql^2}{EJ} \qquad \boxed{\varphi = -\frac{5}{16}\frac{Ql^2}{EJ}}$$

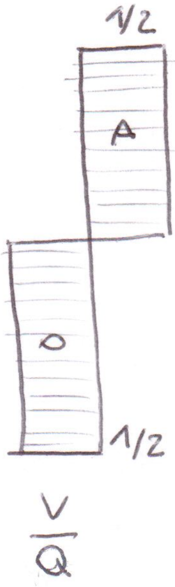
Asta $\overline{AB}$

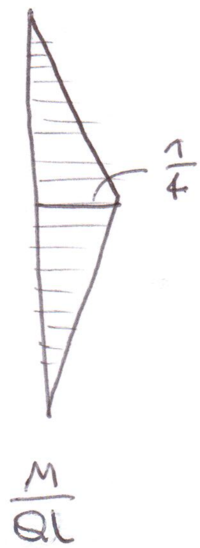




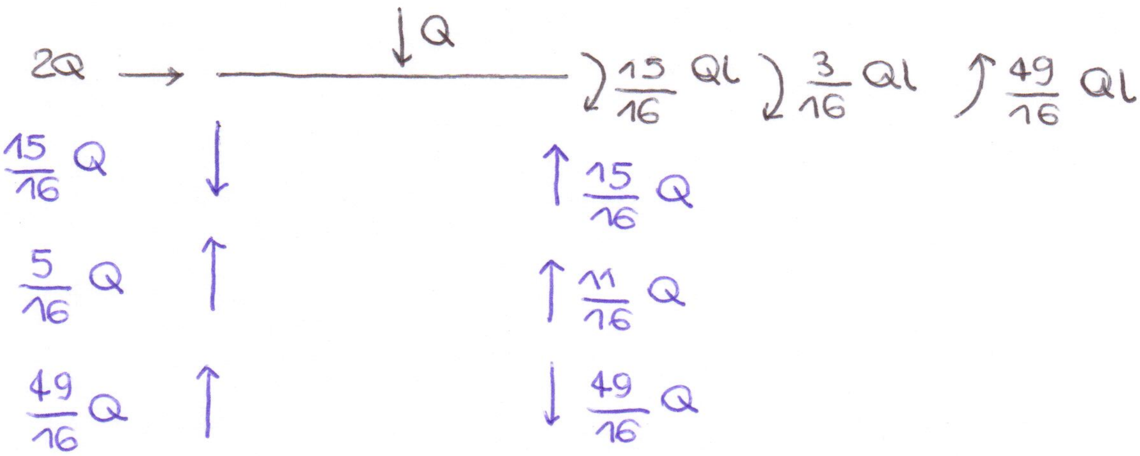
Asta $\overline{BD}$

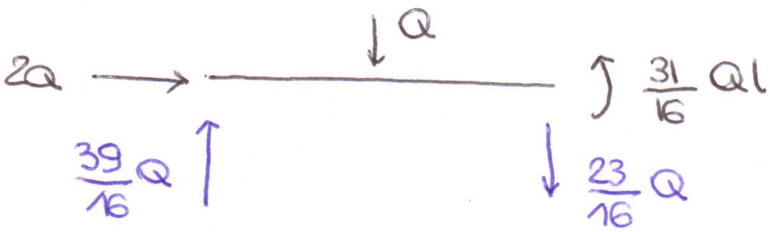


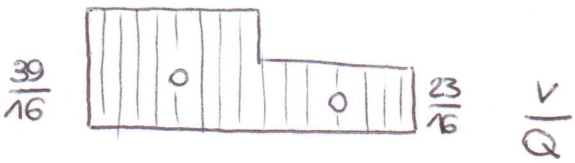


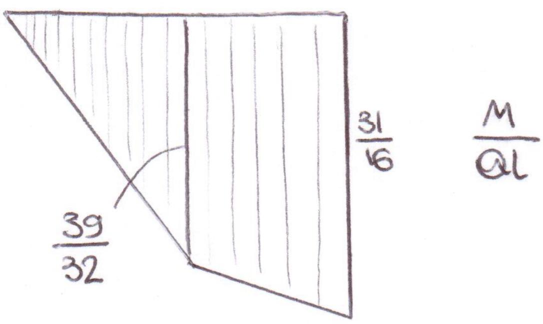


Asta CE

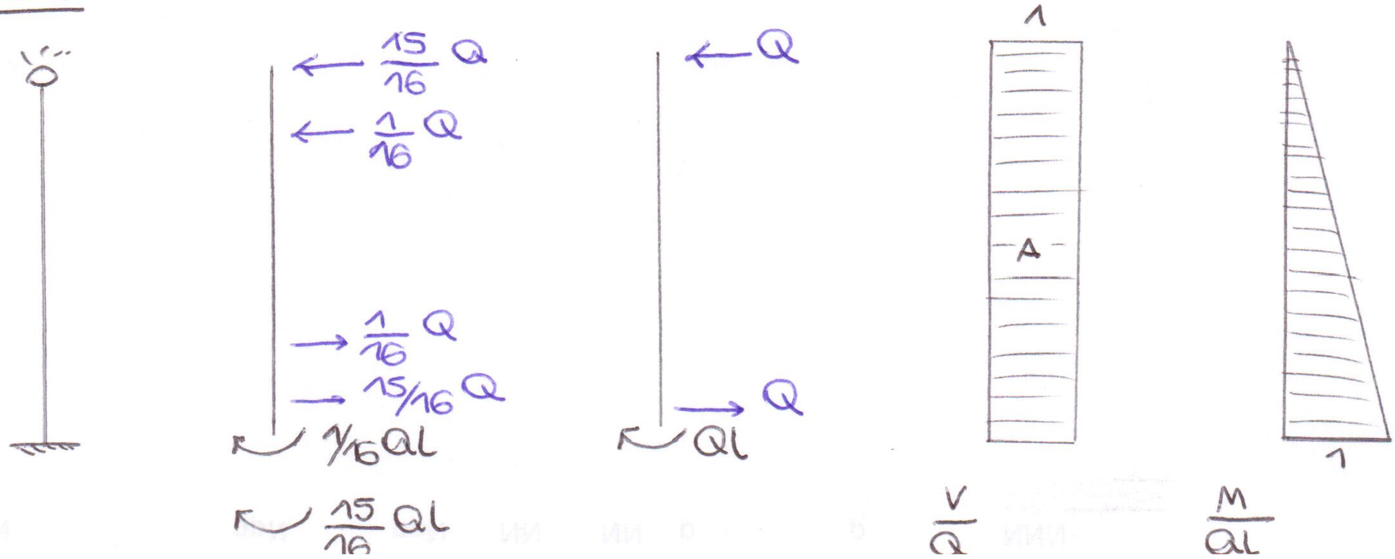




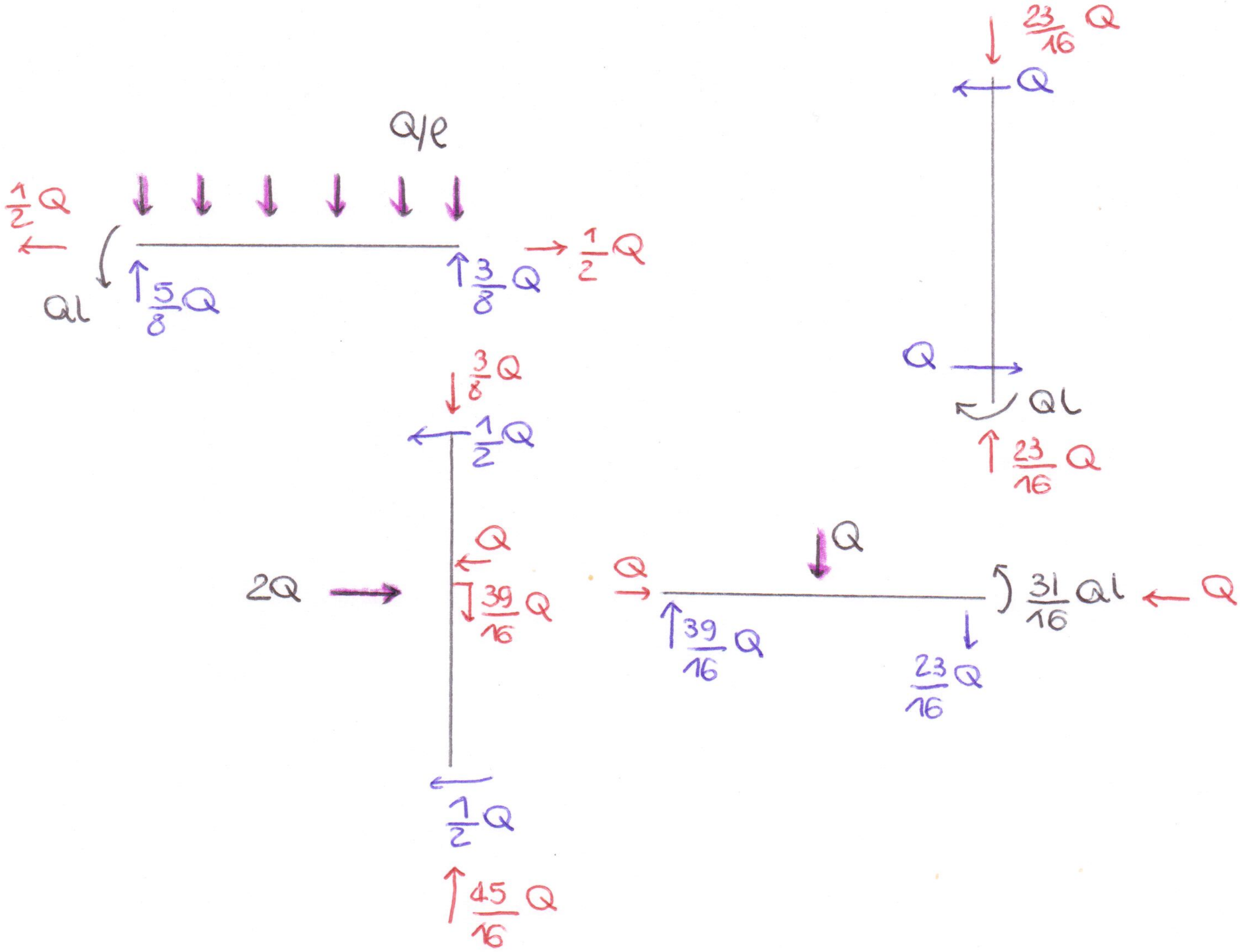




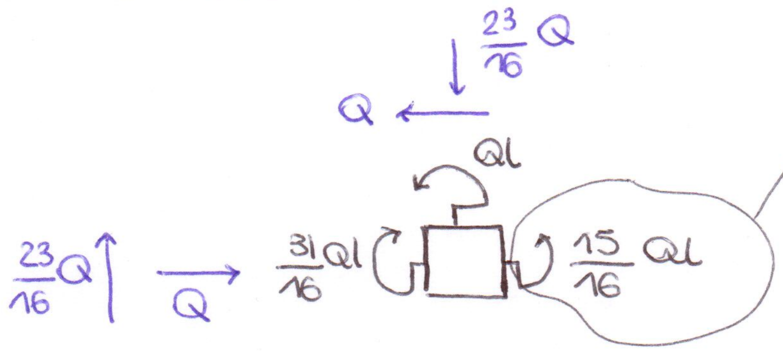
Asta EF



Equilibrio globale



Equilibrio al nodo



contributo molla:

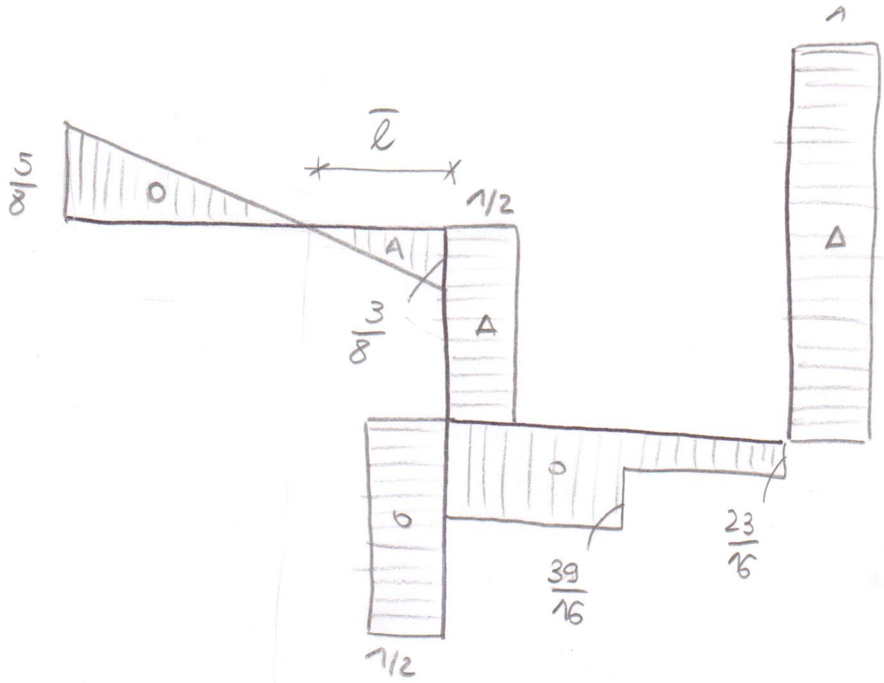
$$\frac{3EJ}{l}\cdot\frac{5}{16}\frac{Ql^2}{EJ}$$

$$\Sigma M=0 \rightarrow \frac{31}{16}Ql - Ql - \frac{15}{16}Ql = 0$$

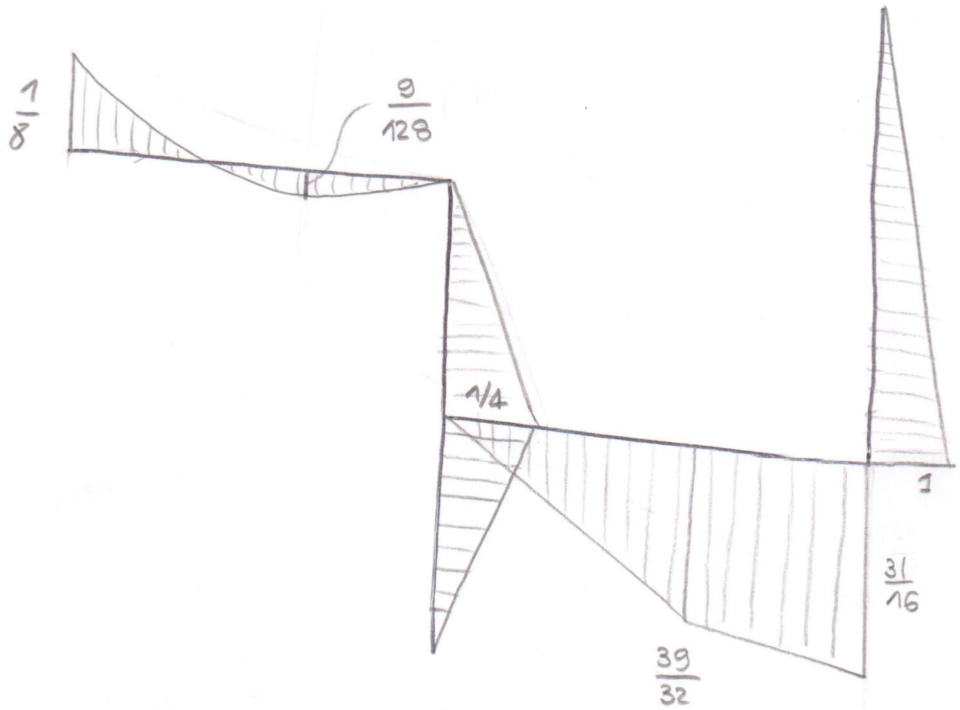
$$\Sigma V=0 \rightarrow \frac{23}{16}Q - \frac{23}{16}Q = 0$$

ok

$$\Sigma H=0 \rightarrow Q - Q = 0$$



$\frac{V}{Q}$



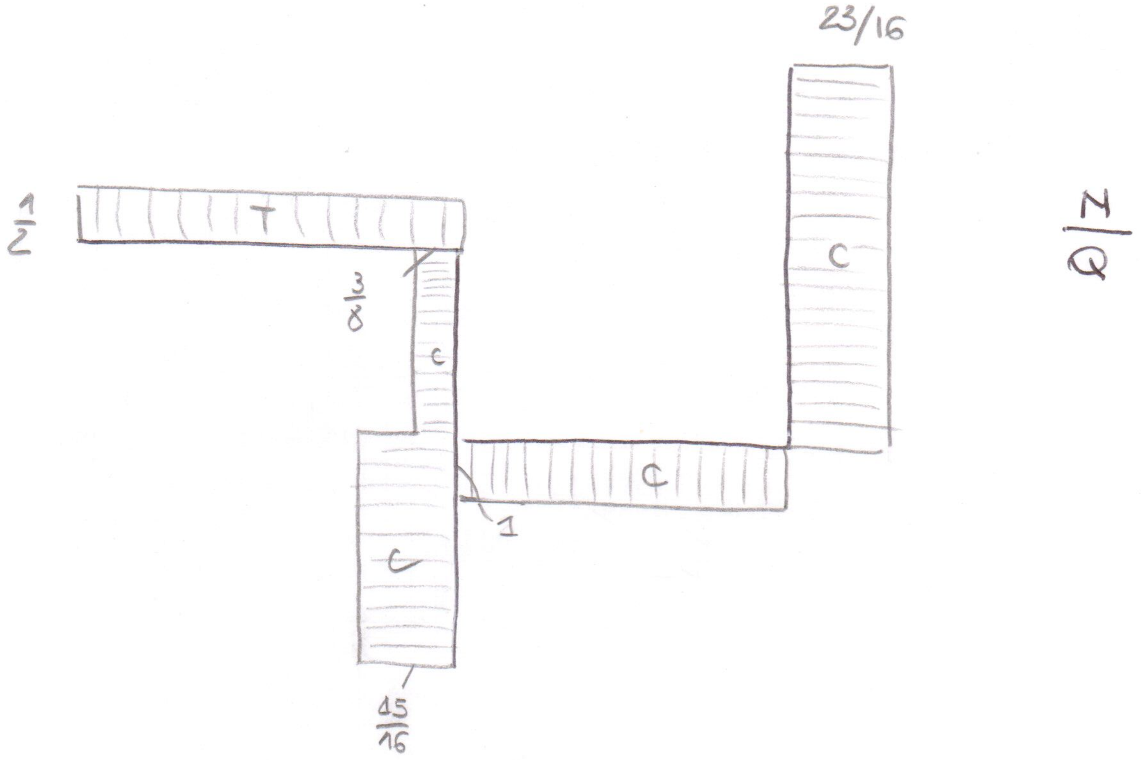
$\frac{M}{Ql}$

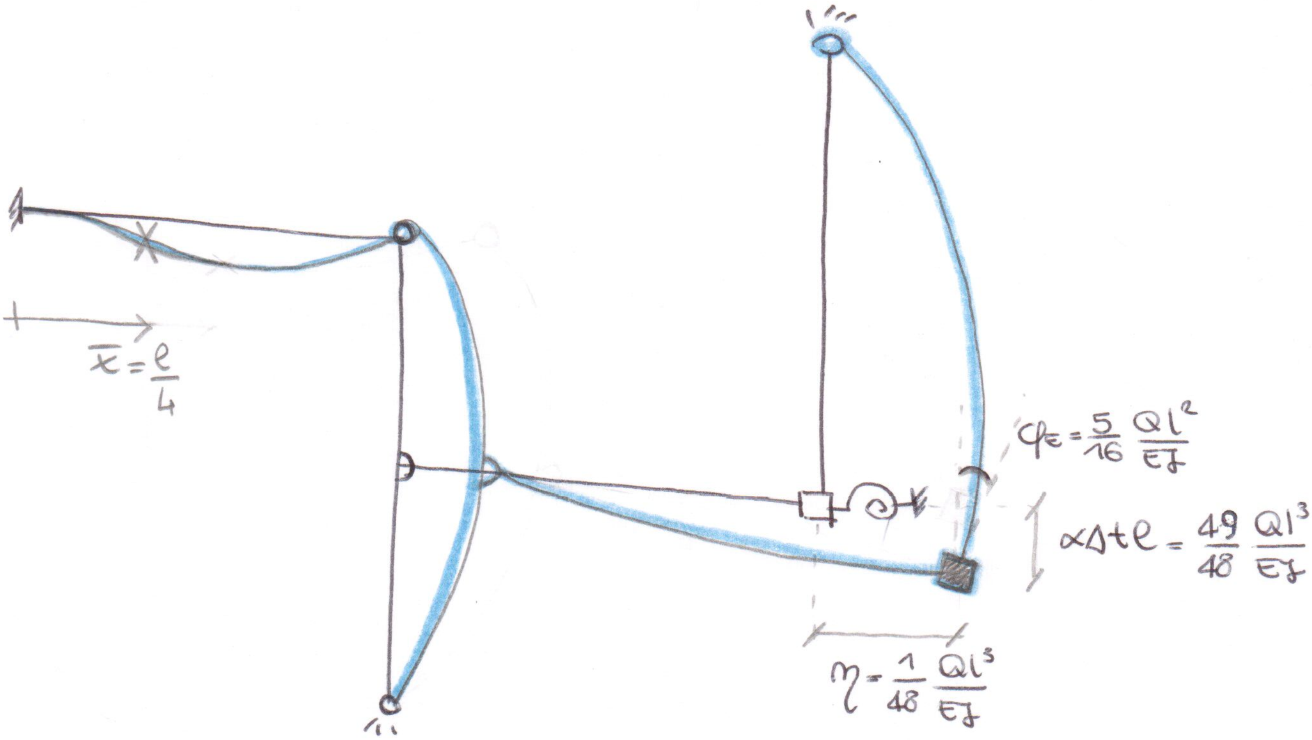
$$\bar{\ell} = \frac{3}{8}\ell$$

$$M^{+}_{max} = \frac{3}{8}pl \cdot \frac{3}{8}\ell - p \cdot \frac{3}{8}\ell \cdot \frac{3}{16}\ell$$

$$= \frac{9}{64}pl^2 - \frac{9}{128}pl^2$$

$$= \frac{9}{128}pl^2$$





$$M(\bar{x})=0 \rightarrow \frac{5}{8}pl\bar{x} - \frac{pl^2}{8} - \frac{p\bar{x}^2}{2} = 0 \qquad \bar{x} = \frac{l}{4}$$

$$5l\bar{x} - l^2 - 4x^2 = 0$$

$$4x^2 - 5lx + l^2 = 0$$

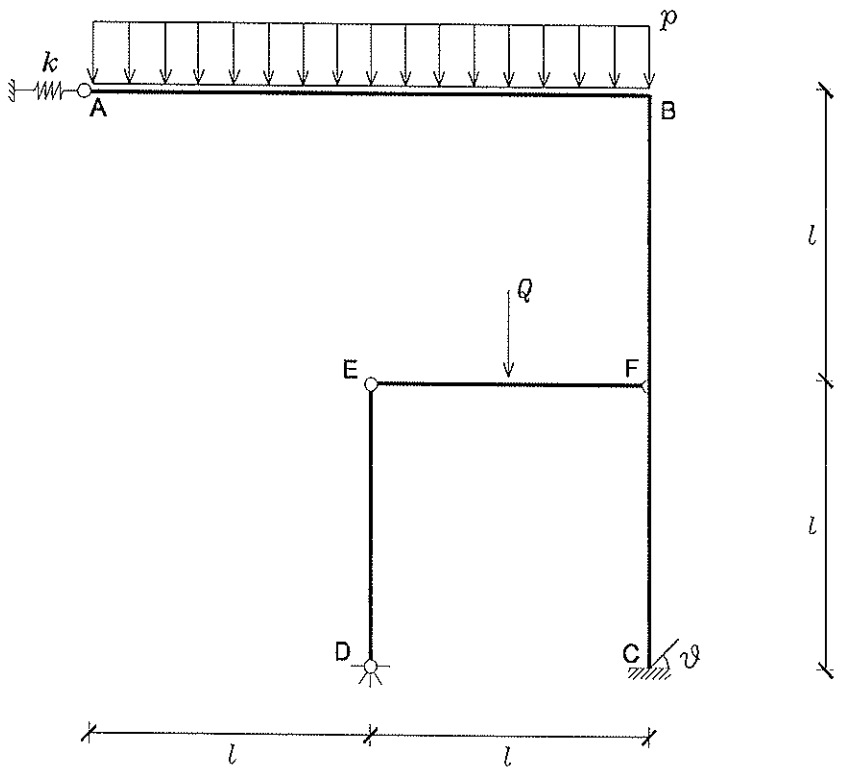
$$x_{1,2} = \frac{5l \pm \sqrt{25l^2 - 16l^2}}{8} = \begin{cases} 1\,l \\ \frac{1}{4}\,l \end{cases}$$

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 12 SETTEMBRE 2011

DOCENTE: **ING. FAUSTO MINELLI** ESERCITATORE: **ING. ADRIANO REGGIA**

Esercizio



$$Q = \frac{9}{8}\text{pl}$$

$$k = 2\frac{EJ}{\text{l}^3}$$

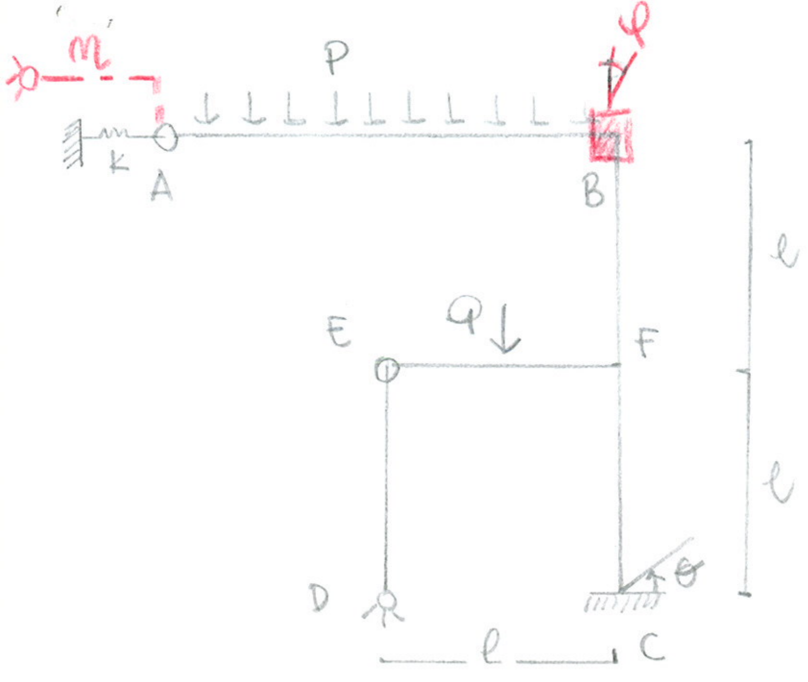
$$\vartheta = \frac{5pl^3}{4EJ}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

T.E. 12/09/2011

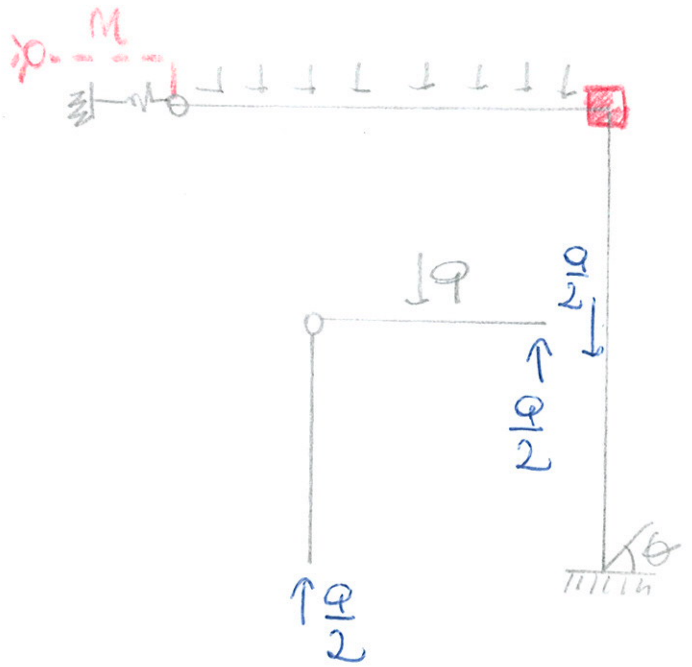


$$Q = \frac{9}{8} pl$$

$$\theta = \frac{5}{4} \frac{pl^3}{EJ}$$

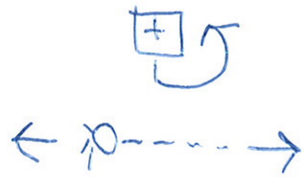
$$K = 2 \frac{EJ}{l^3}$$

(*) Appendice isostatica DEF:



$$\frac{q}{2} = \frac{9}{16} pl$$

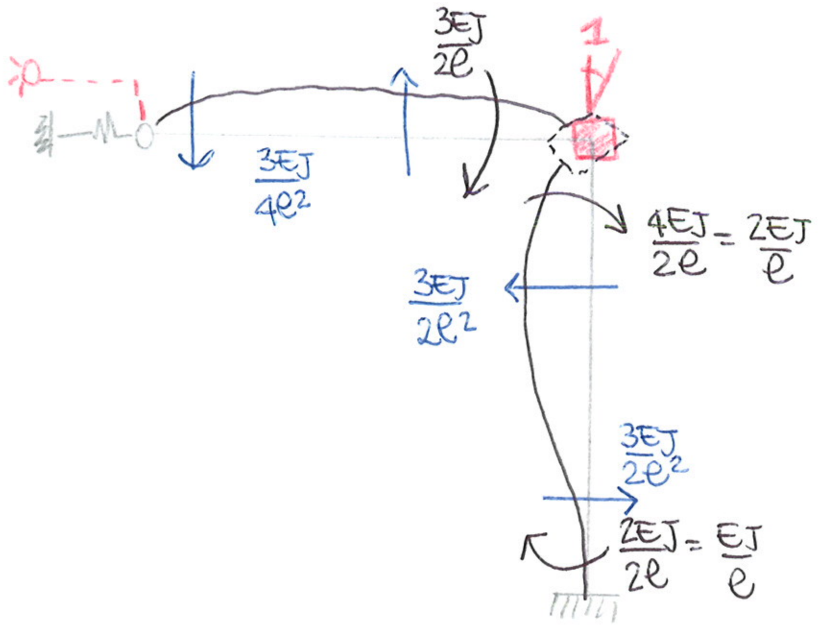
(*) CONVENZIONI



(*) Sistema risolvente:

$$\begin{cases} m_{BB} \varphi_B + m_{B\eta} \eta + m_{B0} = 0 \\ h_{B\eta} \varphi_B + h_{\eta\eta} \eta + h_{\eta 0} = 0 \end{cases}$$

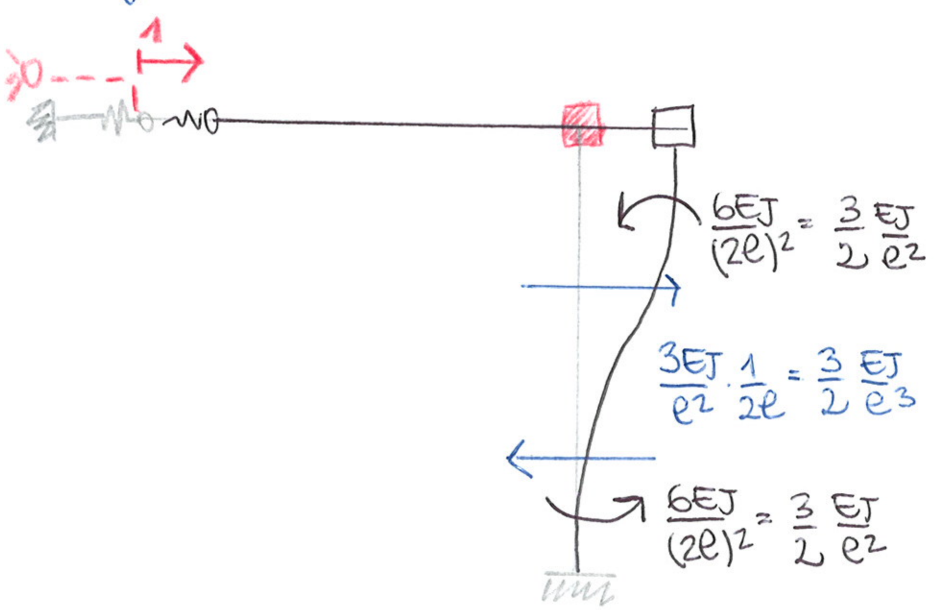
① $\varphi_B = 1 \quad \eta = \rho = \theta = 0$:



$$M_{BB} = \frac{3}{2}\frac{EJ}{\ell} + \frac{2EJ}{\ell} = \frac{(3+4)}{2\ell}EJ = \frac{7}{2}\frac{EJ}{\ell}$$

$$h_{\eta B} = \frac{3}{2}\frac{EJ}{\ell^2}$$

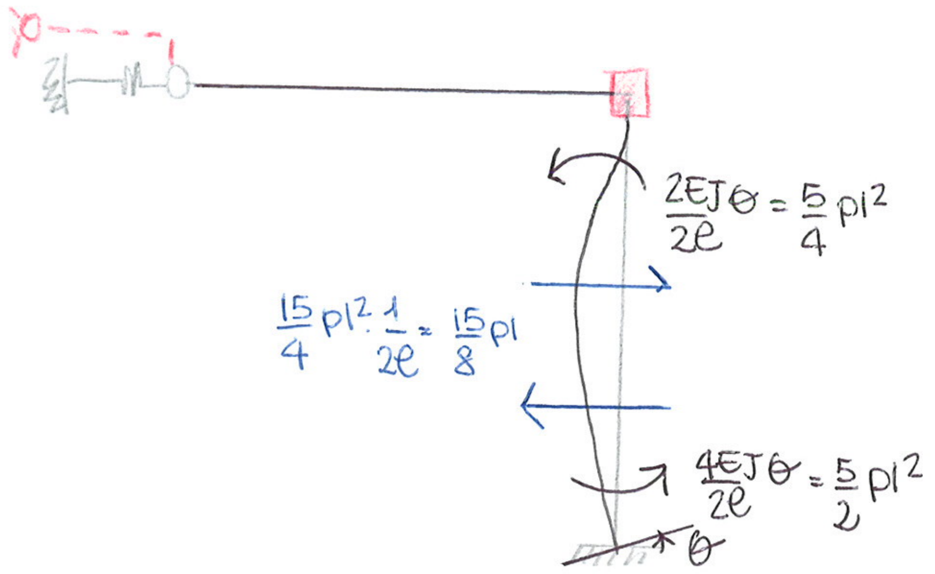
② $\eta = 1 \quad \varphi_B = \rho = \theta = 0$:



$$M_{B\eta} = -\frac{3}{2}\frac{EJ}{\ell^2}$$

$$h_{\eta\eta} = -\frac{3}{2}\frac{EJ}{\ell^3} - k = -\frac{3}{2}\frac{EJ}{\ell^3} - \frac{2EJ}{\ell^3} = \frac{(-3-4)}{2\ell^3}EJ = -\frac{7}{2}\frac{EJ}{\ell^3}$$

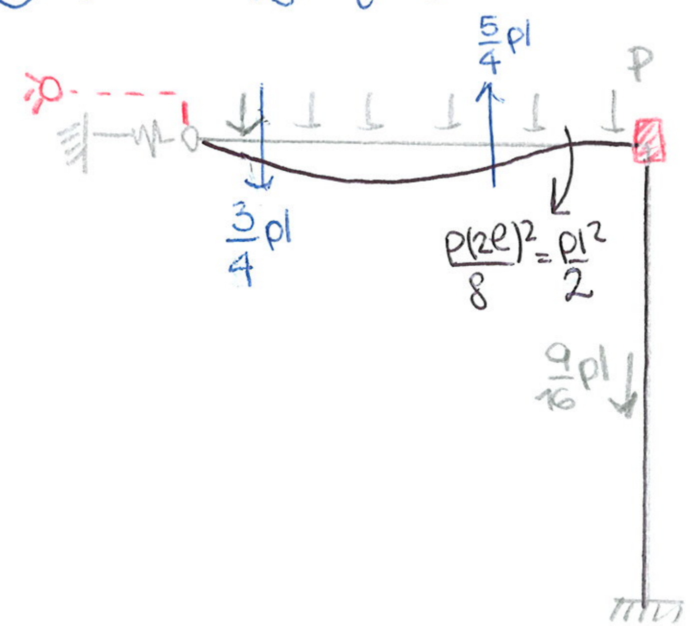
③ $\theta \neq 0 \quad \varphi_B = \eta = p = 0$



$$M_{BO}^{\theta} = -\frac{5}{4} p l^2$$

$$h_{\eta O}^{\theta} = -\frac{15}{8} p \ell$$

④ $p \neq 0 \quad \varphi_B = \eta = \theta = 0$:



$$M_{BO}^{p} = +\frac{p l^2}{2}$$

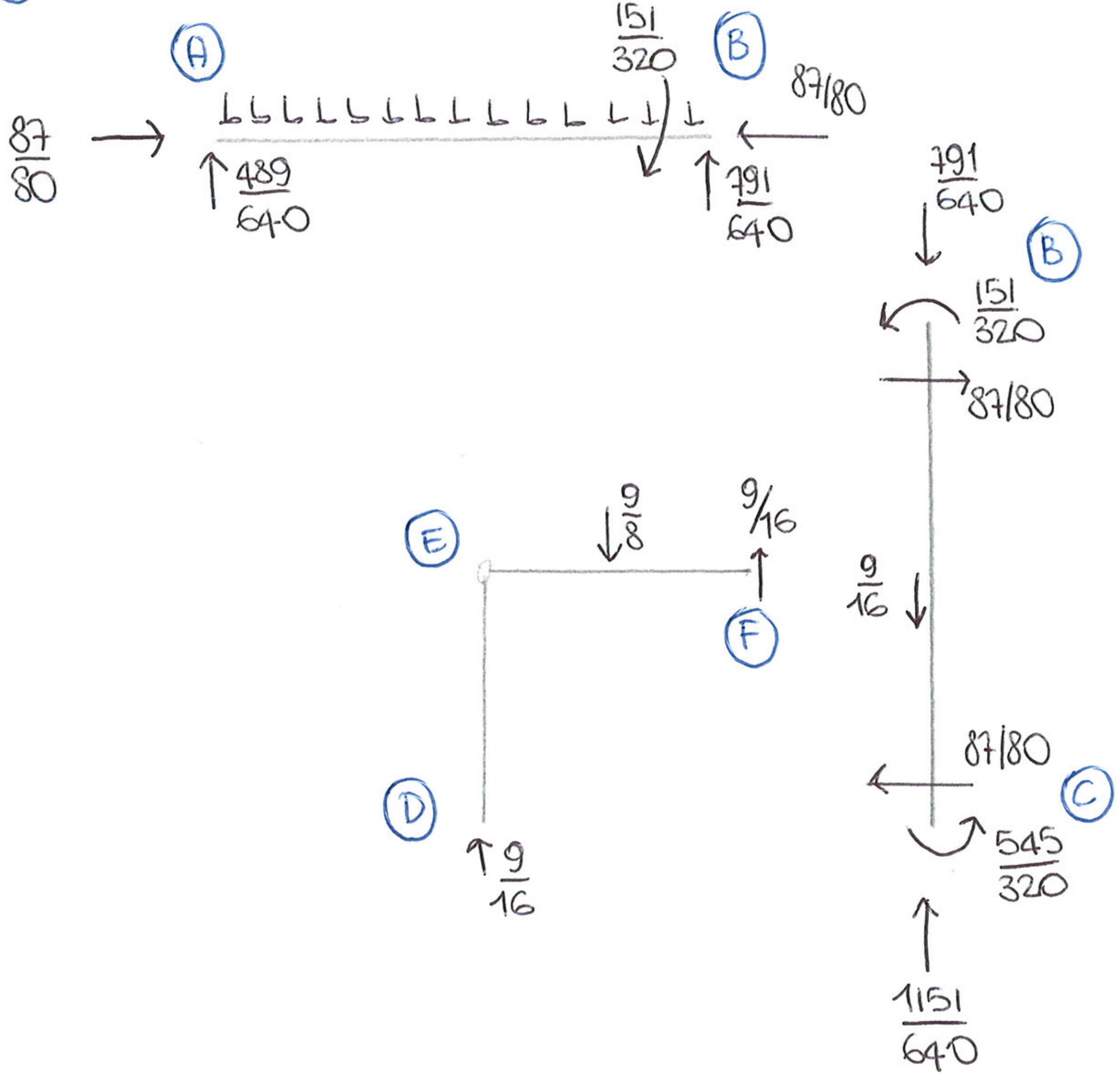
$$h_{\eta O}^{p} = 0$$

(*) **Sistema risolvente:**

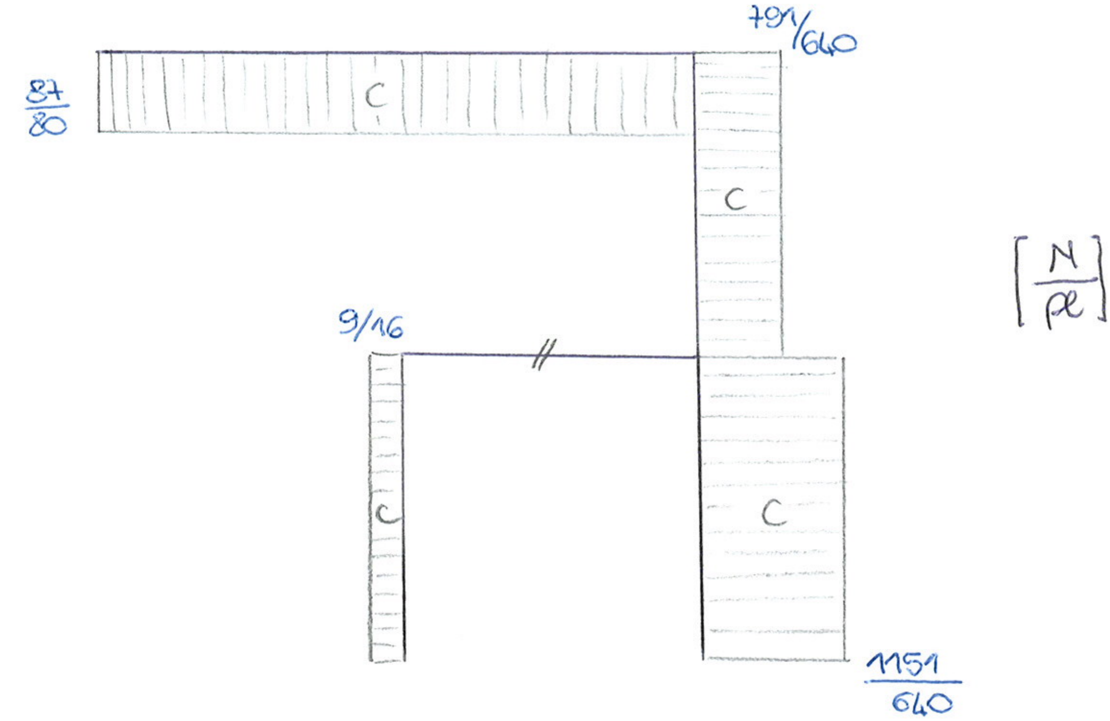
$$\begin{cases} \frac{7}{2}\frac{EJ}{l}\varphi_B - \frac{3}{2}\frac{EJ}{l^2}\eta - \frac{5}{4}pl^2 + \frac{pl^2}{2} = 0 \\ \frac{3}{2}\frac{EJ}{l^2}\varphi_B - \frac{7}{2}\frac{EJ}{l^3}\eta - \frac{15}{8}pl = 0 \end{cases}$$

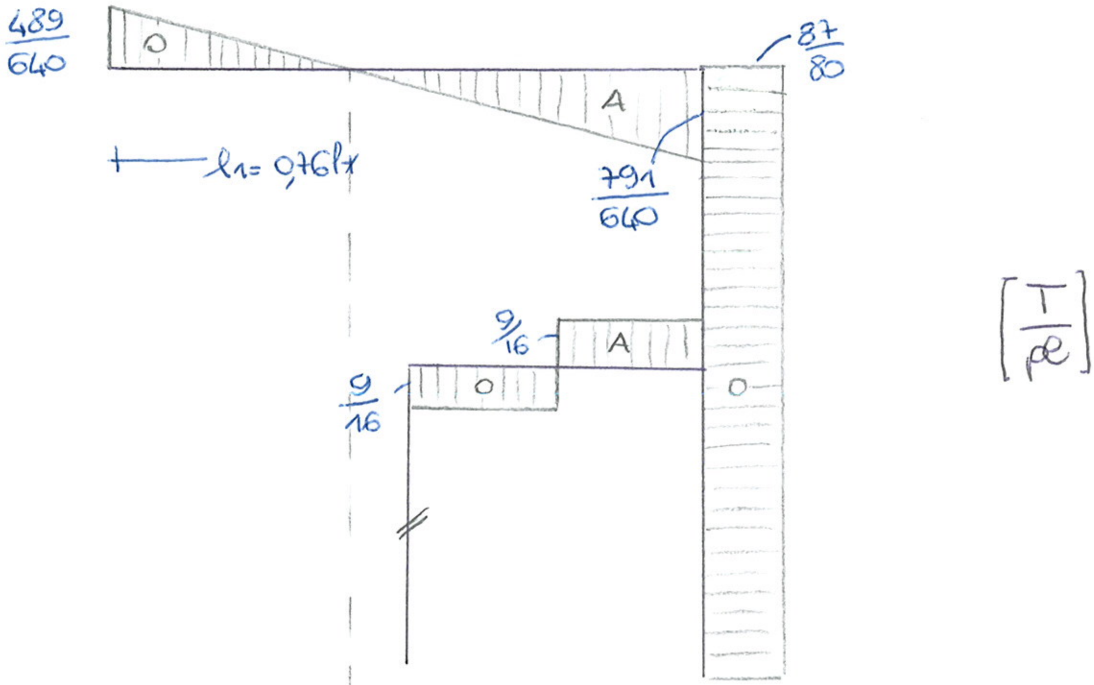
$$\begin{cases} \eta = -\frac{87}{160}\frac{pl^4}{EJ} \\ \varphi_B = -\frac{3}{160}\frac{pl^3}{EJ} \end{cases}$$

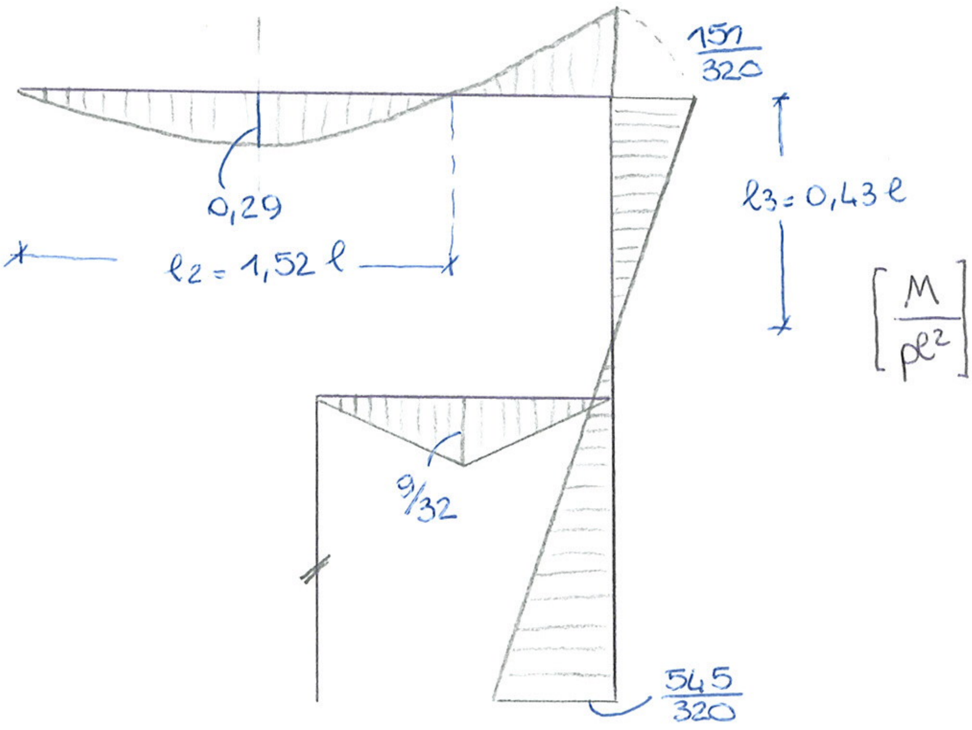
(*) **Equilibrio globale:**



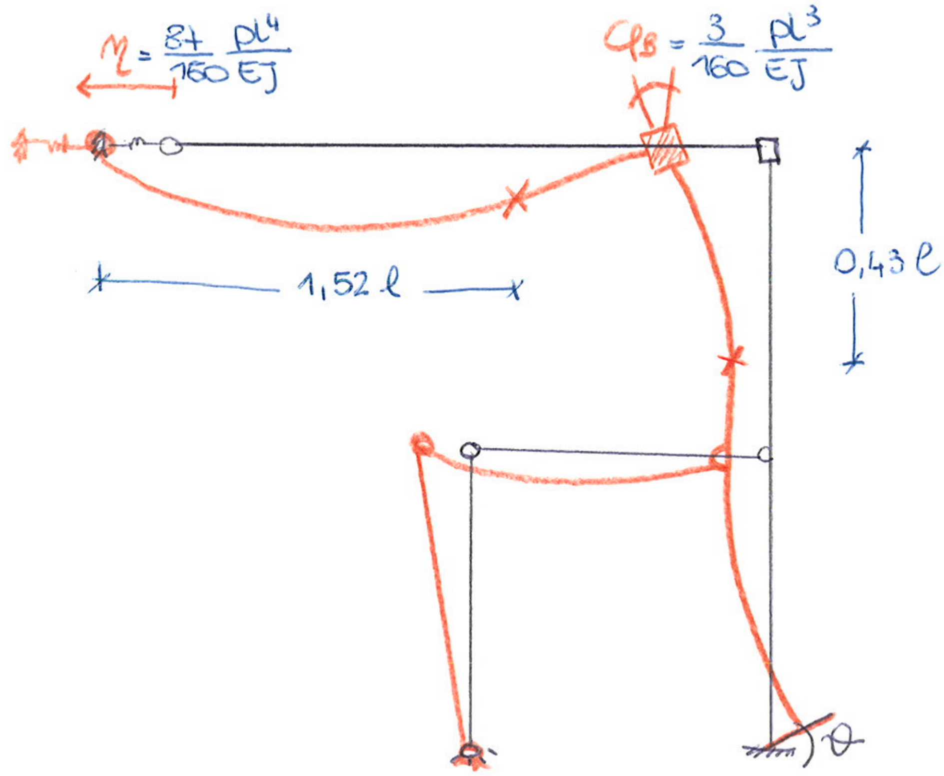
⊛ Grafici:


791/640
87/80
C
C
[N/pl]
9/16
C
C
1151/640


489/640
O
A
87/80
791/640
l1 = 0,76 l
[T/pl]
9/16
A
9/16
O
O


151/320
0,29
l3 = 0,43 l
l2 = 1,52 l
[M/pl²]
9/32
545/320

⊛ Deformata:



⊛ Posizione flessi:

. Calcolo ℓ_1 →

$$\frac{489}{640} p\ell - p\ell_1 = 0$$

$$\ell_1 = \frac{489}{640}\ell = 0{,}76\,\ell$$

. Calcolo ℓ_2 →

$$\frac{489}{640} p\ell \cdot \ell_2 - p \cdot \frac{\ell_2^2}{2} = 0$$

$$\ell_2 = \frac{489}{640} \cdot 2\ell = \frac{489}{320}\ell = 1{,}52\,\ell$$

. Calcolo ℓ_3 →

$$\frac{151}{320} p\ell^2 - \frac{87}{80} p\ell \cdot \ell_3 = 0$$

$$\ell_3 = \frac{151}{320} \cdot \frac{80}{87}\ell = 0{,}43\,\ell$$

. Calcolo $M_{max}(0{,}76\,\ell)$ →

$$\frac{489}{640} p\ell \frac{489}{640}\ell - p\left(\frac{489}{640}\ell\right)^2 \frac{1}{2} =$$

$$= \left(\frac{489}{640}\right)^2 p\ell^2 - \left(\frac{489}{640}\right)\frac{1}{2} pl^2$$

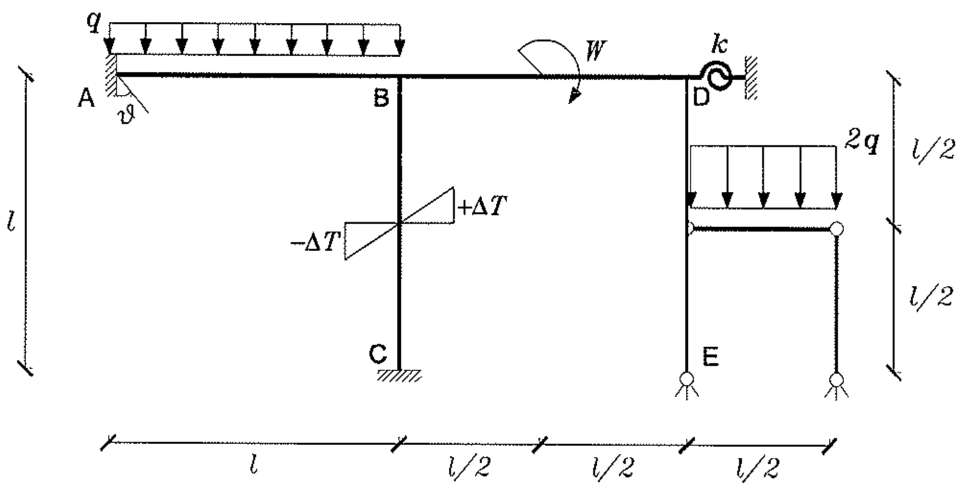
$$= \frac{239121}{819200} pl^2 = 0{,}29\, p\ell^2$$

Tecnica delle Costruzioni

Tema Esame del 25 novembre 2011

Docente: **Ing. Fausto Minelli** Esercitatore: **Ing. Francesca Feroldi**

Fila B



$$\frac{\alpha\,\Delta T}{h} = \frac{1}{6}\frac{ql^2}{EJ}$$

$$k = \frac{EJ}{l}$$

$$\vartheta = \frac{29}{24}\,\frac{ql^3}{EJ}$$

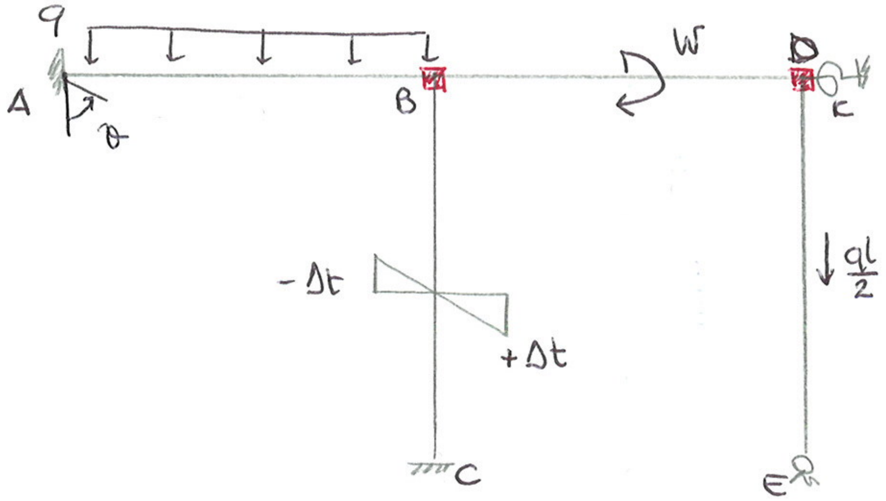
$$W = 2ql^2$$

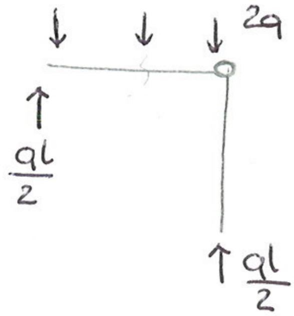
Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

(B) (1)

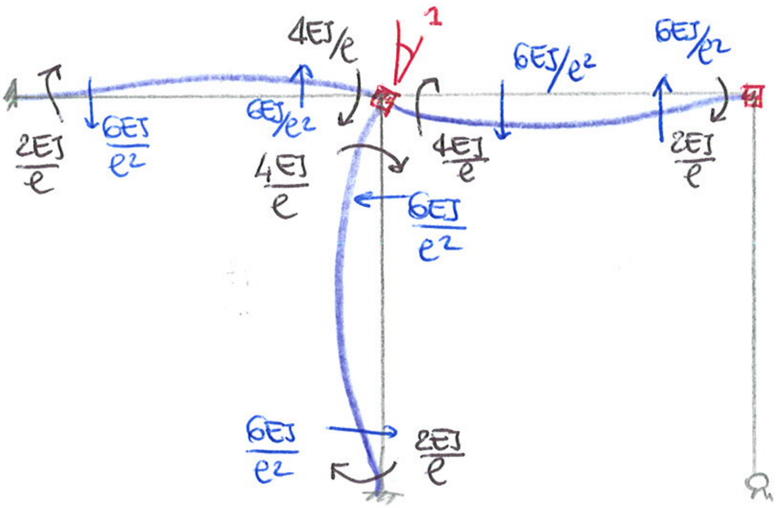


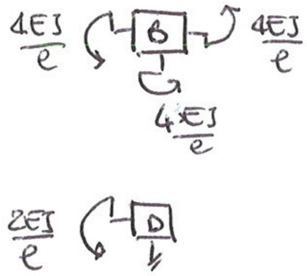


$$\begin{cases} K = \dfrac{EJ}{l} \\ W = 2ql^2 \\ \theta = \dfrac{29}{24}\dfrac{ql^3}{EJ} \\ \dfrac{\alpha \Delta t}{h} = \dfrac{1}{6}\dfrac{ql^2}{EJ} \end{cases}$$

$$\begin{cases} m_{BB} \cdot \varphi_B + m_{BD} \cdot \varphi_D + m_{B0} = 0 \\ m_{DB} \cdot \varphi_B + m_{DD} \cdot \varphi_D + m_{D0} = 0 \end{cases}$$

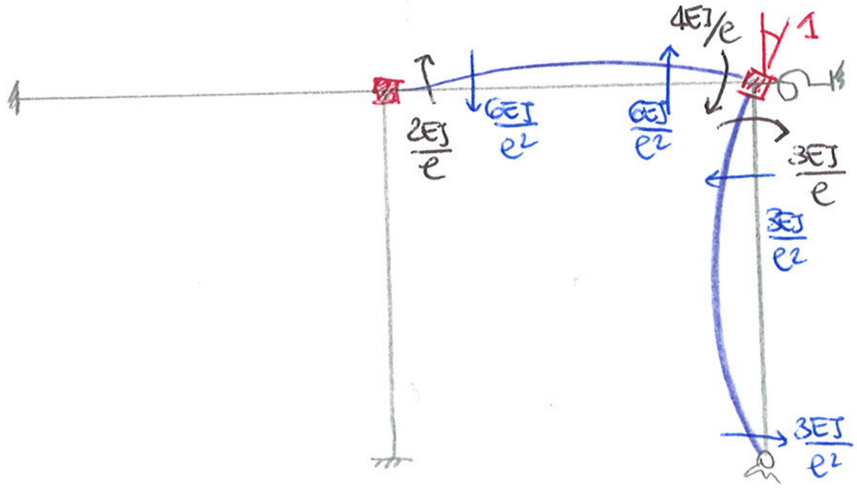
(1) $\varphi_B = 1$

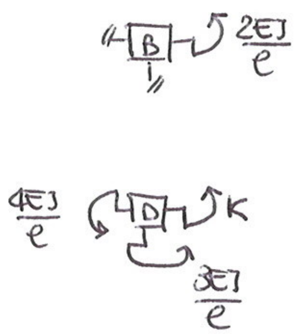




$$\begin{cases} m_{BB} = 12\dfrac{EJ}{l} \\ m_{DB} = \dfrac{2EJ}{l} \end{cases}$$

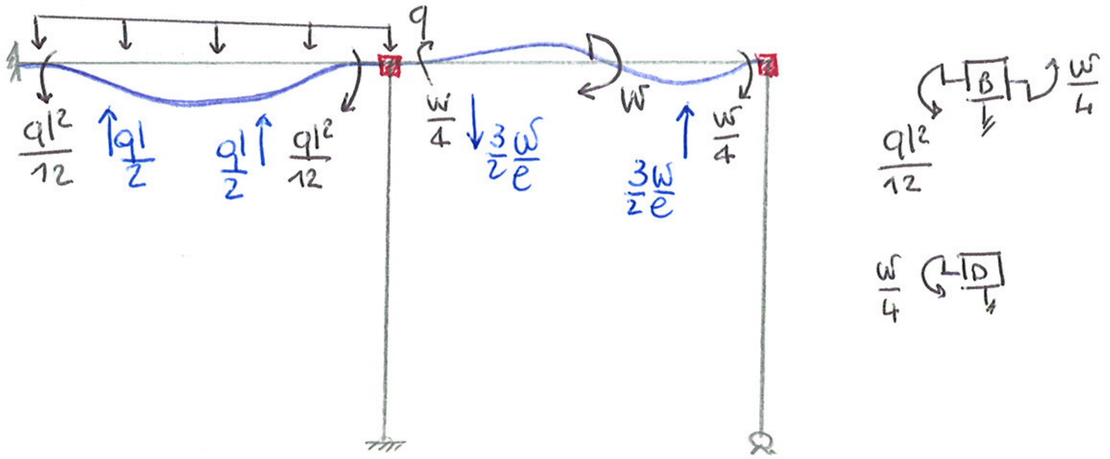
(2) $\varphi_D = 1$





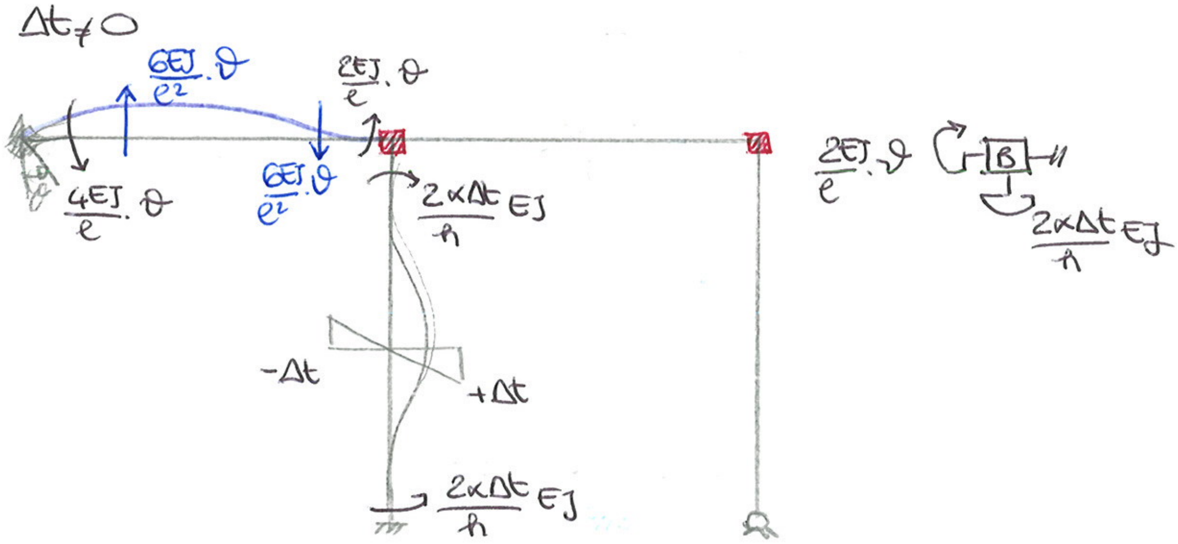
$$\begin{cases} m_{BD} = \dfrac{2EJ}{l} \\ m_{DD} = 7EJ/l + K \end{cases}$$

③ $q \neq 0 \qquad W \neq 0$



$$\begin{cases} m_{BO} = \dfrac{ql^2}{12} + \dfrac{W}{4} \\ m_{CO} = \dfrac{W}{4} \end{cases}$$

④ $\vartheta \neq 0 \qquad \Delta t \neq 0$



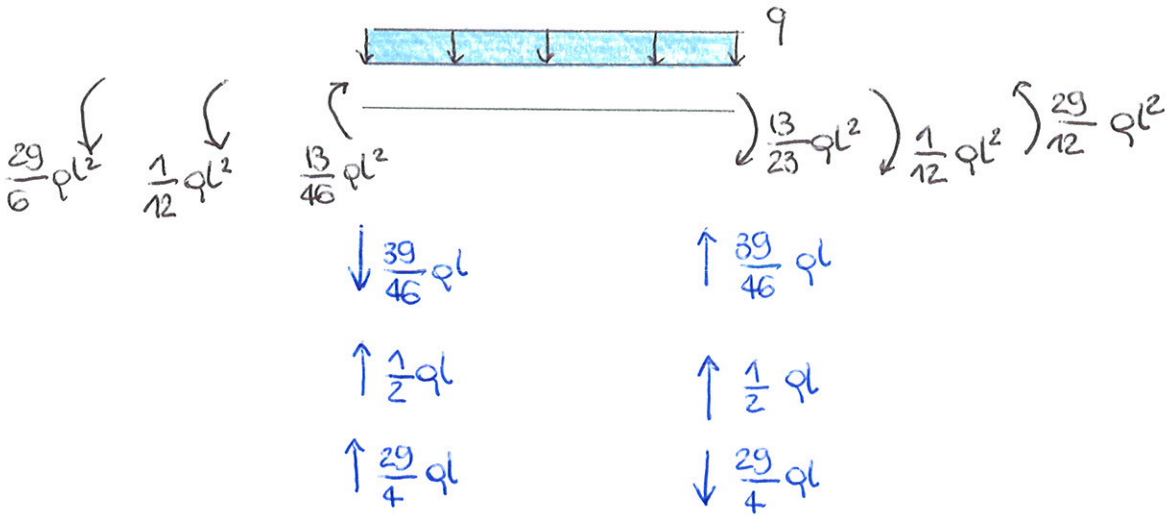
$$\begin{cases} m_{BO} = \dfrac{2\alpha\Delta t}{h} EJ - \dfrac{2EJ}{l} \cdot \vartheta \\ m_{CO} = 0 \end{cases}$$

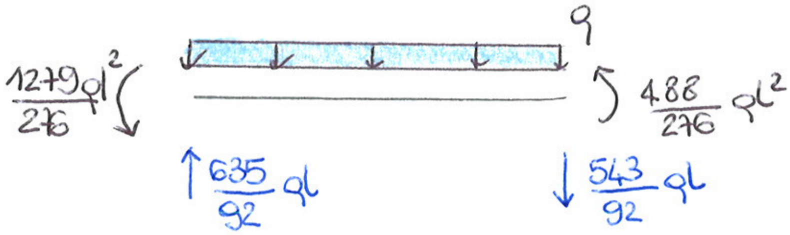
Sistema:

$$\begin{cases} 12\dfrac{EJ}{l} \cdot \varphi_B + \dfrac{2EJ}{l} \cdot \varphi_D + \dfrac{ql^2}{12} + \dfrac{W}{4} + \dfrac{2\alpha\Delta t}{h} EJ - \dfrac{2EJ}{l} \cdot \vartheta = 0 \\ \dfrac{2EJ}{l} \cdot \varphi_B + (7+k)\dfrac{EJ}{l} \cdot \varphi_D + \dfrac{W}{4} = 0 \end{cases}$$

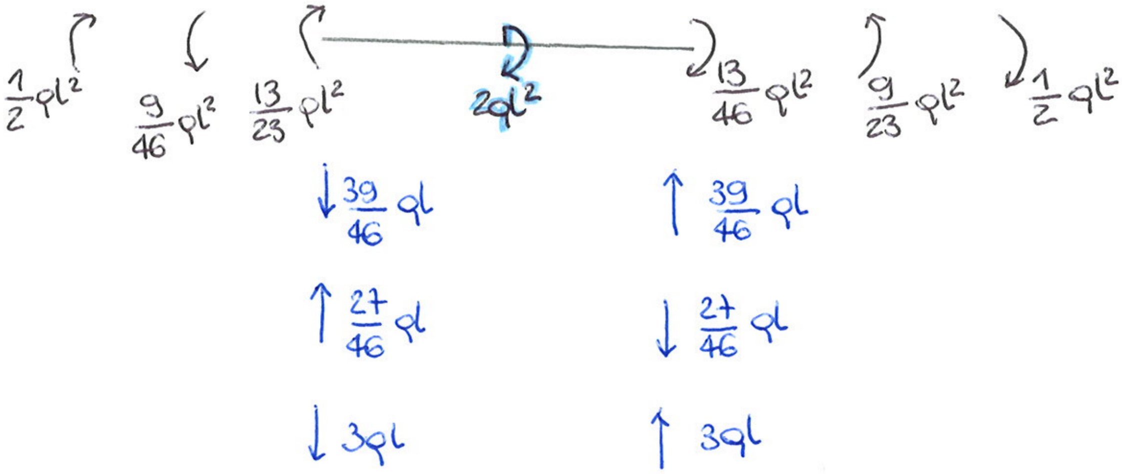
$$\begin{cases} \boxed{\varphi_B = \dfrac{13}{92}\dfrac{ql^3}{EJ}} \\ \boxed{\varphi_D = -\dfrac{9}{92}\dfrac{ql^3}{EJ}} \end{cases}$$

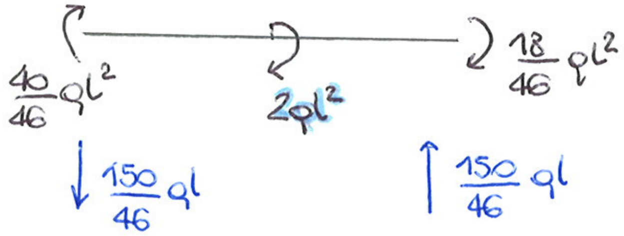
Asta AB



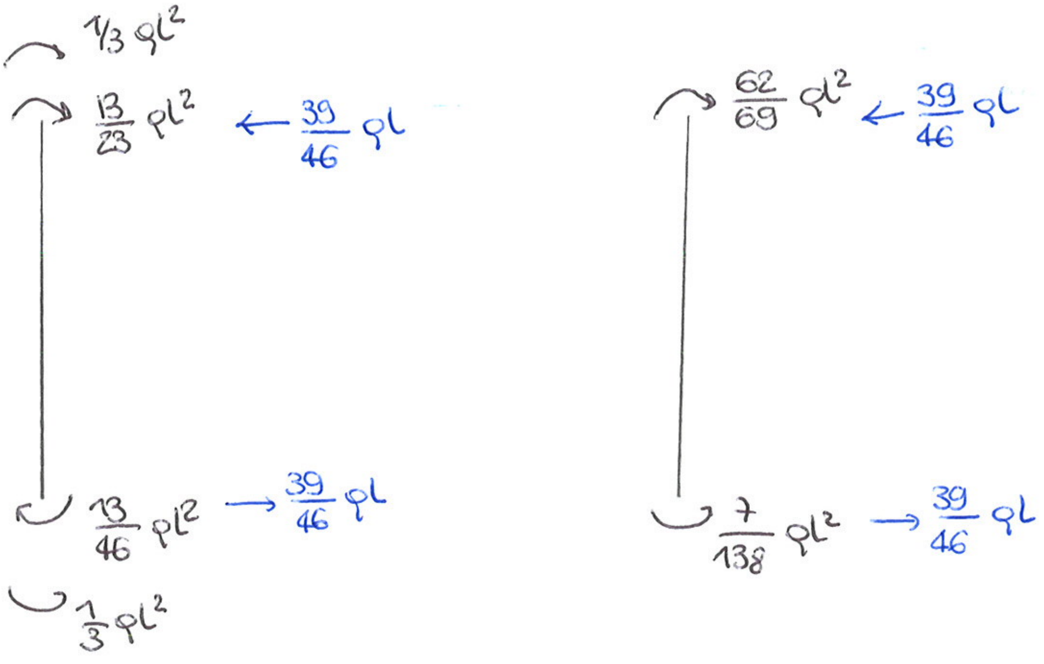


Asta BD

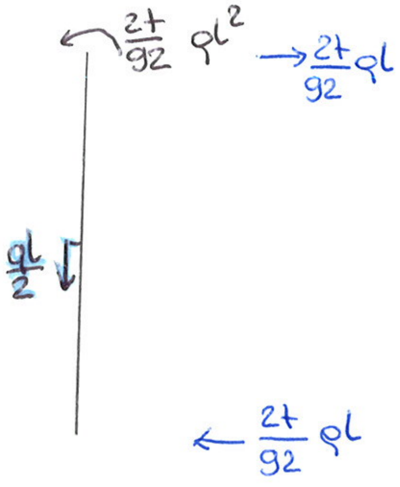




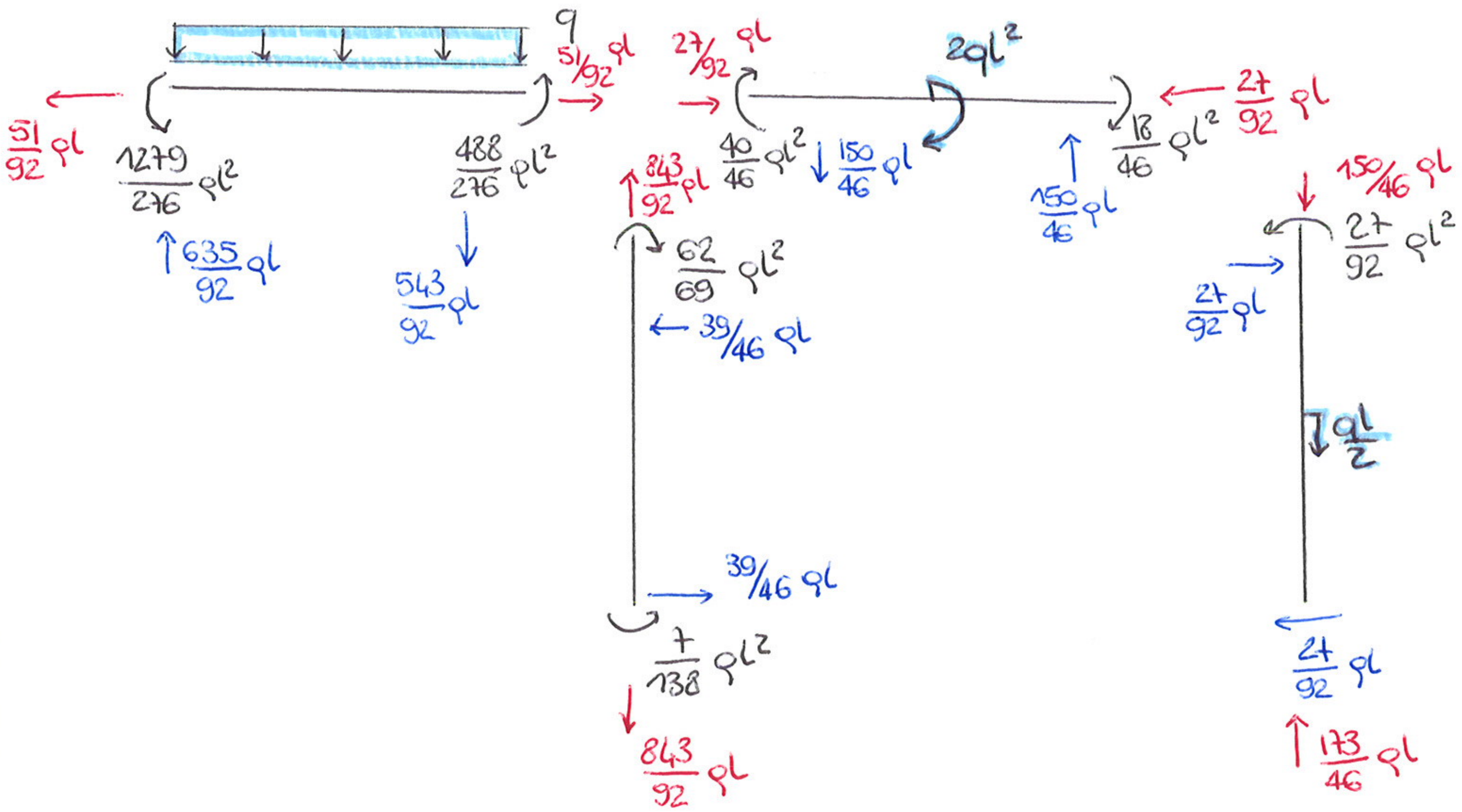
Asta BC



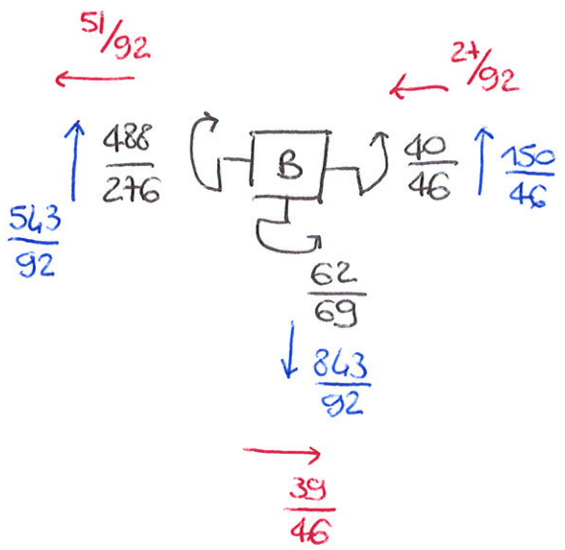
Asta DE

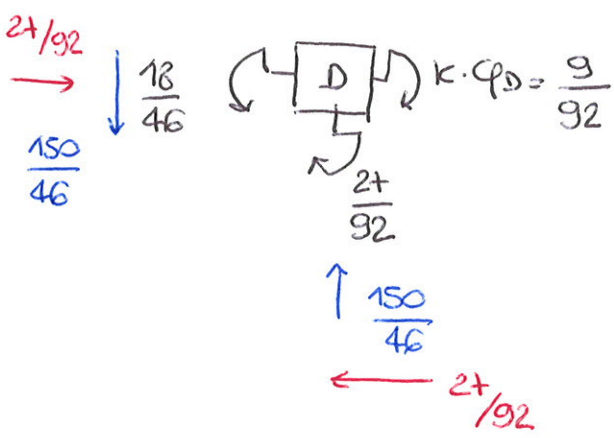


Equilibrio globale

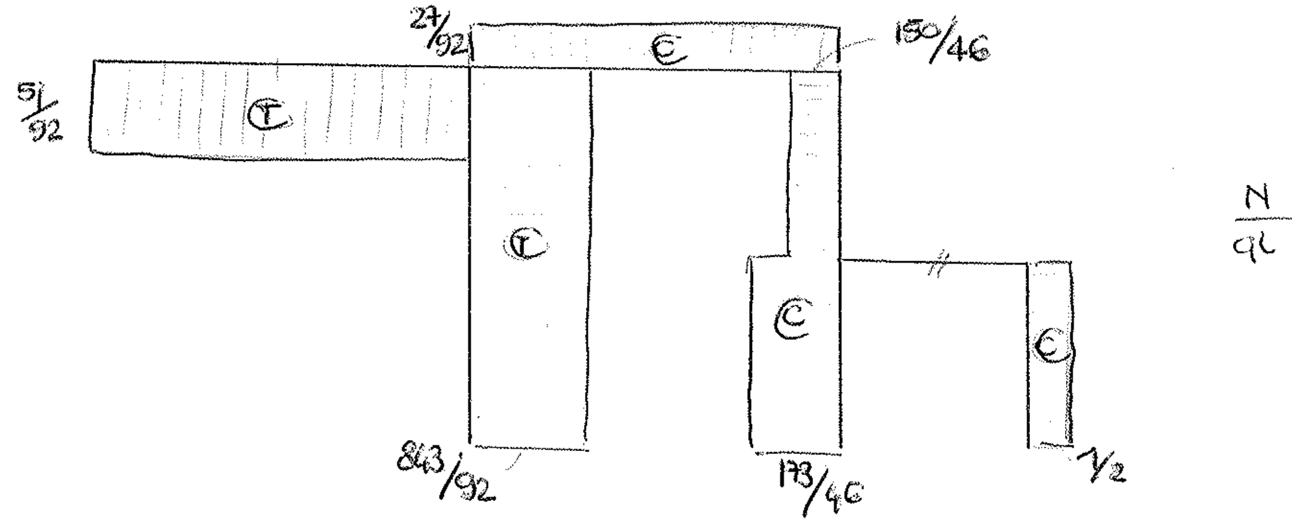


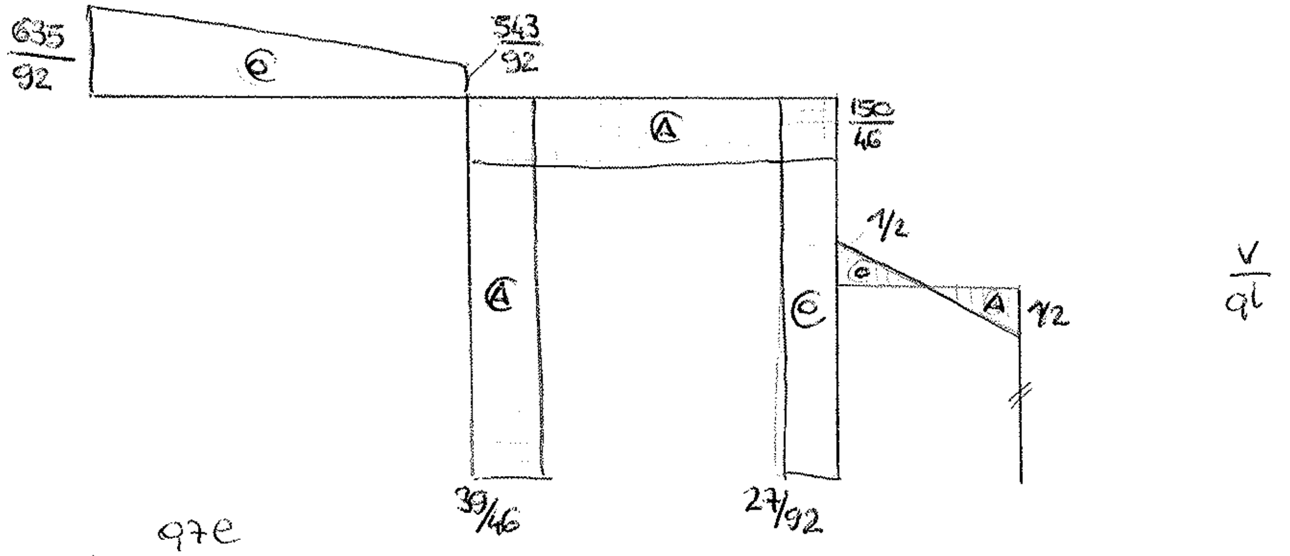
Equilibrio di nodi:

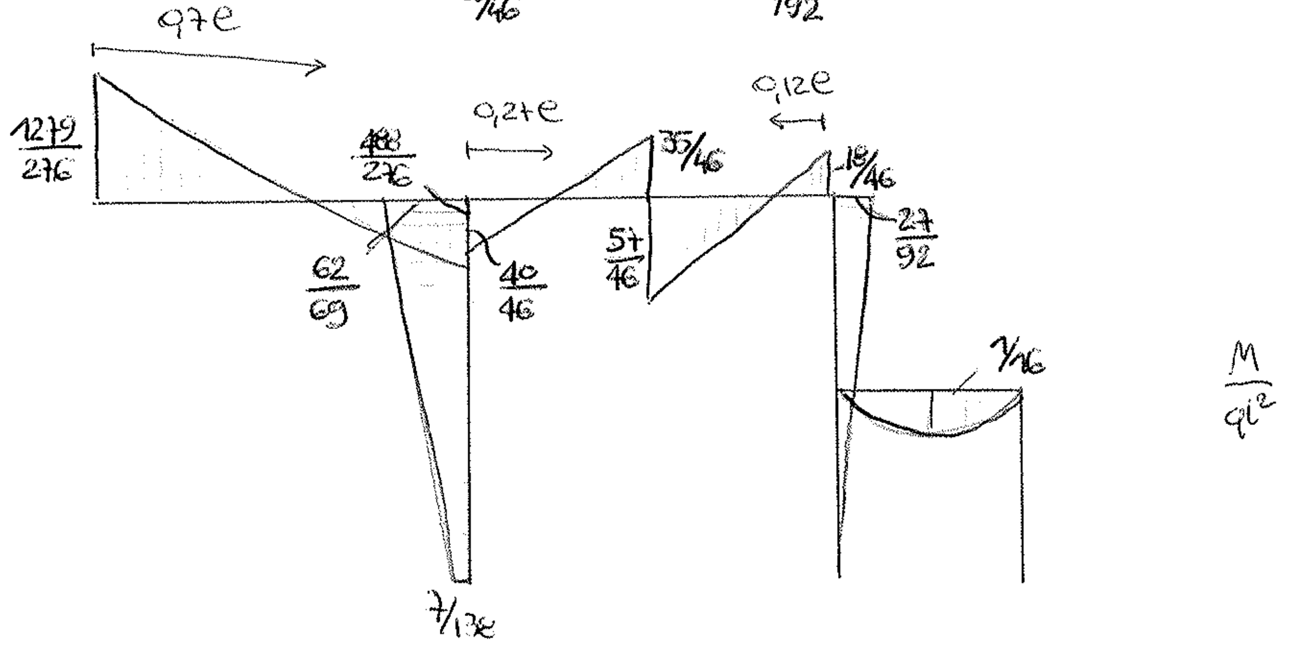




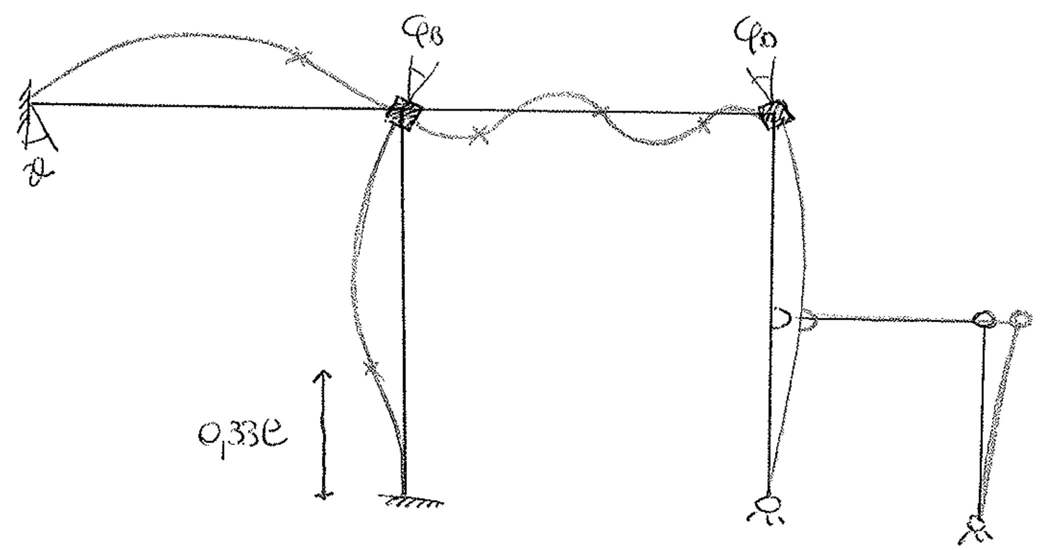
Diagrammi


27/92
150/46
51/92
843/92
173/46
1/2
N/ql

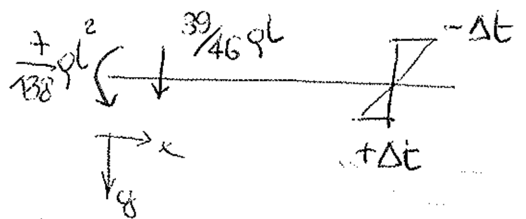

635/92
543/92
150/46
1/2
1/2
39/46
27/92
V/ql


0,7e
0,2+e
0,12e
1279/276
488/276
35/46
18/46
62/69
40/46
57/46
27/92
7/46
7/138
M/ql²

Deformata



$$y''(x) = -\frac{M(x)}{EJ} \pm \frac{2\alpha\Delta t}{h}$$



$$y''(x) = -\frac{1}{EJ}\left(-\frac{7}{138}ql^2 - \frac{39}{46}qlx\right) - \frac{1}{3}\frac{ql^2}{EJ}$$

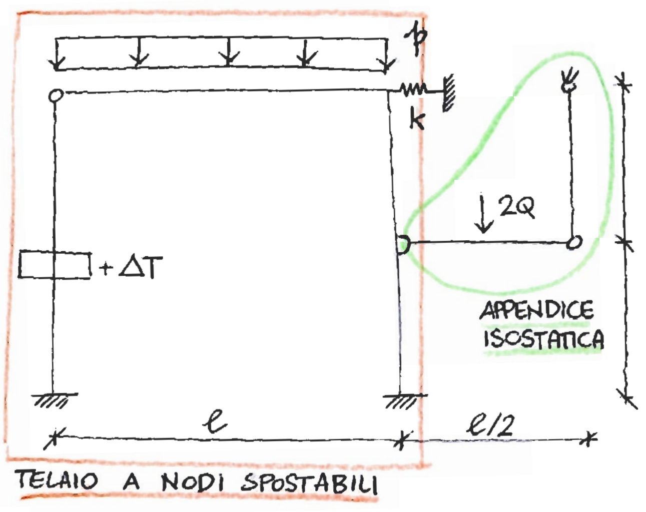
$$y''(x) = \frac{7}{138}ql^2 - \frac{1}{3}ql^2 + \frac{39}{46}qlx \qquad y''(x) > 0$$

$$-\frac{39}{138}ql^2 + \frac{39}{46}qlx > 0$$

$$x > \frac{13}{46}\cdot\frac{46}{39}l$$

$$x > 0{,}33l$$

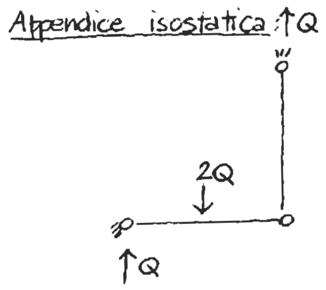
TEMA ESAME 'TECNICA delle COSTRUZIONI' TEMA ESAME 16/01/2012

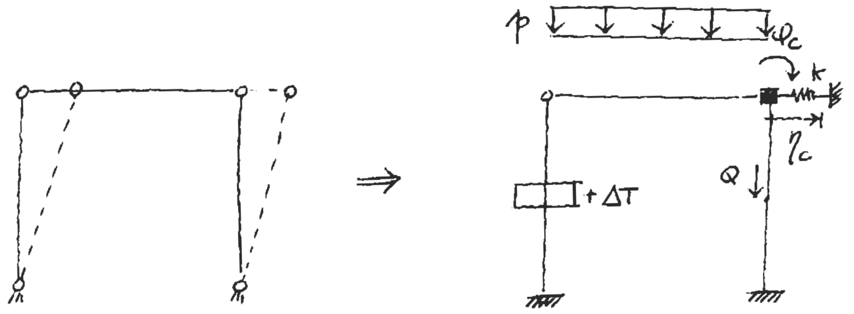


$$\alpha \Delta T = \frac{11}{8}\frac{pL^3}{EJ}$$

$$k = \frac{3EJ}{\ell^3}$$

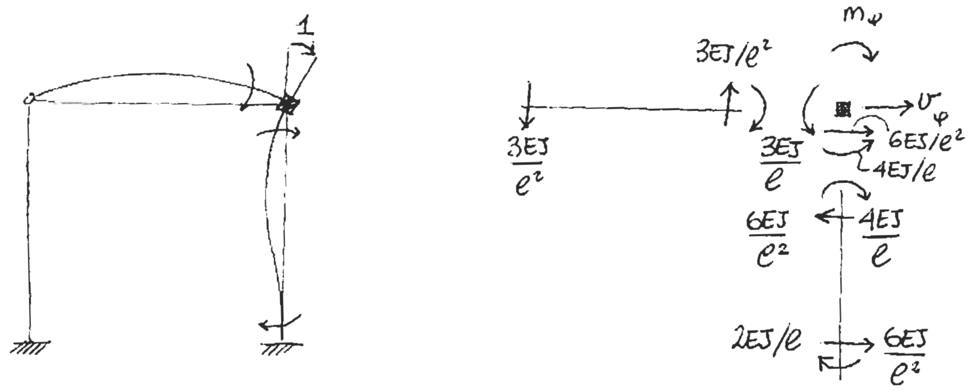
$$Q = \frac{3}{2} p \ell$$





2 INCOGNITE: φ_c, η_c

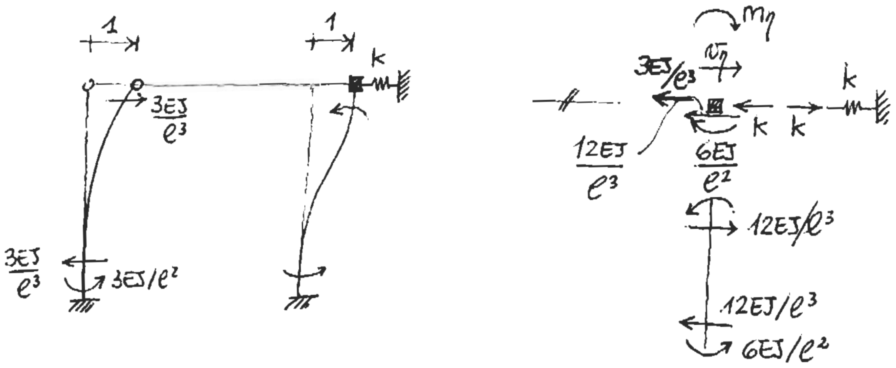
$\varphi_c = 1$)



$$m_\varphi = \frac{7EJ}{\ell}$$

$$v_\varphi = -6EJ/\ell^2$$

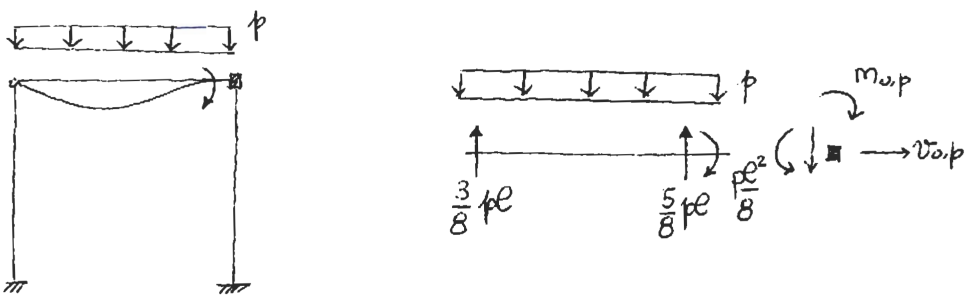
$\eta_c = 1$)



$$m_\eta = -\frac{6EJ}{\ell^2}$$

$$v_\eta = \frac{15EJ}{\ell^3} + k = \frac{18EJ}{\ell^3}$$

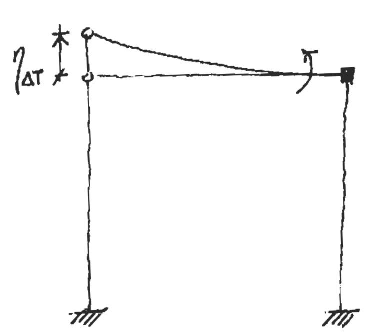
p)

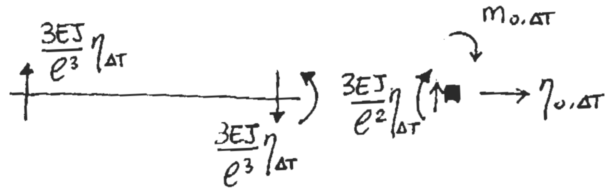


$$m_{o,p} = p\ell^2/8$$

$$v_{o,p} = \varnothing$$

ΔT)





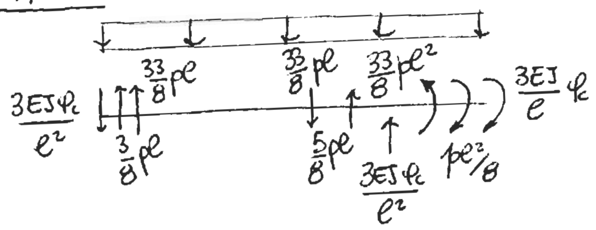
$$m_{0,\Delta T} = -\frac{3EJ}{\ell^2}\cdot\eta_{\Delta T} = \ldots \quad \eta_{\Delta T} = \alpha\Delta T\ell = \frac{11}{8}\cdot\frac{p\ell^4}{EJ} \ldots = \ldots -\frac{33}{8}p\ell^2$$

SISTEMA RISOLVENTE:

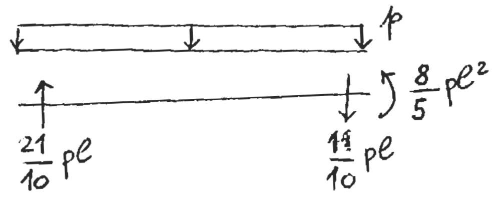
$$\begin{cases} \frac{7EJ}{\ell}\varphi_c - \frac{6EJ}{\ell^2}\eta_c + \frac{p\ell^2}{8} - \frac{33}{8}p\ell^2 = 0 \quad \Rightarrow \quad \frac{7EJ}{\ell}\varphi_c - \frac{6EJ}{\ell^2}\eta_c = 4p\ell^2 \\ -\frac{6EJ}{\ell^2}\varphi_c + \frac{18EJ}{\ell^3}\eta_c = 0 \quad \Rightarrow \quad \eta_c = \frac{1}{3}\varphi_c\cdot\ell \end{cases}$$

$$\frac{7EJ}{\ell}\varphi_c - \frac{2EJ}{\ell}\varphi_c = 4p\ell^2 \Rightarrow \boxed{\varphi_c = +\frac{4}{5}\cdot\frac{p\ell^3}{EJ}} \quad e \quad \boxed{\eta_c = +\frac{4}{15}\cdot\frac{p\ell^4}{EJ}}$$

ASTA BC

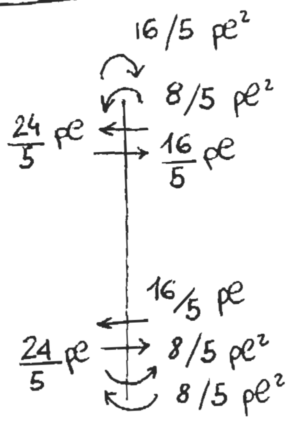


$\Rightarrow$

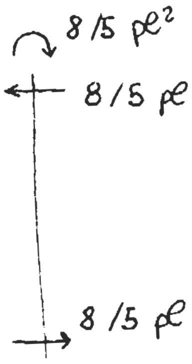


$$\frac{3EJ}{\ell}\varphi_c = \frac{60}{23}p\ell^2 \qquad \frac{3EJ}{\ell^2}\varphi_c = \frac{60}{23}p\ell$$

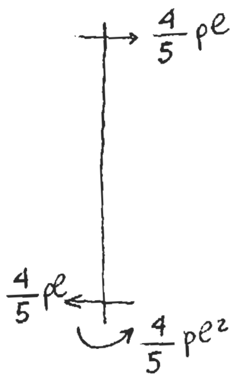
ASTA CD



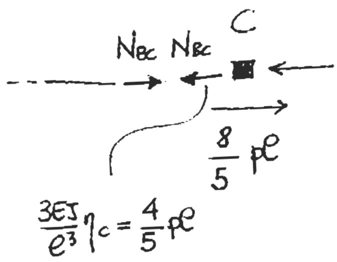
$\Rightarrow$



ASTA AB



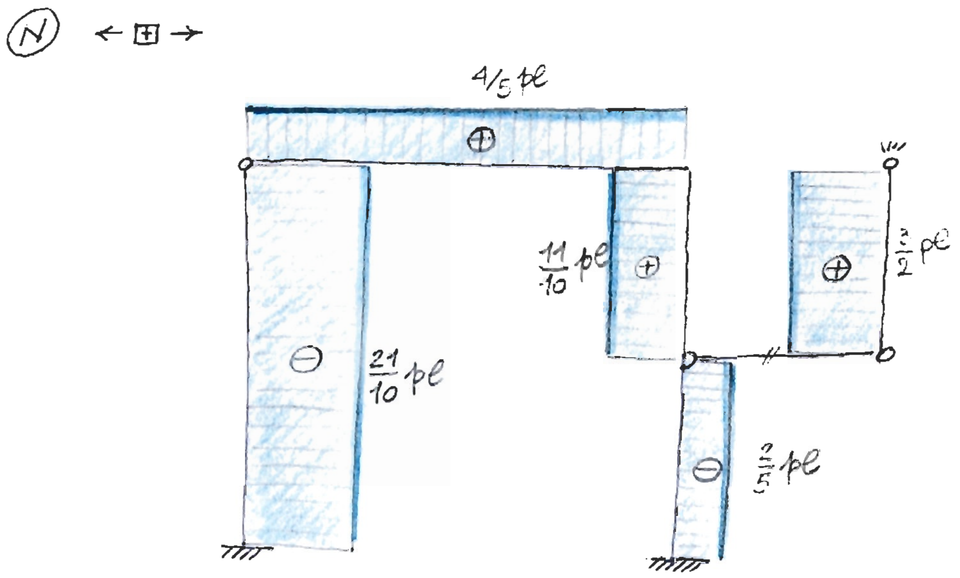
EQUILIBRIO AL NODO C:

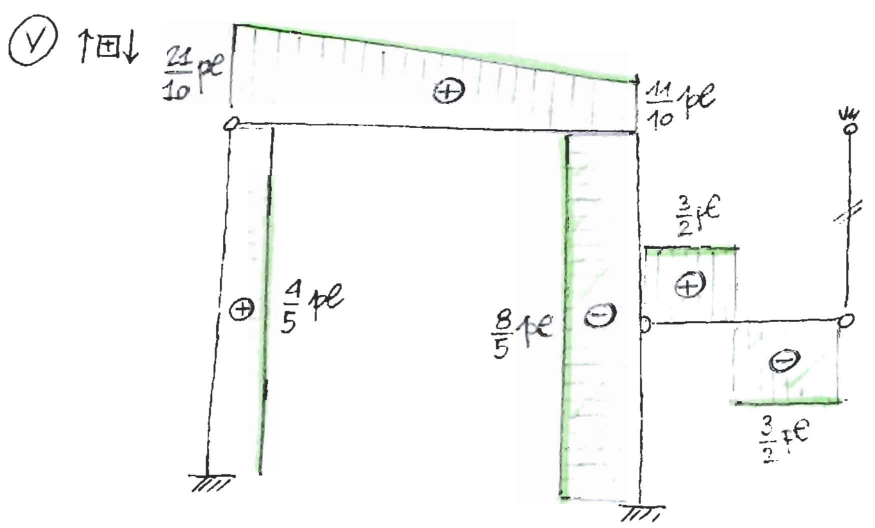


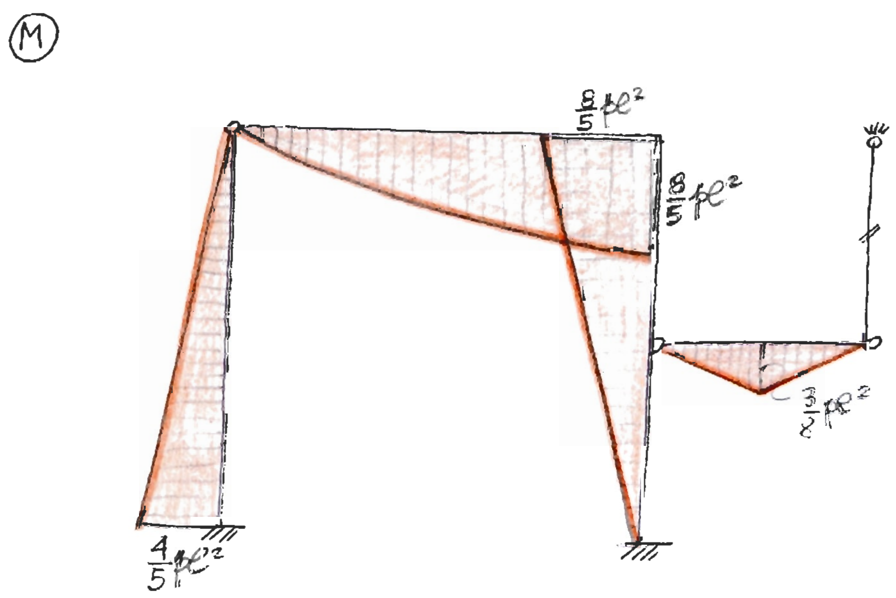
$$k\eta_c = \frac{3EJ}{\ell^3}\cdot\frac{4}{15}\cdot\frac{p\ell^4}{EJ} = \frac{4}{5}p\ell \quad \Rightarrow \quad N_{BC} = \frac{4}{5}p\ell;$$

CONFERMA L'EQUILIBRIO DEL NODO B

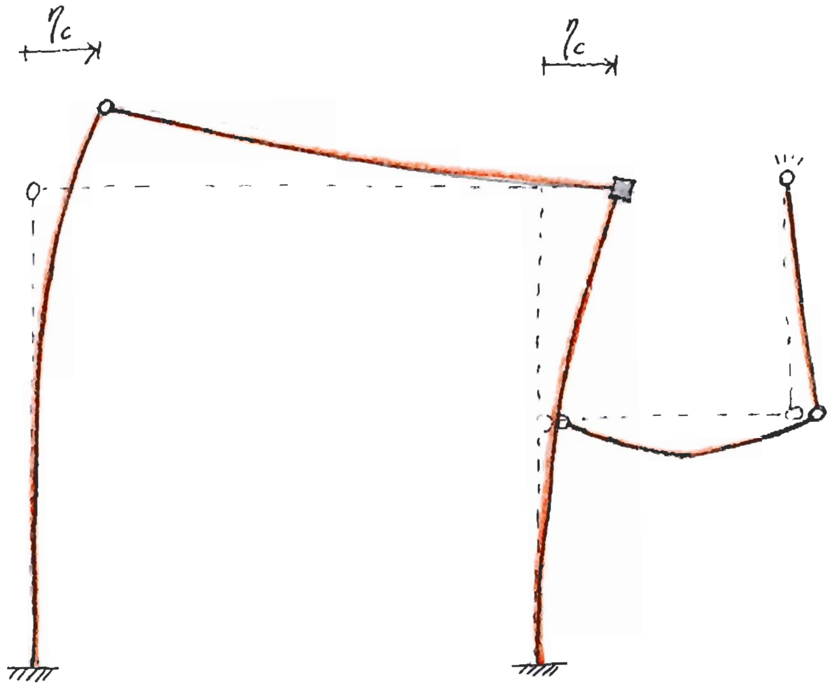
DIAGRAMMI AZIONI INTERNE







DEFORMATA QUALITATIVA

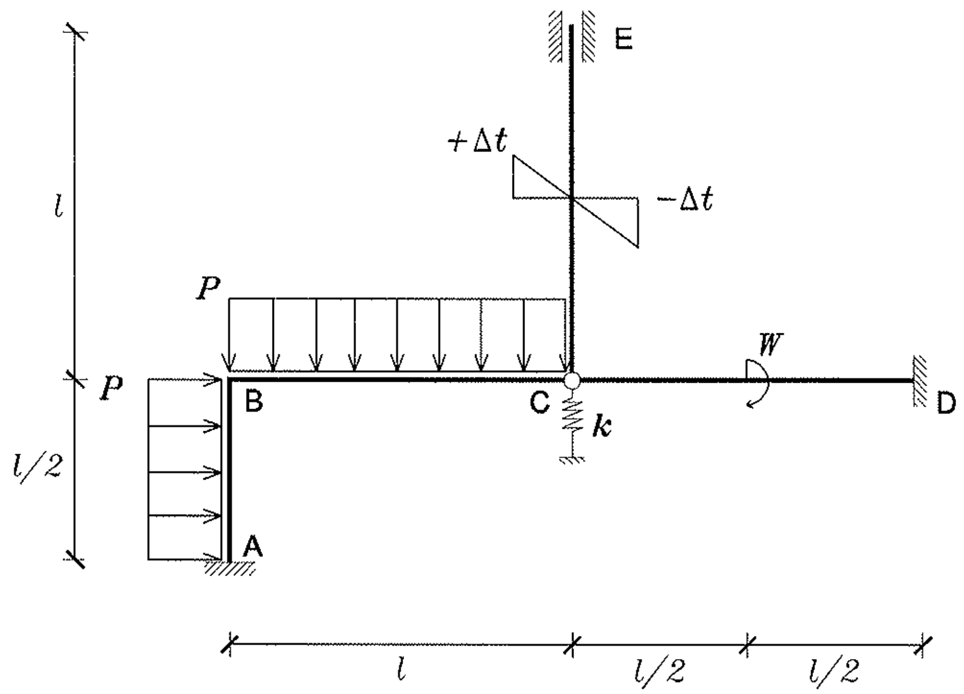


FONDAMENTI DI PROGETTAZIONE STRUTTURALE
ESAME 30/01/2012, 3h

Ing. F. Minelli

Nome:………………….. *Cognome:* ………………………….. *n.matr.:* …………

ESERCIZIO 1 (20 punti)



$$W = \frac{1}{18} P l^2$$

$$k = 3\frac{EJ}{l^3}$$

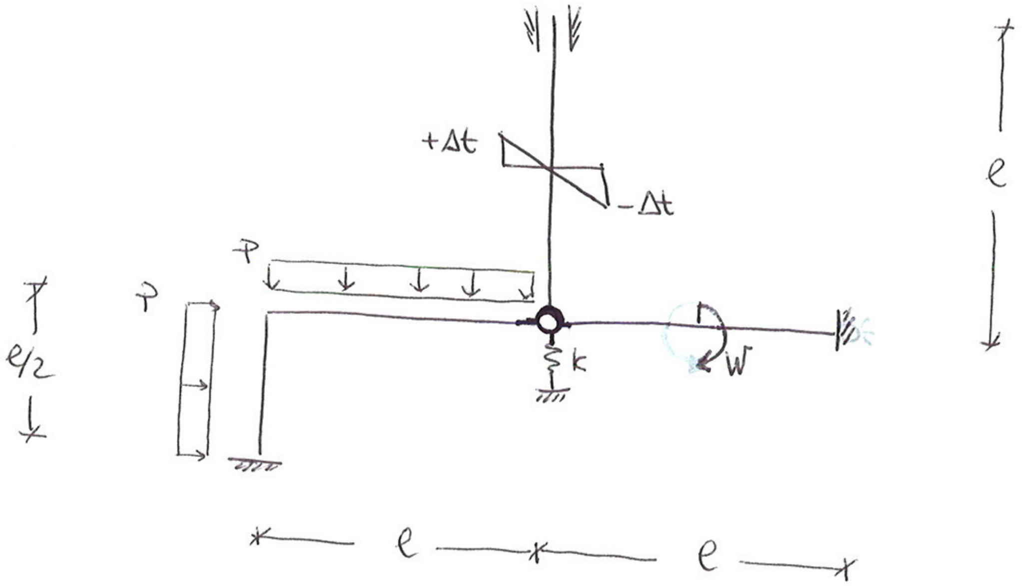
$$\frac{\alpha \Delta t}{h} = \frac{5}{12}\frac{Pl^2}{EJ}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi)
2. Taglio
3. Azione assiale
4. Deformata qualitativa con posizione dei flessi

T.E. 30/01/2012

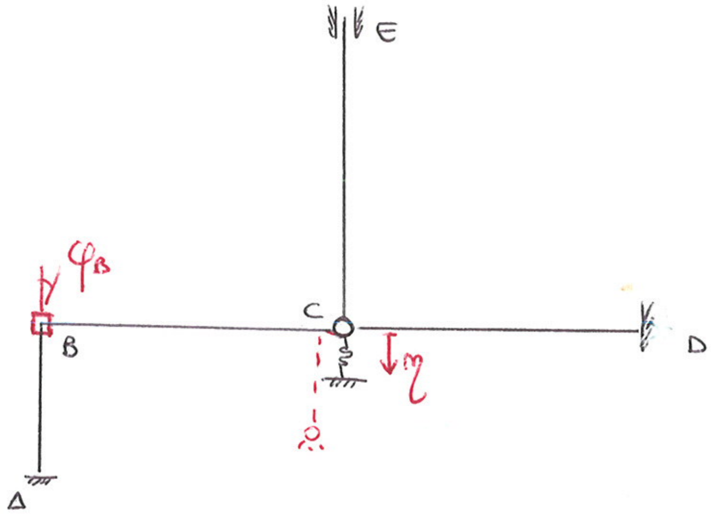


$$\frac{\alpha \Delta t}{h} = \frac{5}{12}\frac{pl^2}{EJ}$$

$$K = \frac{3EJ}{l^3}$$

$$W = \frac{1}{18}pl^2$$

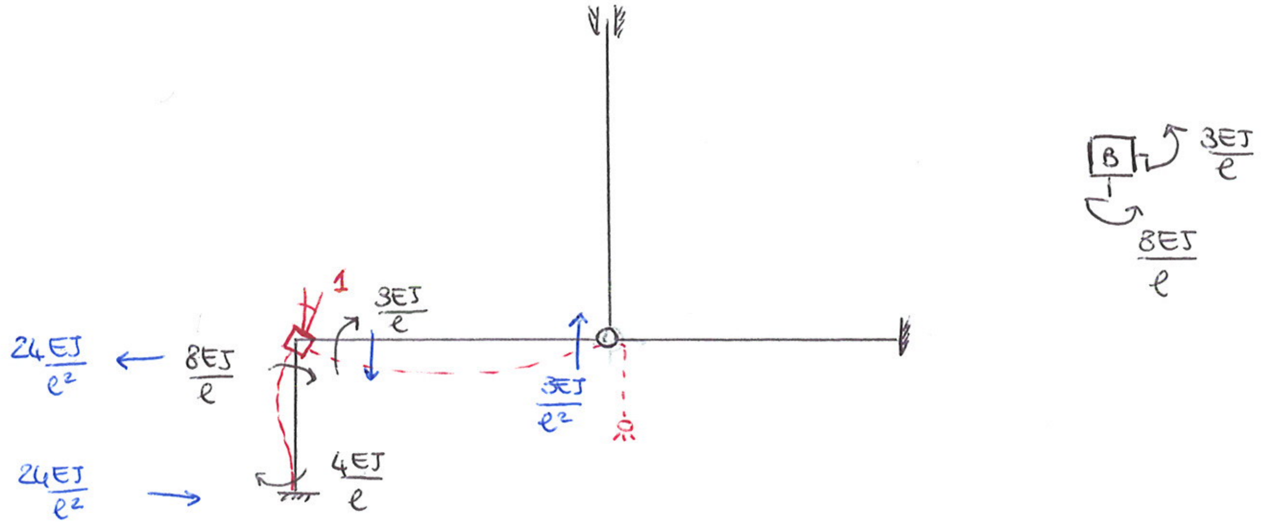
Struttura a nodi spostabili



$$\begin{cases} m_{BB}\cdot\varphi_B + m_{B\eta}\cdot\eta + m_{B0} = 0 \\ r_{\eta B}\cdot\varphi_B + r_{\eta\eta}\cdot\eta + r_{\eta 0} = 0 \end{cases}$$

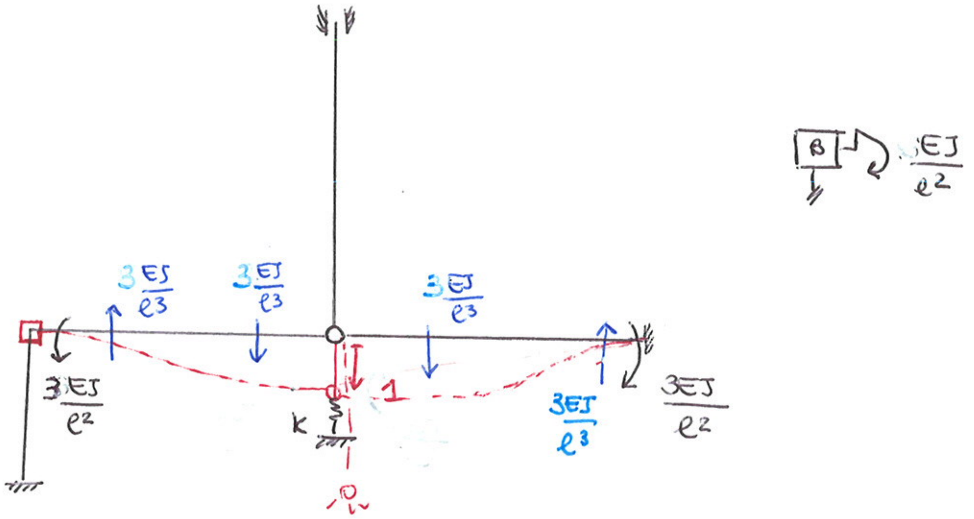
Convenzioni di segno:

① $\varphi_B = 1$



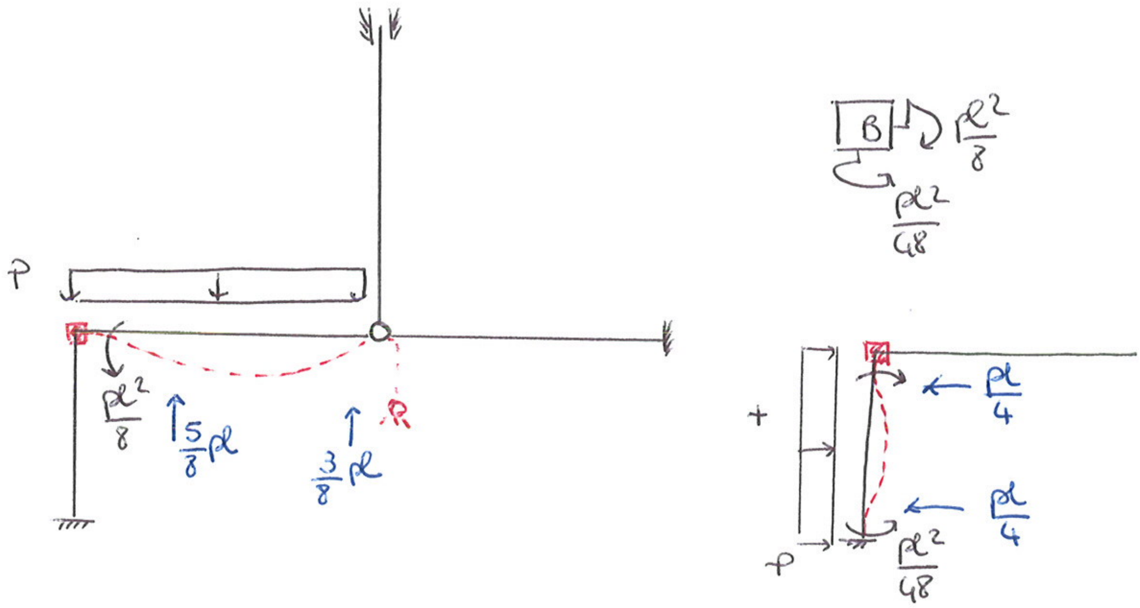
$$\begin{cases} m_{BB} = 11 \dfrac{EJ}{\ell} \\ k_{\eta B} = -\dfrac{3EJ}{\ell^2} \end{cases}$$

② $\eta = 1$



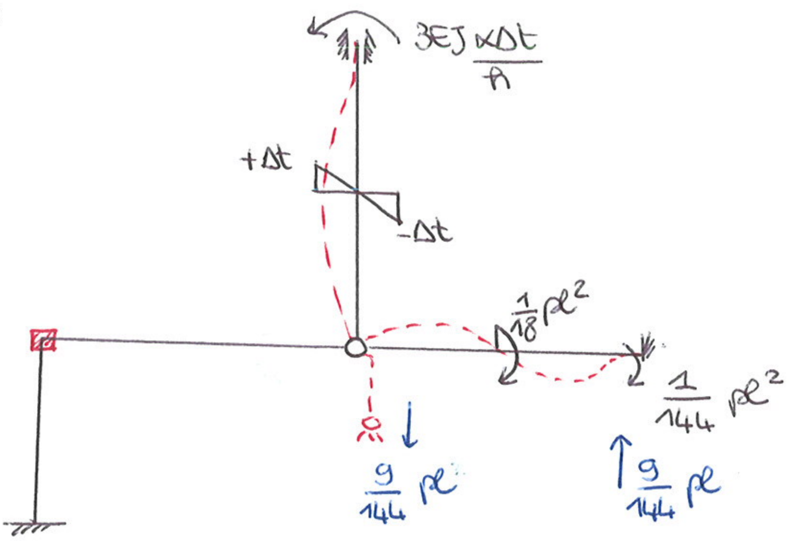
$$\begin{cases} m_{B\eta} = -3 \dfrac{EJ}{\ell^2} \\ k_{\eta\eta} = 6 \dfrac{EJ}{\ell^3} + K \end{cases}$$

③ $p \neq 0$



$$\begin{cases} m_{B0} = -\frac{5}{48}\,pl^2 \\ R_{\eta 0} = -\frac{3}{8}\,pl \end{cases}$$

④ $\Delta t \neq 0 \quad w \neq 0$

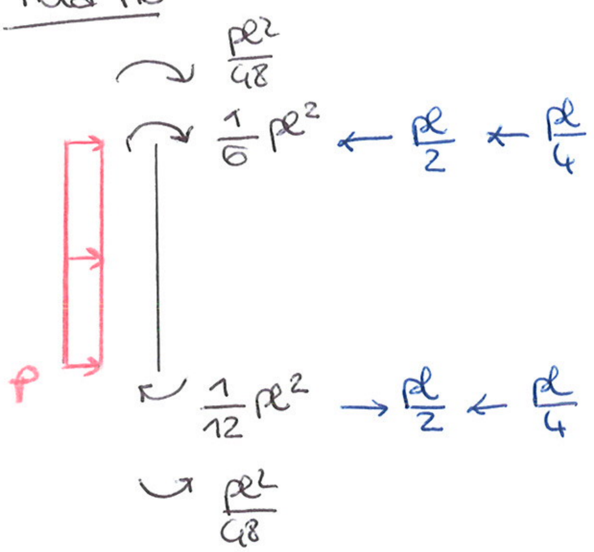


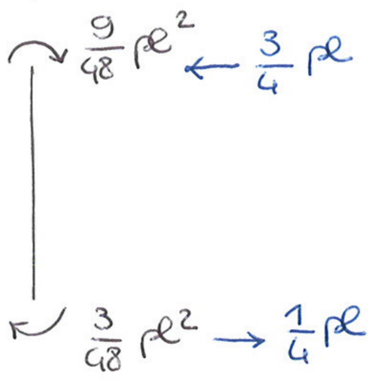
$$\begin{cases} m_{B0} = 0 \\ R_{\eta 0} = \frac{9}{144}\,pl \end{cases}$$

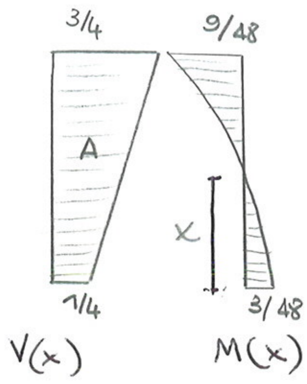
sistema risolvente:

$$\begin{cases} \frac{11EJ}{l}\varphi_B - \frac{3EJ}{l^2}\eta - \frac{5}{48}pl^2 = 0 \\ -\frac{3EJ}{l^2}\varphi_B + \frac{9EJ}{l^3}\eta - \frac{3}{8}pl + \frac{9}{144}pl = 0 \end{cases} \qquad \begin{cases} \varphi_B = \frac{1}{48}\frac{pl^3}{EJ} \\ \eta = \frac{1}{24}\frac{pl^4}{EJ} \end{cases}$$

Asta AB



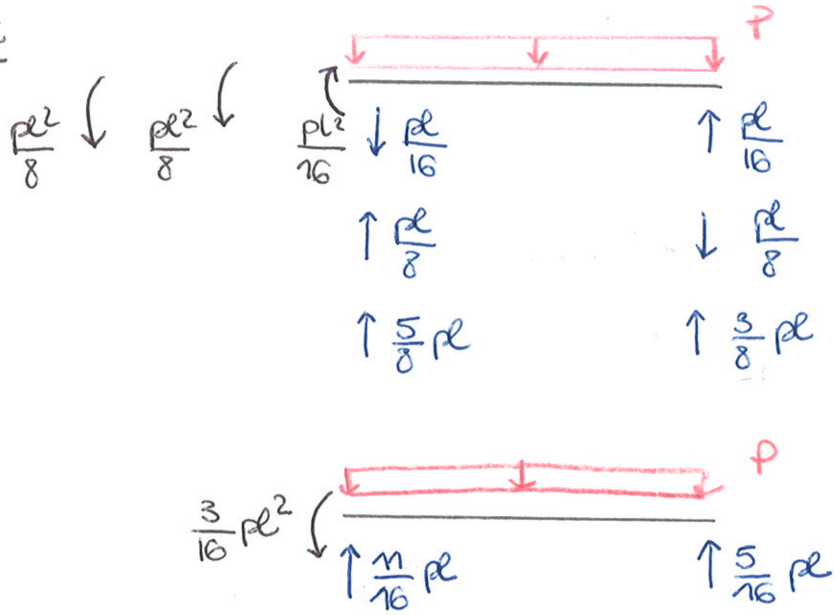


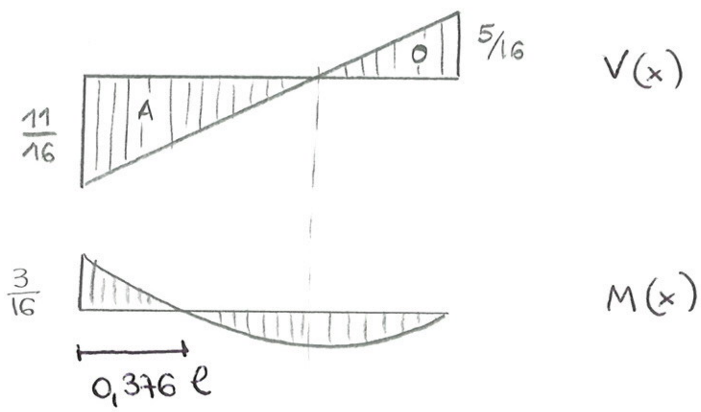


$$M(x) = 0 \rightarrow \frac{3}{48} p l^2 - \frac{1}{4} p l x - \frac{p x^2}{2} = 0$$

$$x = 0{,}183\, l$$

Asta BC

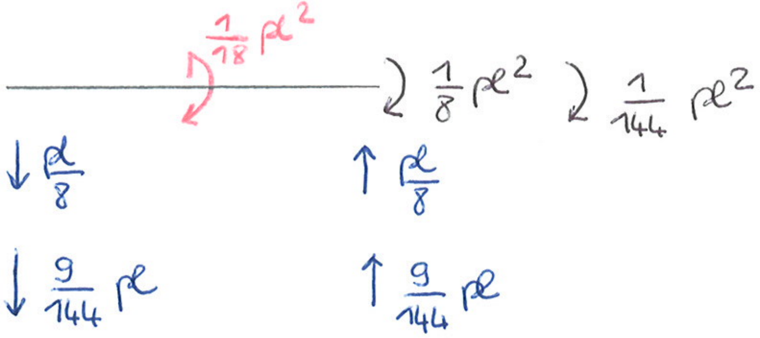


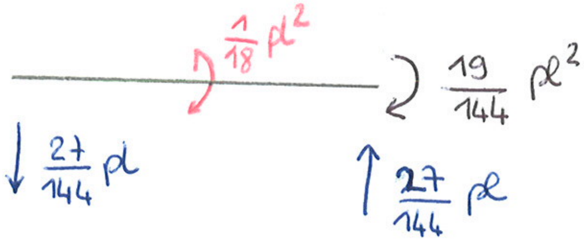


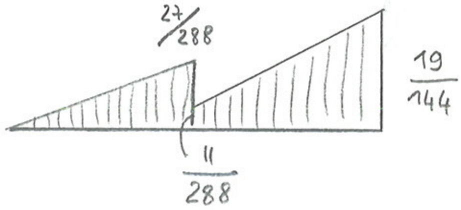
$$M(\bar{x}) = 0 \rightarrow \frac{11}{16} p l \bar{x} - \frac{3}{16} p l^2 - p \frac{\bar{x}^2}{2} = 0$$

$$\bar{x} = \frac{-\frac{11}{16} \pm \sqrt{0{,}472 - 4 \cdot (-1/2)(-3/16)}}{(-1)} = \begin{cases} +1\, l \\ 0{,}376\, l \end{cases}$$

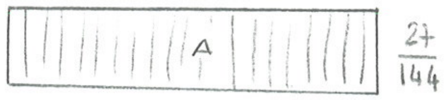
Asta CD





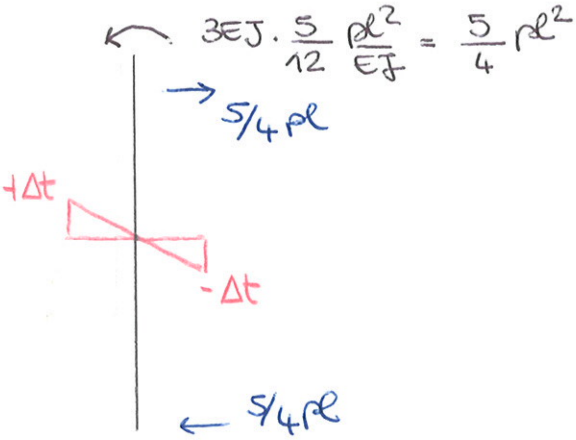


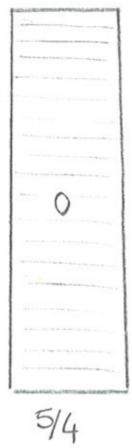
M(x)



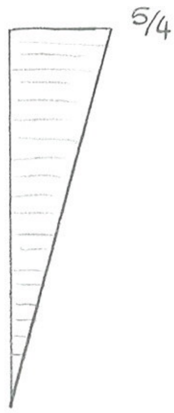
V(x)

Asta CE

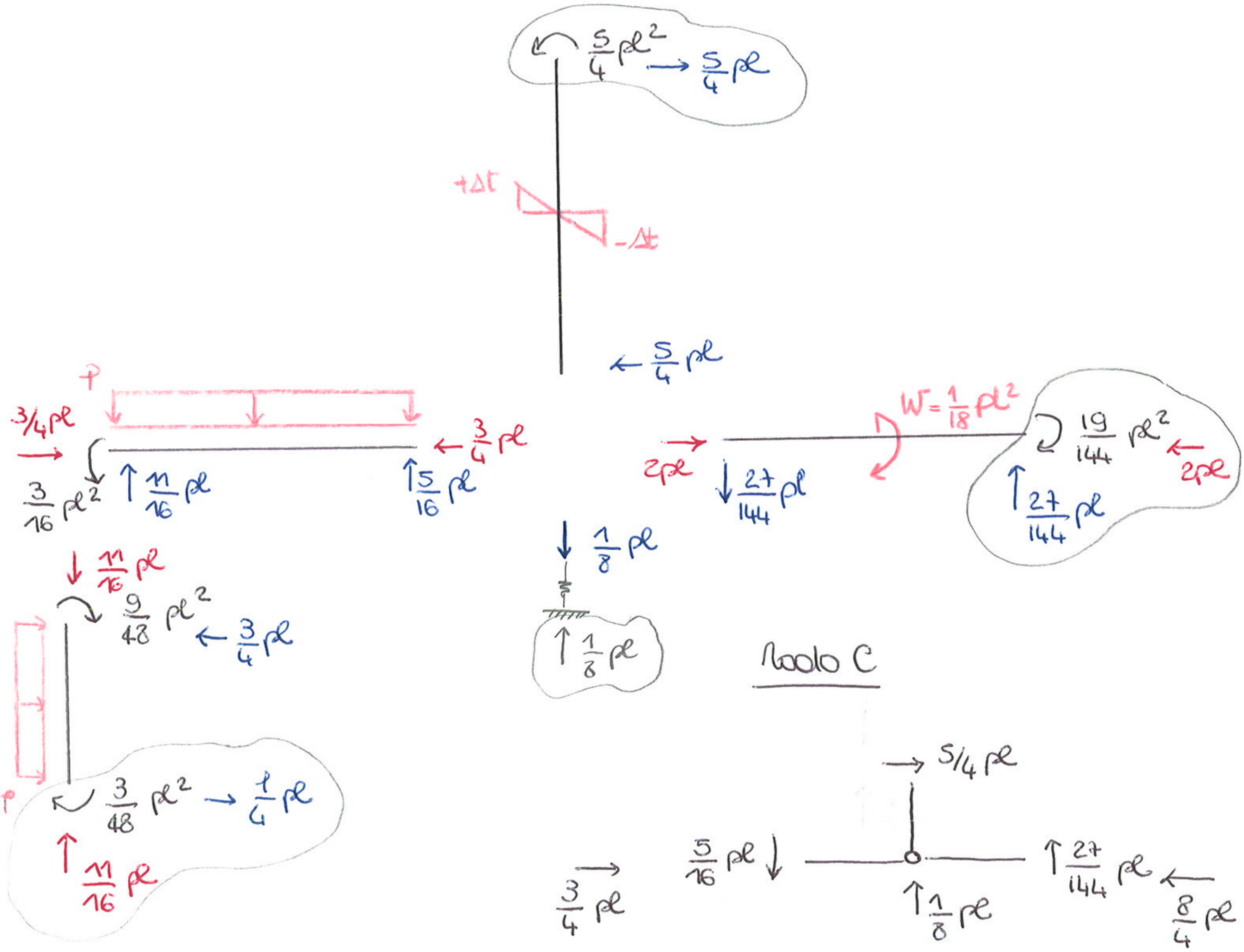




V(x)



M(x)



Equilibrio globale

$$\Sigma V = 0 \rightarrow \frac{11}{16}p\ell - p\ell + \frac{27}{144}p\ell + \frac{1}{8}p\ell = 0$$

$$\frac{99-144+27+18}{144}p\ell = 0 \quad \text{ok}$$

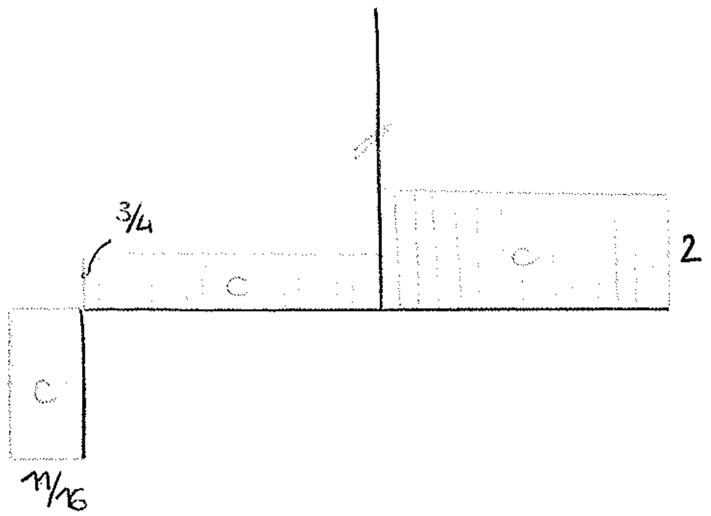
$$\Sigma H = 0 \rightarrow \frac{p\ell}{2} + \frac{1}{4}p\ell + \frac{5}{4}p\ell - \frac{8}{4}p\ell = 0$$

$$\frac{2+1+5-8}{4}p\ell = 0 \quad \text{ok}$$

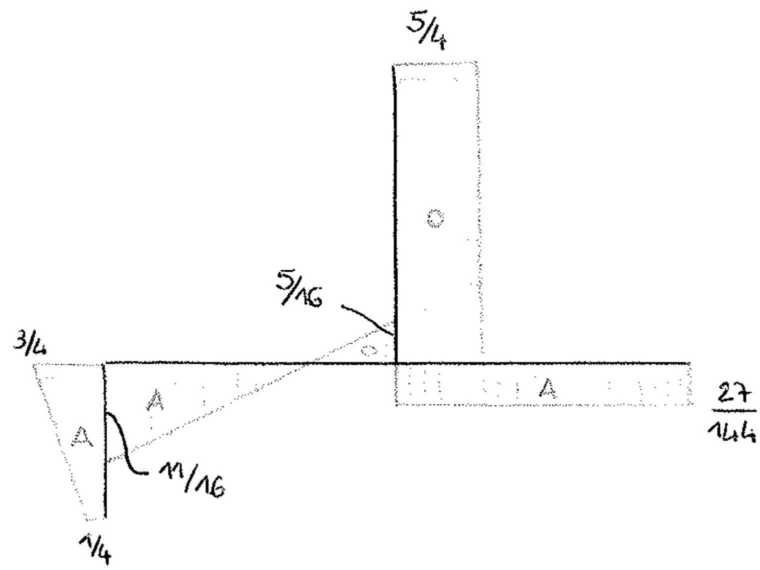
$$\Sigma M(A) = 0 \rightarrow \frac{3}{48}p\ell^2 - \frac{1}{8}p\ell^2 - \frac{1}{8}p\ell^2 - \frac{54}{144}p\ell^2 + \frac{19}{144}p\ell^2 - \frac{5}{4}p\ell^2 + \frac{5}{4}p\ell + \frac{1}{18}p\ell^2 +$$

$$+ \frac{p\ell^2}{2} - \frac{p\ell^2}{8} = 0 \qquad \frac{9-36-54+19-180+180+8+72-18}{144}p\ell^2 = 0 \quad \text{ok}$$

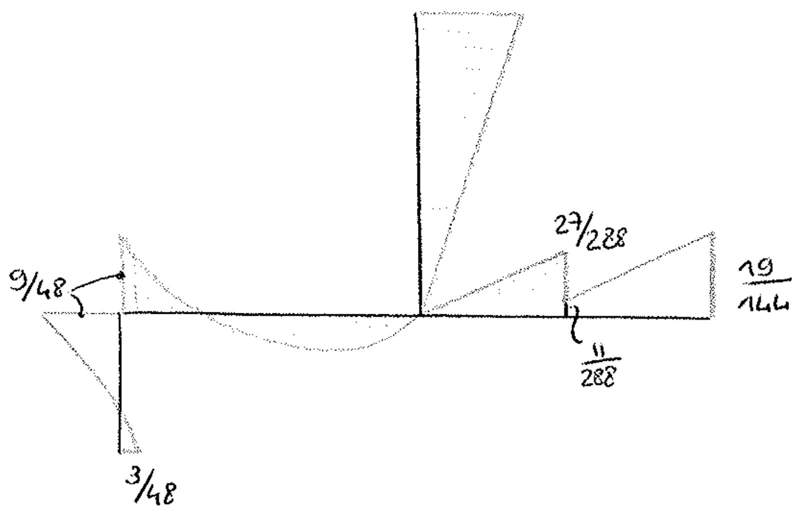
Diagrammi



$\left[\frac{N}{pl}\right]$

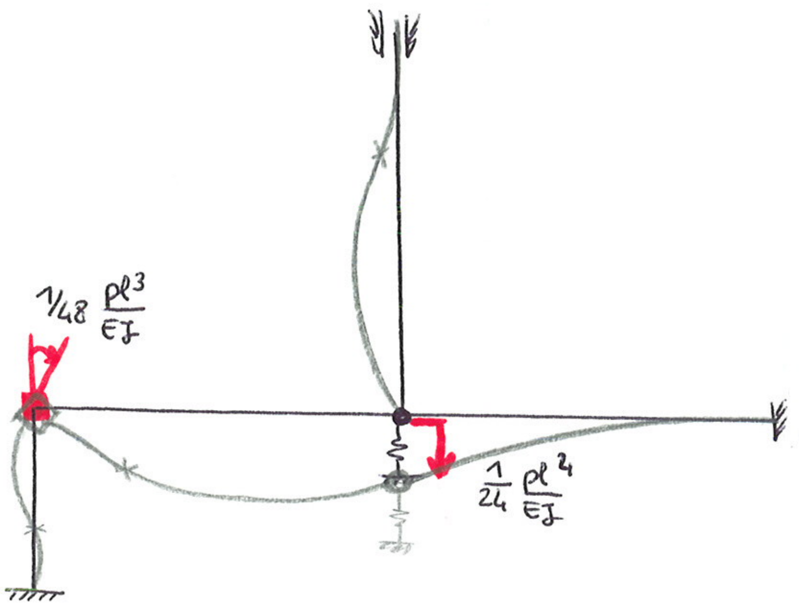


$\left[\frac{V}{pl}\right]$

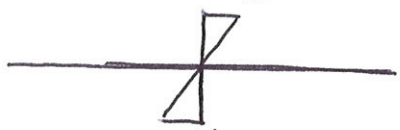


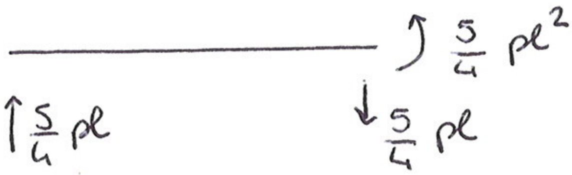
$\left[\frac{M}{pl^2}\right]$

Deformata



Calcolo deformata dell' asta CE:





$$y''(x) = -\frac{M(x)}{EJ} \pm \frac{2\alpha \Delta t}{t}$$

$$y''(x) = -\frac{1}{EJ}\left(\frac{5}{4}pl\,x\right) + \frac{5}{6}\frac{pl^2}{EJ}$$

$$-\frac{5}{4}pl\frac{x}{EJ} + \frac{5}{6}\frac{pl^2}{EJ} > 0$$

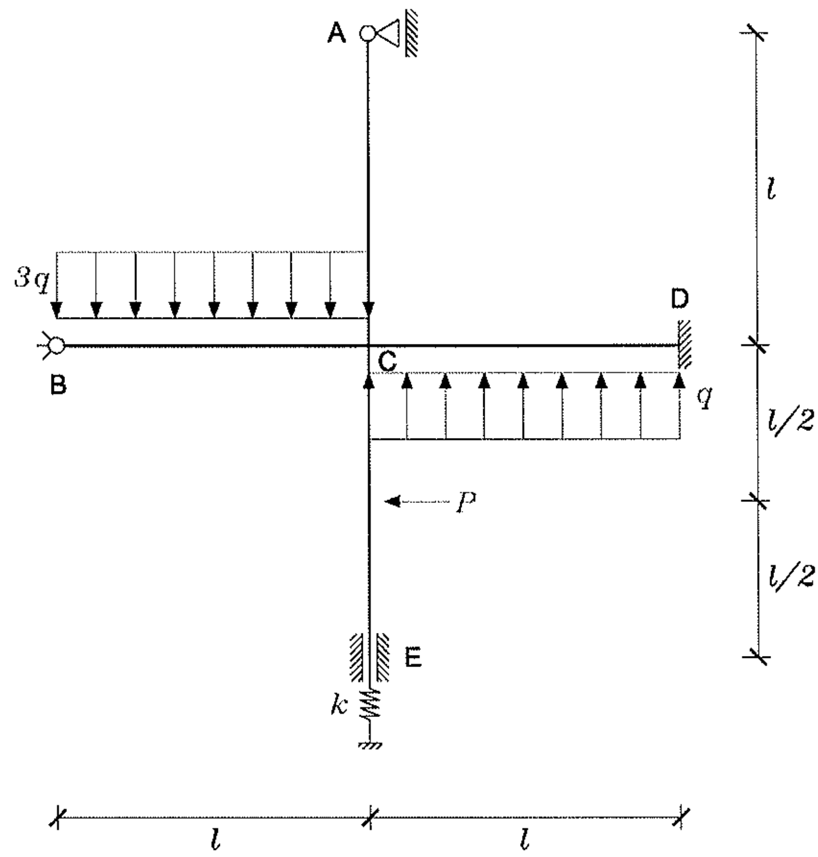
$$-\frac{5}{4}x > -\frac{5}{6}l$$

$$x < \frac{2}{3}l \qquad x < 0{,}66\,l \quad \text{curvatura positiva}$$

TECNICA DELLE COSTRUZIONI
TEMA ESAME DEL 02 APRILE 2012

DOCENTE: **ING. FAUSTO MINELLI** ESERCITATORE: **ING. FRANCESCA FEROLDI**

Esercizio



$P = ql$

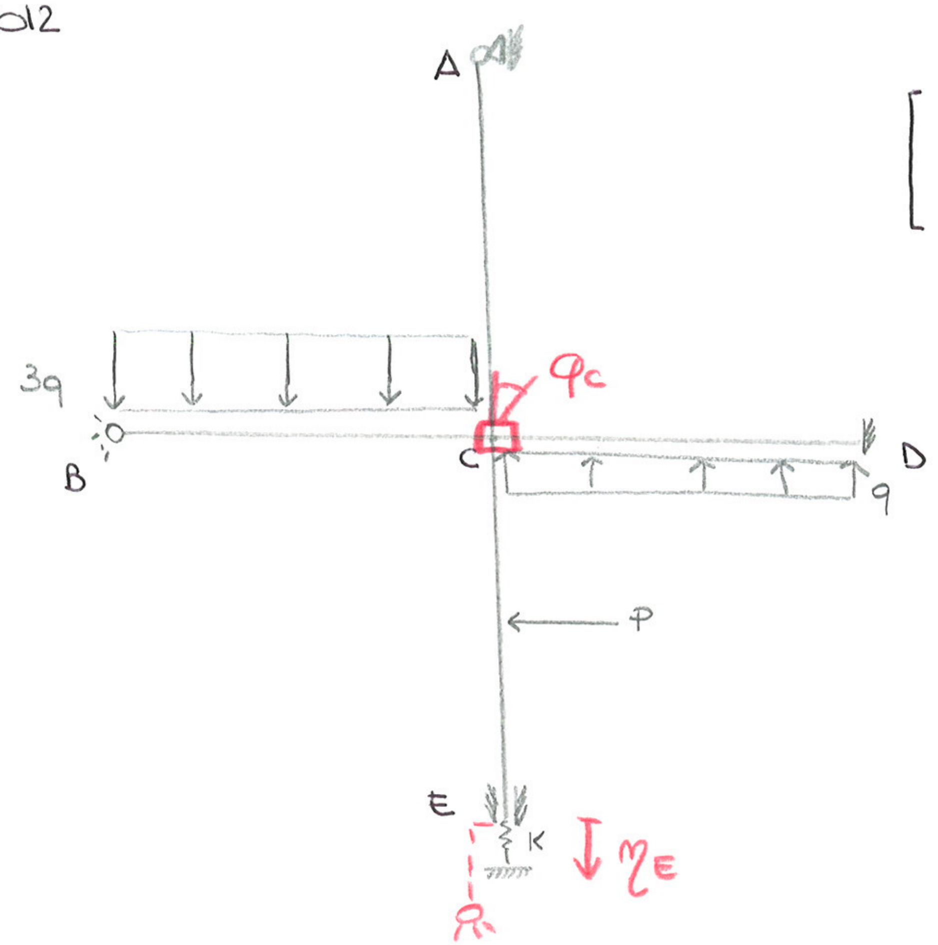
$k = 3\frac{EJ}{l^3}$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

T.E. 02/04/2012 ①

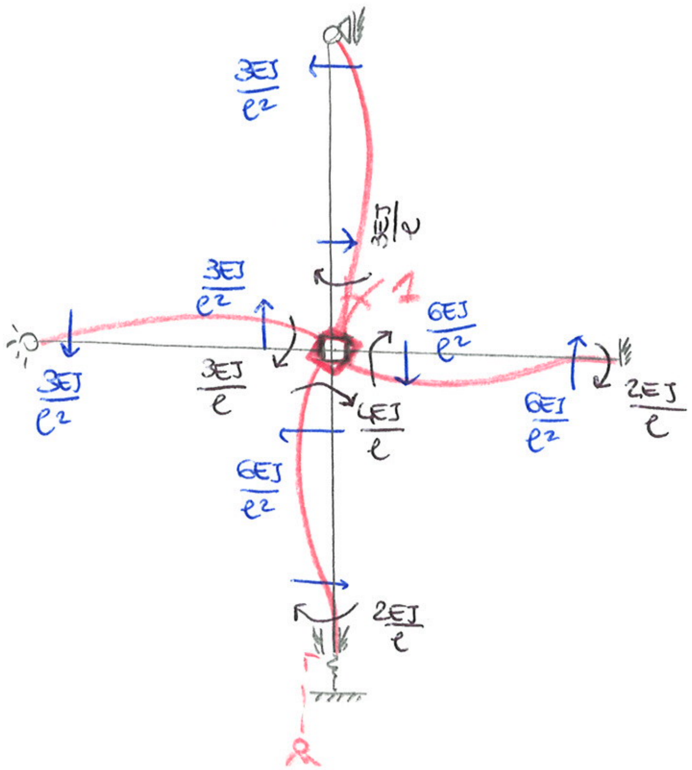


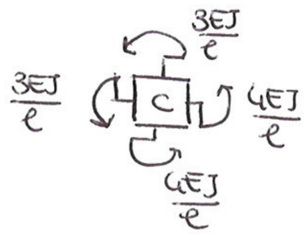
$$\begin{cases} P = q \cdot \ell \\ K = 3\dfrac{EJ}{\ell^3} \end{cases}$$

$$\begin{cases} \varphi_C \cdot m_{CC} + m_{CE} \cdot \eta_E + m_{C0} = 0 \\ \varphi_C \cdot R_{EC} + \eta_E \cdot R_{EE} + R_{E0} = 0 \end{cases}$$

Convenzioni di segno

① $\varphi_C = 1$

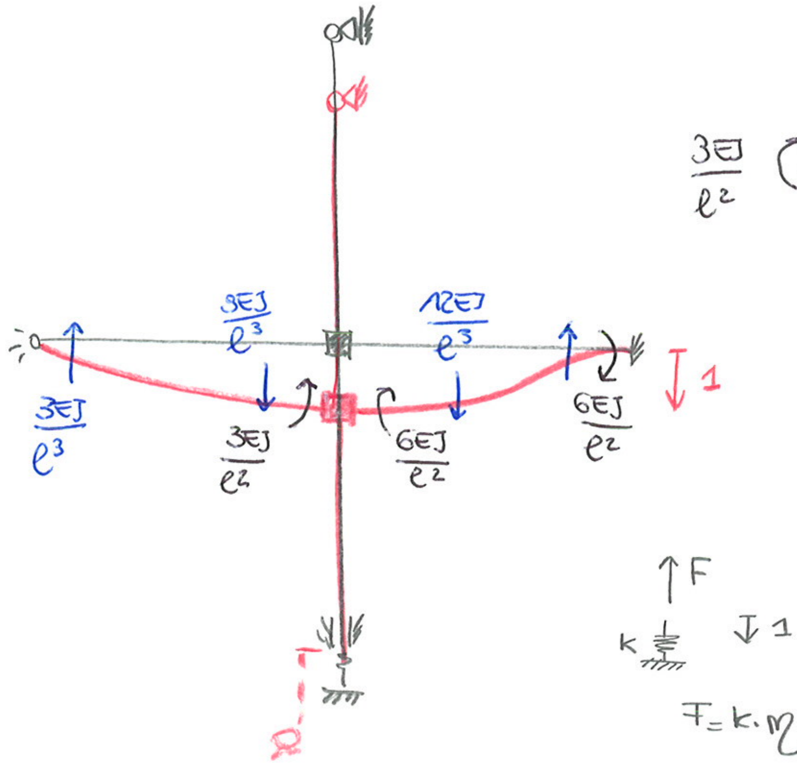


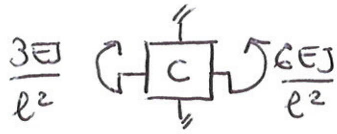


$m_{CC} = 14\dfrac{EJ}{\ell}$

$R_{EC} = \dfrac{3EJ}{\ell^2}$

② $\eta = 1$



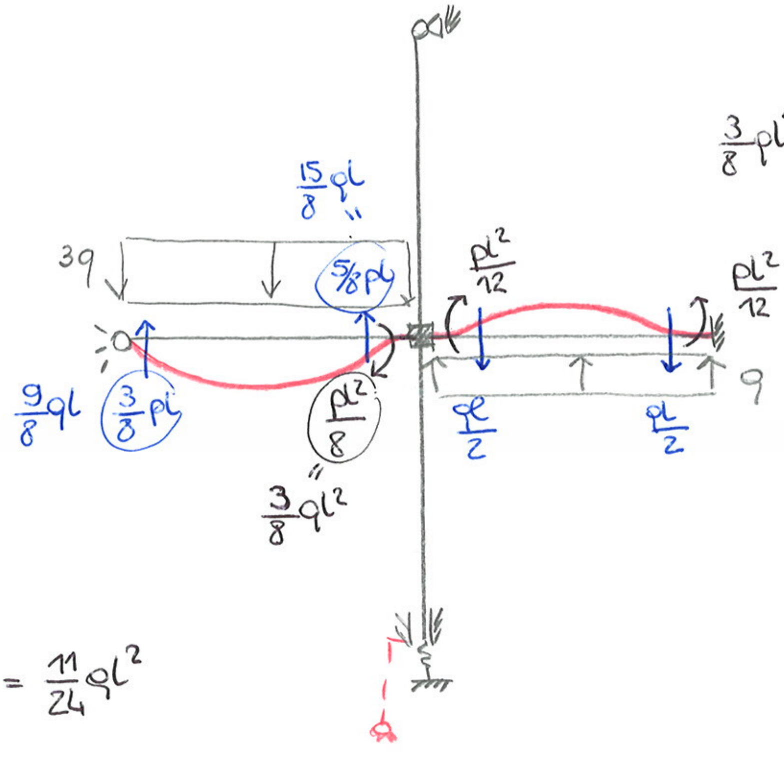


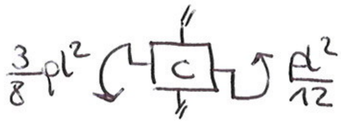
$F = k \cdot \eta$

$F = \frac{3EJ}{l^3}$

$$\begin{cases} m_{c\eta} = \frac{3EJ}{l^2} \\ r_{\eta\eta} = \frac{15EJ}{l^3} + \frac{3EJ}{l^3} = 18\frac{EJ}{l^3} \end{cases}$$

③ $q \neq 0$



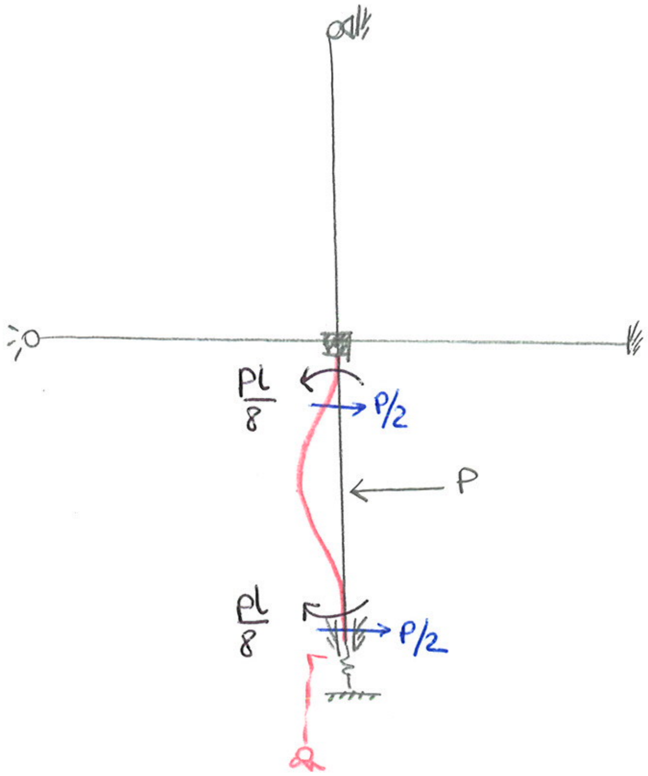


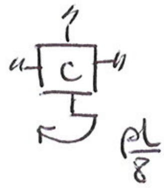
$\uparrow \frac{15}{8}ql \quad \downarrow \frac{1}{2}ql$

$\frac{15-4}{8} = \frac{11}{8}$

$$\begin{cases} m_{c0} = \left(\frac{3}{8} + \frac{1}{12}\right)ql^2 = \frac{11}{24}ql^2 \\ r_{\eta 0} = -\frac{11}{8}ql \end{cases}$$

④ $P \neq 0$





$$\begin{cases} m_{C0} = -\frac{pl}{8} \\ R_{\eta 0} = \phi \end{cases}$$

Sistema risolvente

$$\begin{cases} \varphi_C \cdot 14\frac{EJ}{l} + \eta \cdot \frac{3EJ}{l^2} + \frac{11}{24}ql^2 - \frac{pl}{8} = 0 \\ \varphi_C \cdot \frac{3EJ}{l^2} + \eta \cdot \frac{18EJ}{l^3} - \frac{11}{8}ql = 0 \end{cases}$$

$\Big) \; P = q \cdot l$

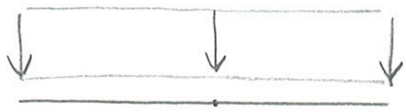
$$\frac{11}{24}ql^2 - \frac{ql^2}{8}$$

$$\frac{11-3}{24} = \frac{8}{24} = \frac{1}{3}$$

$$\begin{cases} \varphi_C \cdot 14\frac{EJ}{l} + \eta \cdot \frac{3EJ}{l^2} + \frac{1}{3}ql^2 = 0 \\ \varphi_C \cdot \frac{3EJ}{l^2} + \eta \cdot \frac{18EJ}{l^3} - \frac{11}{8}ql = 0 \end{cases}$$

$$\begin{cases} \varphi_C = -\frac{1}{24}\frac{ql^3}{EJ} \\ \eta = \frac{1}{12}\frac{ql^4}{EJ} \end{cases}$$

Asta BC



$\frac{1}{8} + \frac{7}{8} - \frac{3}{8} = \emptyset$

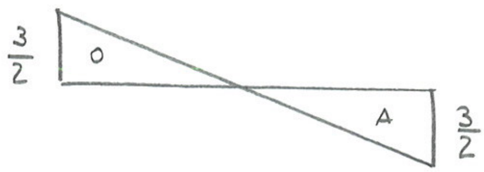
$\frac{3EJ}{24l} \frac{ql^3}{EJ}$ $\quad \frac{1}{4}ql^2 \quad \frac{3}{8}ql^2$

$\frac{1}{8}ql \uparrow \qquad \downarrow \frac{1}{8}ql$

$\frac{1}{4}ql \uparrow \qquad \downarrow \frac{1}{4}ql$

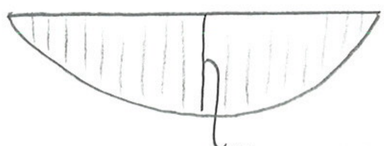
$\frac{9}{8}ql \uparrow \qquad \uparrow \frac{15}{8}ql$

$\frac{3}{2}ql \uparrow \qquad \uparrow \frac{3}{2}ql$



V(x)

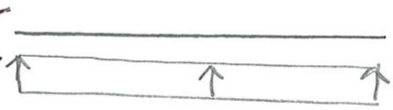
M(x)



$M_{max}(l/2) = \frac{3}{4}ql^2 - \frac{3}{8}ql^2 = \frac{3}{8}ql^2$

Asta CD

$\frac{ql^2}{12} \quad \frac{1}{2}ql^2 \quad \frac{1}{6}ql^2$ $\qquad \frac{1}{12}ql^2 \quad \frac{1}{2}ql^2 \quad \frac{ql^2}{12}$



$\frac{1}{12} + \frac{6}{12} - \frac{2}{12} = \frac{5}{12}$

$\uparrow \frac{1}{4}ql \qquad \downarrow \frac{1}{4}ql$

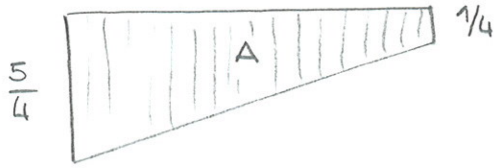
$\frac{1}{12} - \frac{6}{12} + \frac{1}{12} = -\frac{4}{12} = \frac{1}{3}$

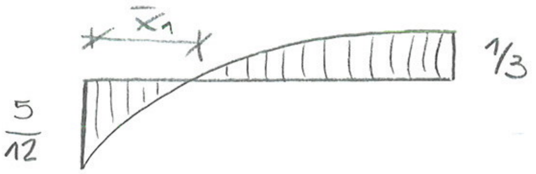
$\downarrow ql \qquad \uparrow ql$

$\downarrow \frac{ql}{2} \qquad \downarrow \frac{ql}{2}$

$\frac{5}{12}ql^2 \qquad \frac{1}{3}ql^2$

$\downarrow \frac{5}{4}ql \qquad \uparrow \frac{1}{4}ql$





$$M(\bar{x}) = 0 \rightarrow \frac{5}{12} ql^2 - \frac{5}{4} ql\bar{x} + q\frac{\bar{x}^2}{2} = 0$$

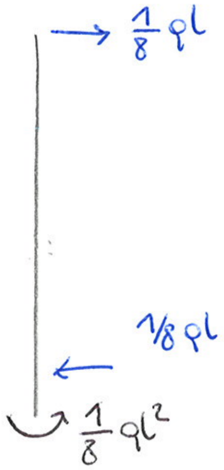
$$x_{1,2} = \frac{5/4 \pm \sqrt{25/16 - 4 \cdot 1/2 \cdot 5/12}}{1} = 0{,}4\, l$$

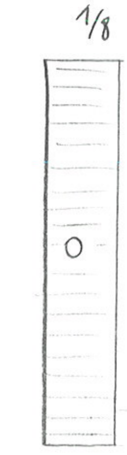
$$\bar{x}_3 = ? \rightarrow \frac{7}{24} ql^2 = \frac{3}{4} ql\bar{x}_3$$

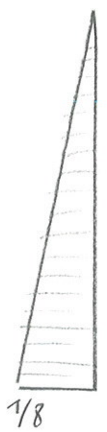
$$\bar{x}_3 = \frac{7}{24} \cdot \frac{4}{3} l$$

$$\bar{x}_3 = 0{,}38\, l$$

Asta AC





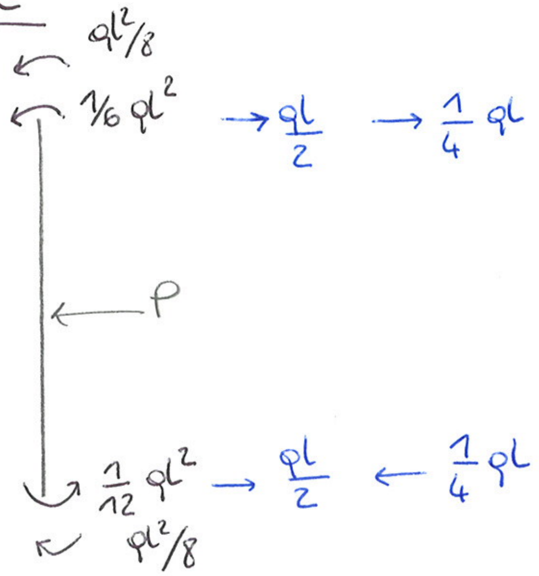


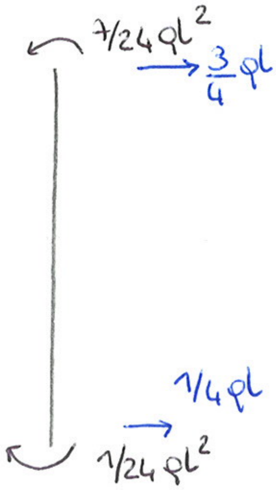
$$\bar{x}_2 = ? \rightarrow \frac{1}{24} ql^2 - \frac{1}{4} ql\bar{x} = 0$$

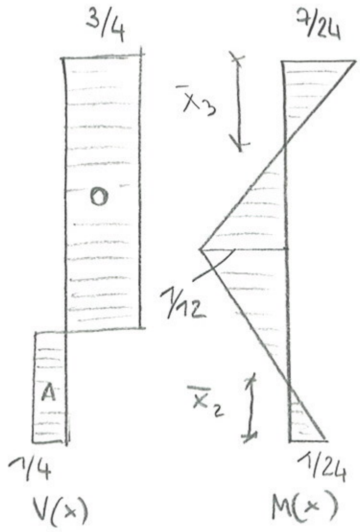
$$\frac{1}{4} ql\bar{x} = \frac{1}{24} ql^2$$

$$\bar{x} = \frac{4}{24} l = 0{,}16\, l$$

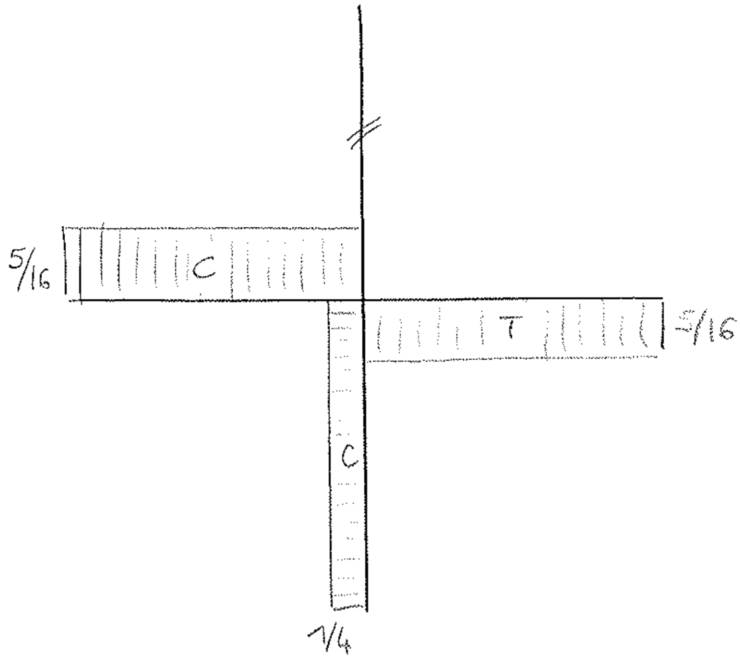
Asta CE



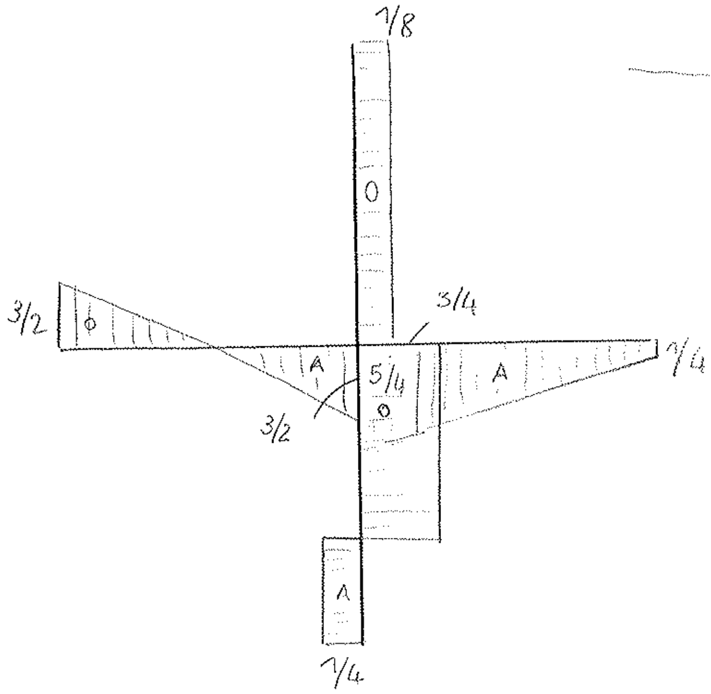


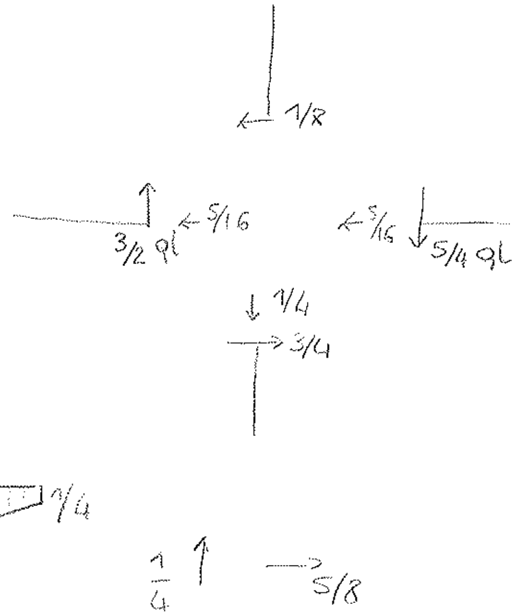


$\frac{N}{ql}$

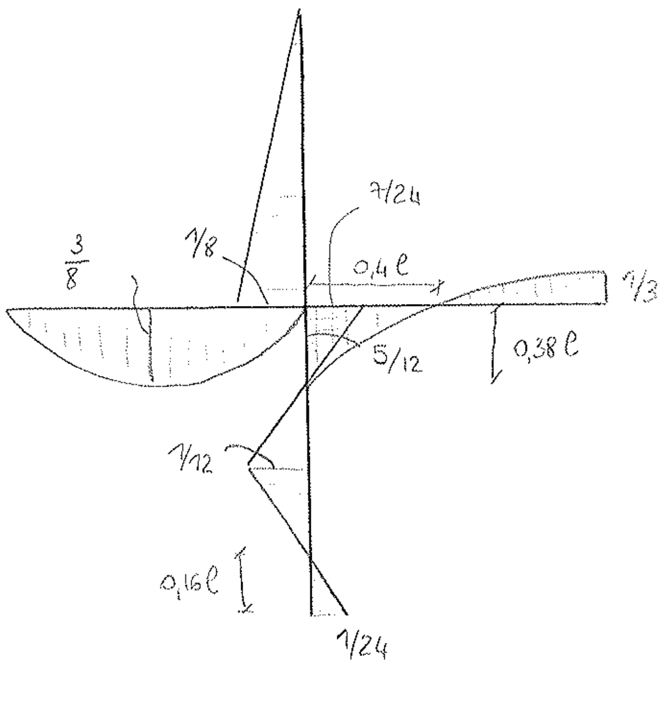


$\frac{V}{ql}$

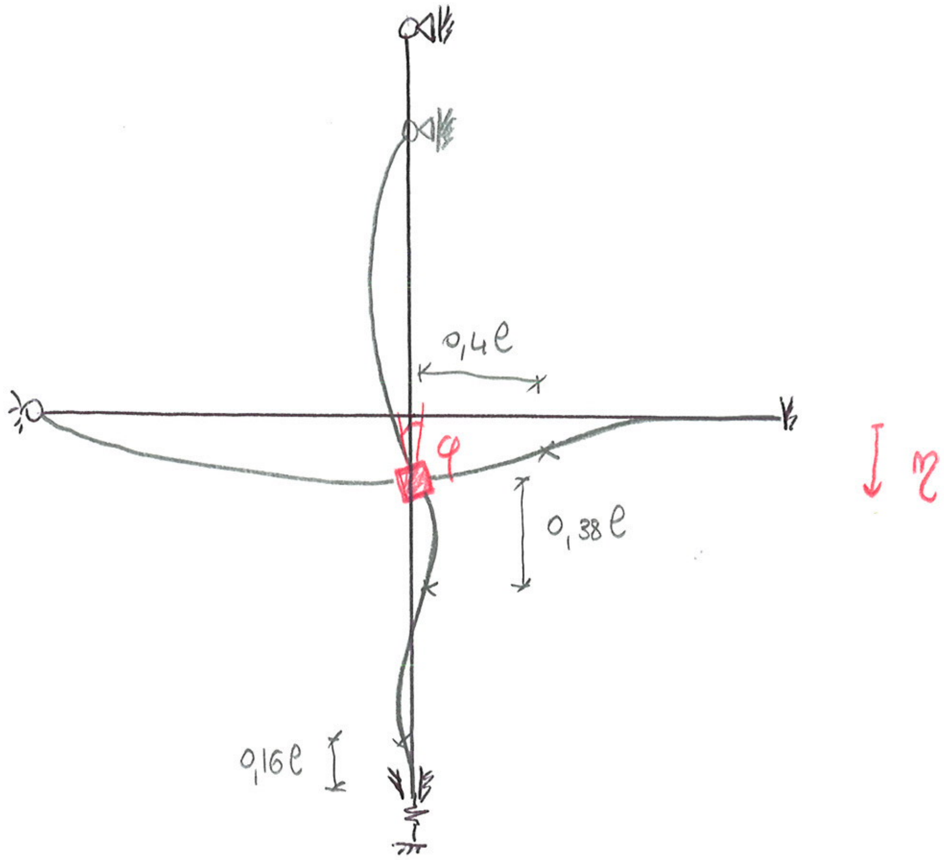




$\frac{M}{ql^2}$



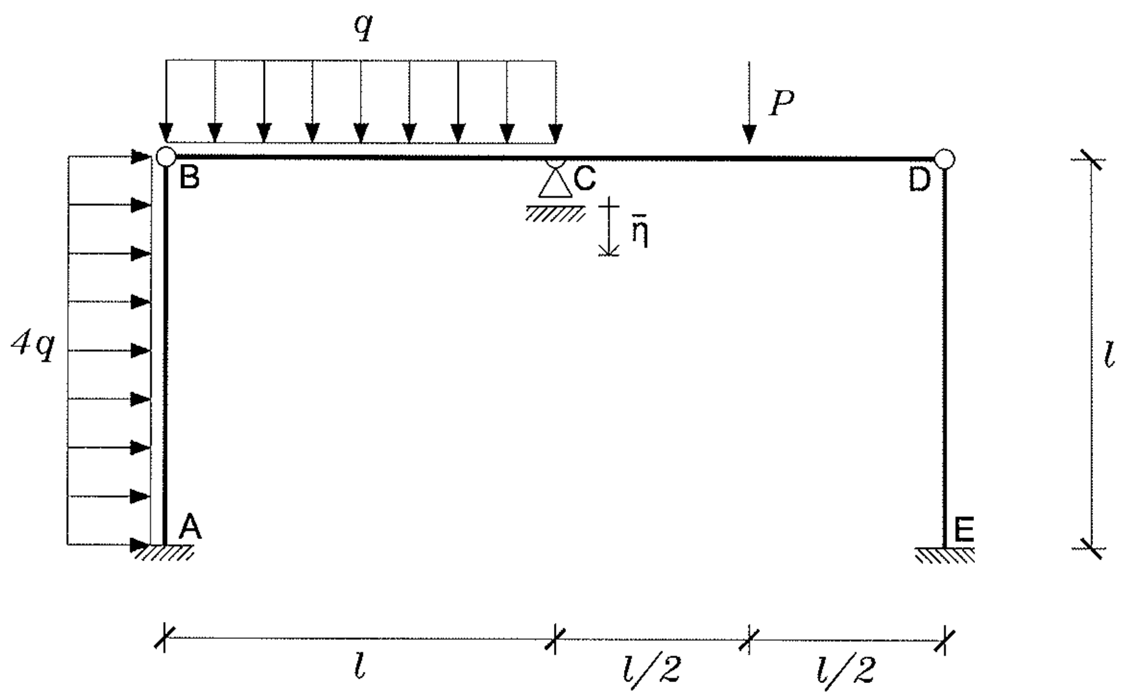
Deformata



TECNICA DELLE COSTRUZIONI
PROVA IN ITINERE DEL 23 APRILE 2012

DOCENTE: **PROF. GIOVANNI PLIZZARI** ESERCITATORE: **ING. FAUSTO MINELLI**

Esercizio



$P = 2\,ql$

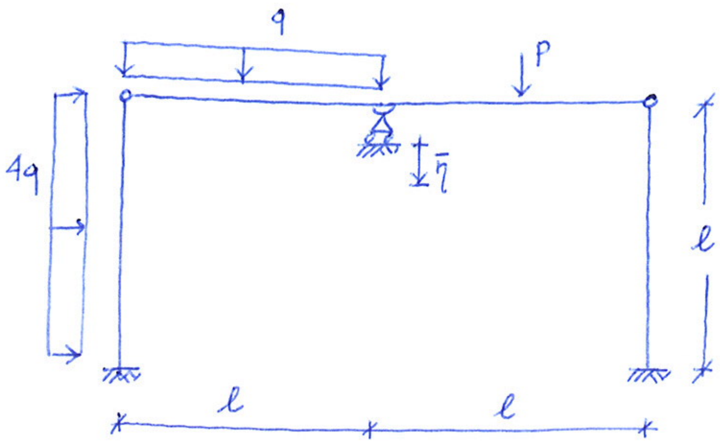
$\bar{\eta} = \dfrac{qL^4}{12\,EJ}$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Tecnica 23/06/2012

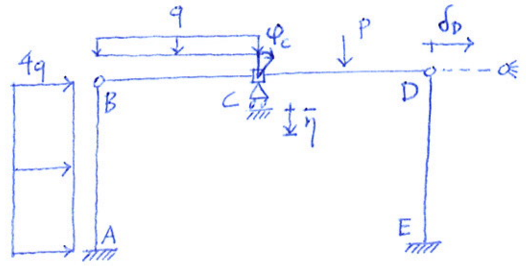


$P = 2qL$

$\bar{\eta} = \dfrac{qL^4}{12EJ}$

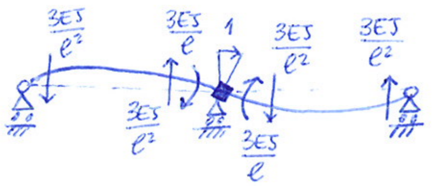
$\dfrac{\alpha \Delta T}{H} = \dfrac{qL^3}{EJ}$

$\Rightarrow$



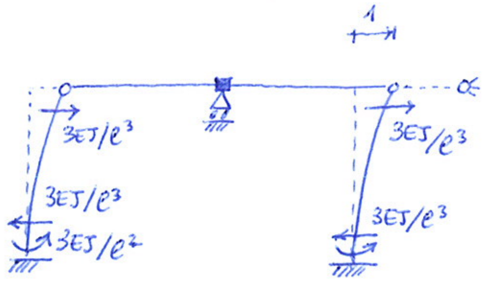
2 incognite nodali: φ_c, δ_D

$\varphi_c)$



$m_\varphi = \dfrac{6EJ}{l}$; $v_\varphi = 0$

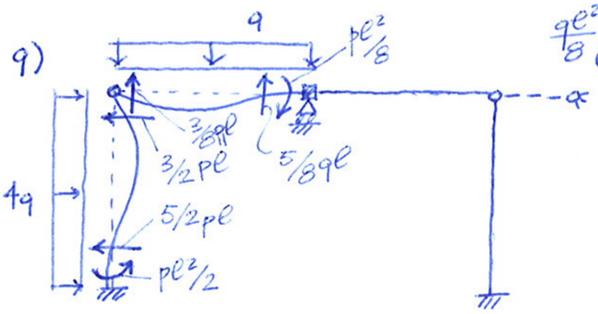
$\delta)$

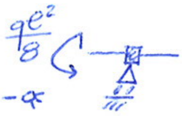


$m_\delta = 0$

$v_\delta = +\dfrac{6EJ}{l^3}$

$q)$

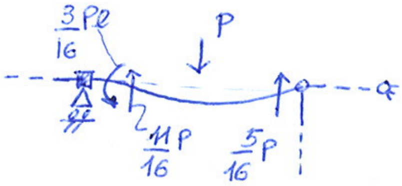




$m_{0,q} = +\dfrac{ql^2}{8}$

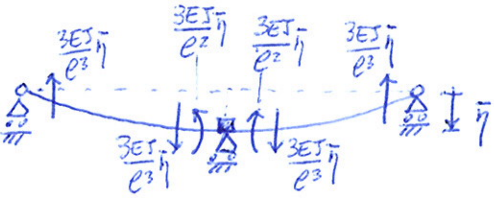
$v_{0,q} = -\dfrac{3}{2} ql$

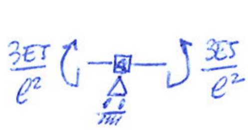
$P)$



$\dfrac{3}{8} ql^2 \Rightarrow m_{0,P} = -\dfrac{3}{8} ql^2$

$\bar{\eta})$





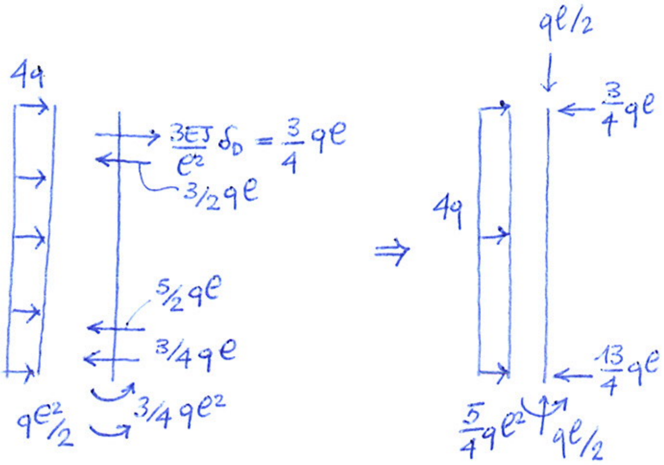
$m_{0,\bar{\eta}} = 0$ $\quad v_{0,\bar{\eta}} = 0$

SISTEMA RISOLVENTE

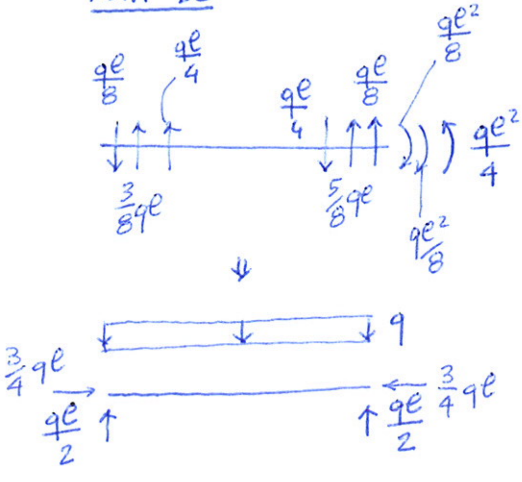
$$\frac{6EJ}{l}\varphi_c = -\frac{ql^2}{8} + \frac{3}{8}ql^2 \Longrightarrow \frac{6EJ}{l}\varphi_c = \frac{ql^2}{4} \Rightarrow \boxed{\varphi_c = \frac{ql^3}{24EJ}}$$

$$\frac{6EJ}{l^3}\delta_D = \frac{3}{2}ql \Rightarrow \boxed{\delta_D = \frac{ql^4}{4EJ}}$$

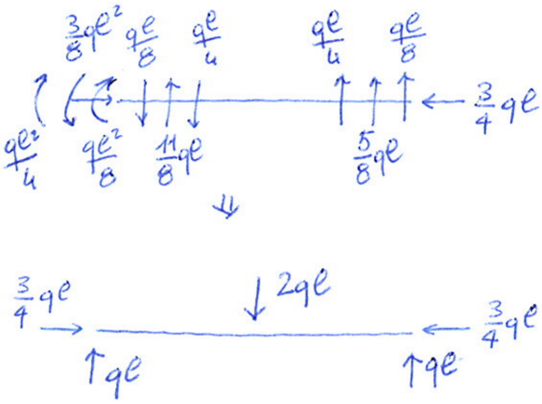
ASTA AB



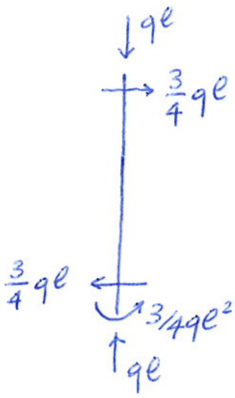
ASTA BC



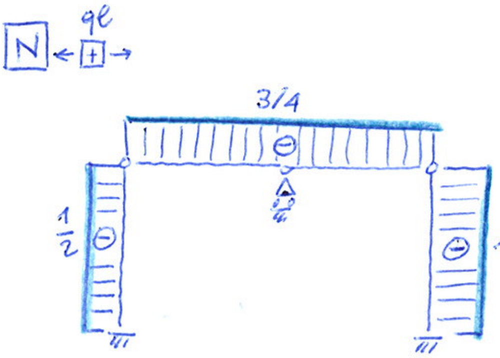
ASTA CD

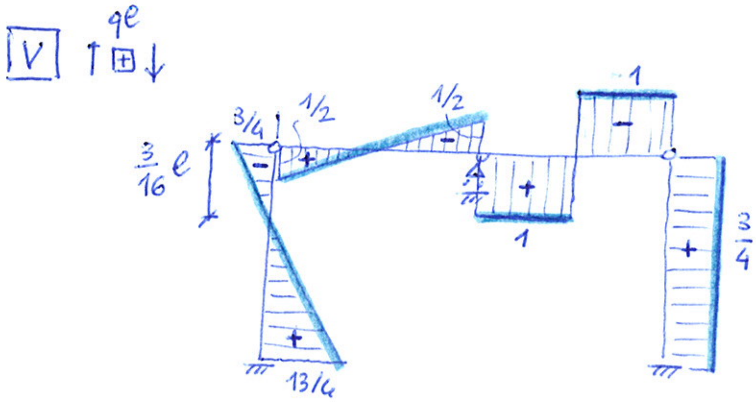


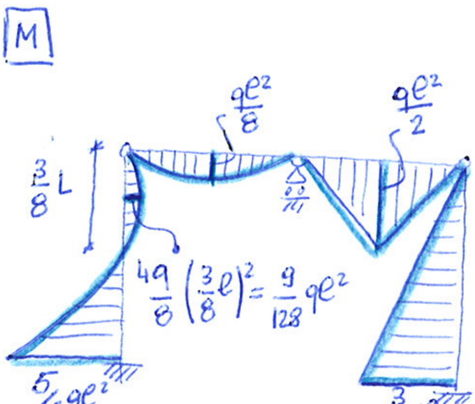
ASTA DE



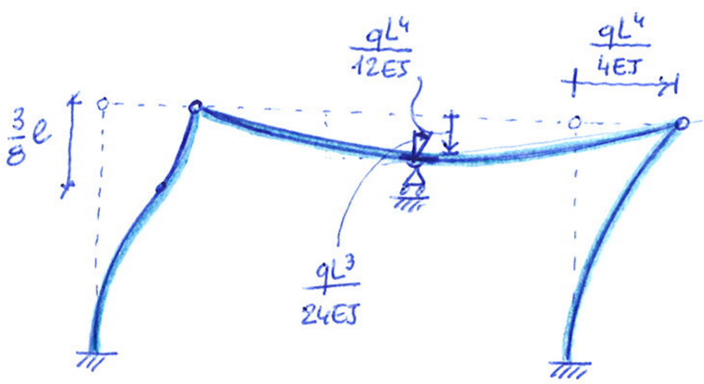
DIAGRAMMI AZIONI INTERNE







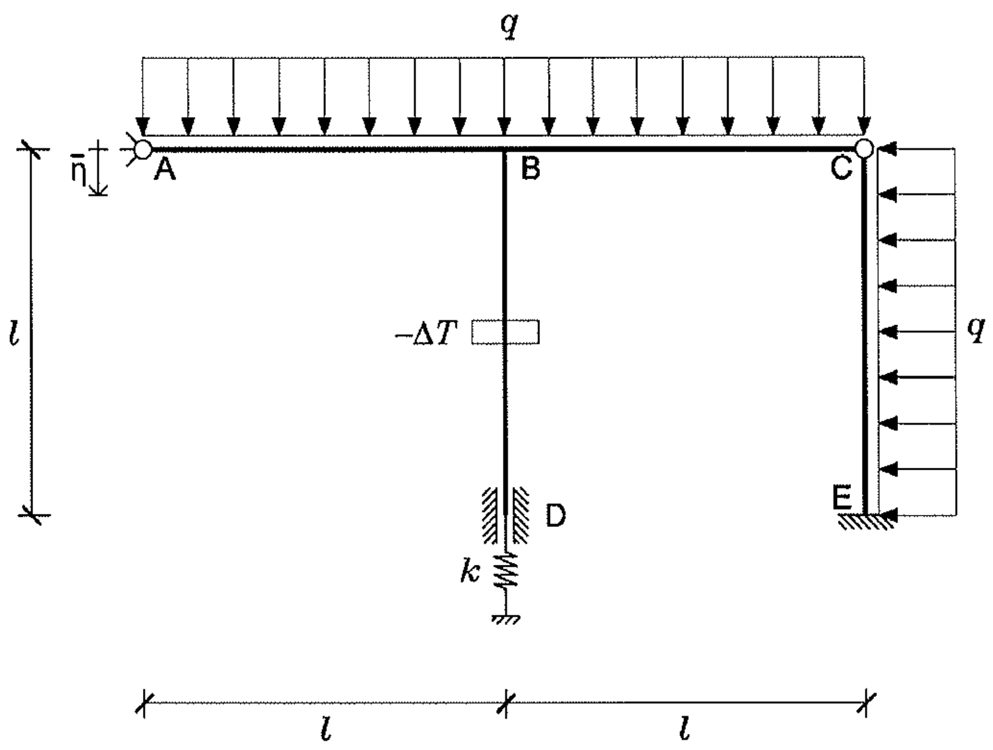
DEFORMATA QUALITATIVA



TECNICA DELLE COSTRUZIONI
APPELLO DEL 18 GIGUNO 2012

DOCENTE: **PROF. GIOVANNI PLIZZARI** ESERCITATORE: **ING. FAUSTO MINELLI**

Esercizio



$$k = 4\frac{EJ}{L^3}$$

$$\bar{\eta} = 2\frac{qL^4}{EJ}$$

$$\alpha\Delta t = \frac{ql^3}{EJ}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

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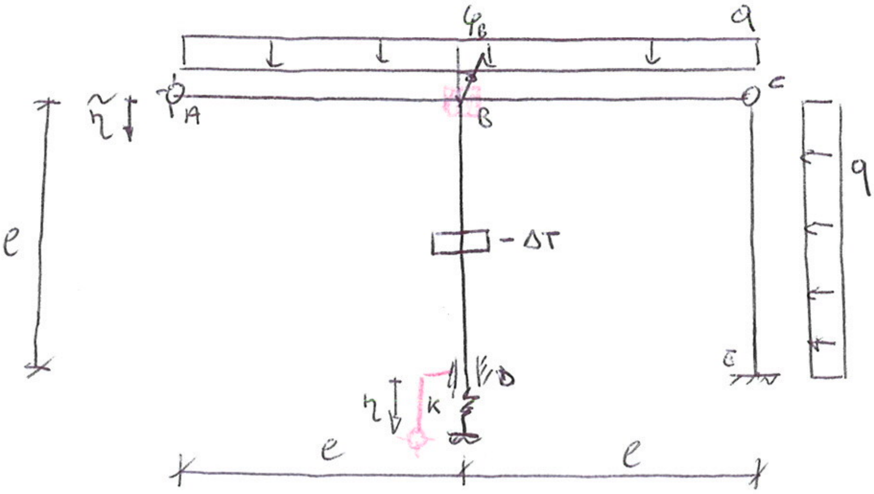
T. E. 18/06/2012

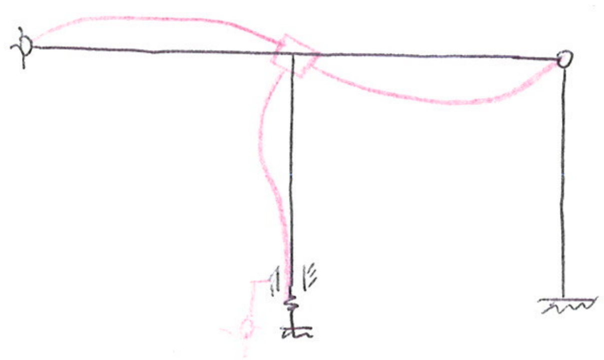


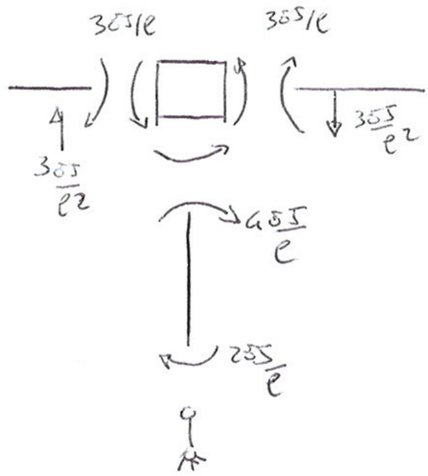
$K = 4\frac{EJ}{\ell^3}$

$\tilde{\eta} = 2\frac{q\ell^4}{EJ}$

$\alpha\Delta T = \frac{q\ell^3}{EJ}$

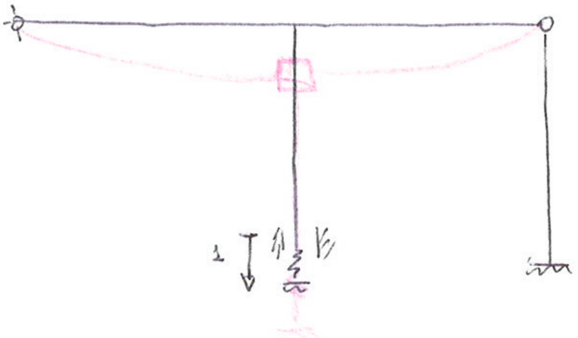
⊙ $\varphi_B = 1$ $\quad \eta = q = \Delta T = \tilde{\eta} = 0$

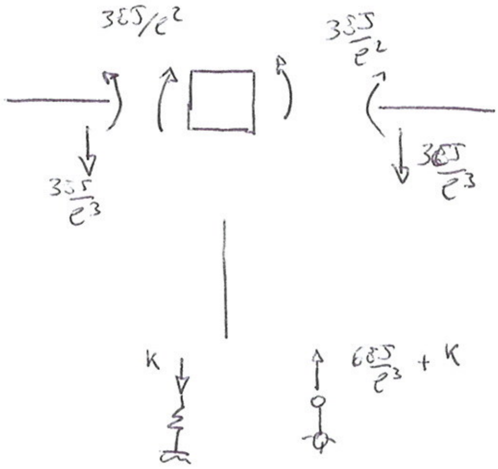




$$\begin{cases} m_\varphi = \frac{3EJ}{\ell} + \frac{3EJ}{\ell} + \frac{4EJ}{\ell} = 10\frac{EJ}{\ell} \\ h_\eta = \frac{3EJ}{\ell^2} - \frac{3EJ}{\ell^2} = 0 \end{cases}$$

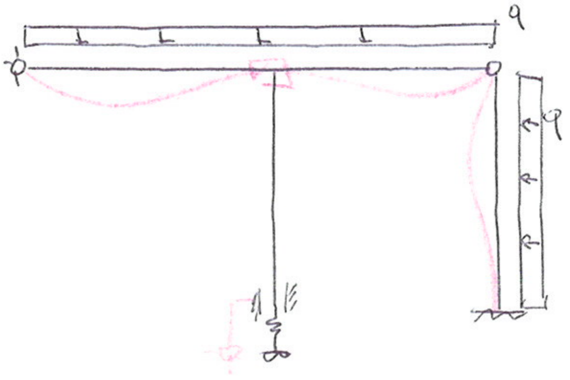
⊙ $\eta = 1$ $\quad \varphi_B = q = \Delta T = \tilde{\eta} = 0$

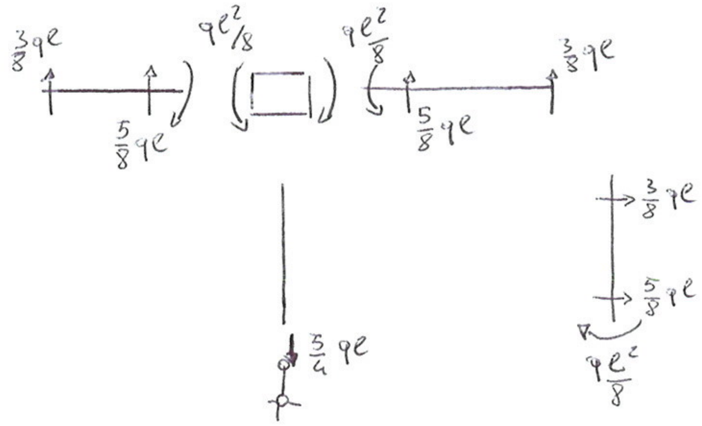




$$\begin{cases} m_\eta = 0 \\ h_\eta = \frac{3EJ}{\ell^3} + \frac{3EJ}{\ell^3} + K = \frac{6EJ}{\ell^3} + \frac{4EJ}{\ell^3} = 10\frac{EJ}{\ell^3} \end{cases}$$

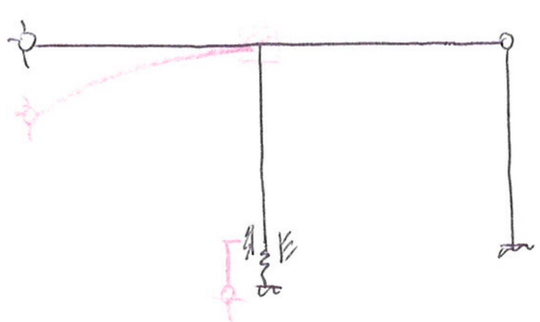
⊙ $q \neq 0 \qquad \varphi_B = \eta = \Delta T = \tilde{\eta} = 0$

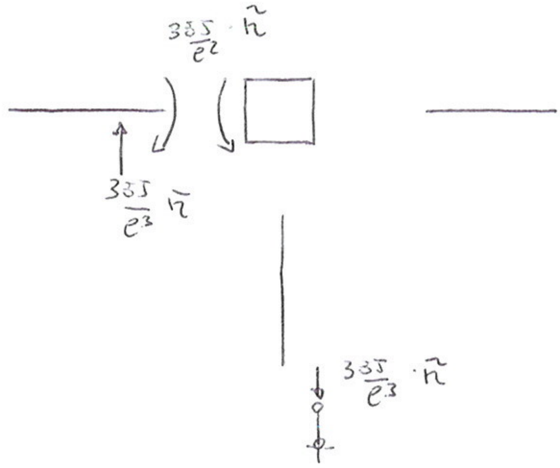




$$\begin{cases} m_{0,q} = \dfrac{q\ell^2}{8} - \dfrac{q\ell^2}{8} = 0 \\ h_{0,q} = -\dfrac{5}{8} q\ell - \dfrac{5}{8} q\ell = -\dfrac{5}{4} q\ell \end{cases}$$

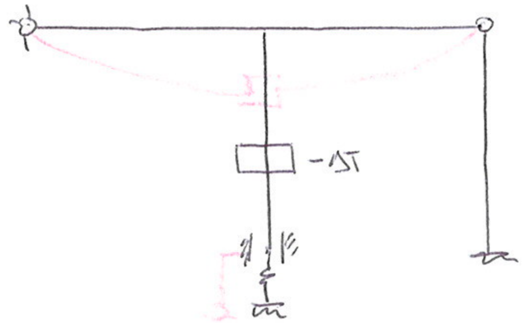
⊙ $\tilde{\eta} \neq 0 \qquad \varphi_B = \eta = \Delta T = q = 0$

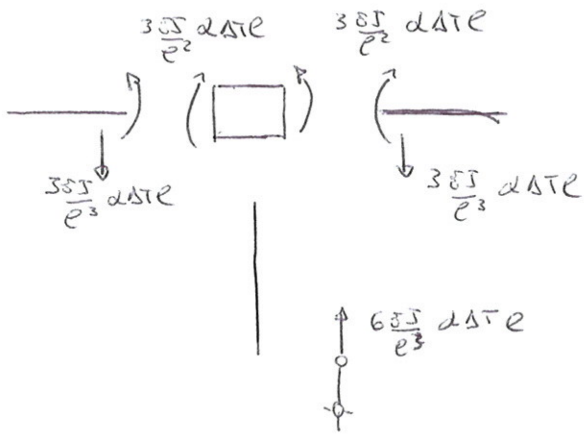




$$\begin{cases} m_{0,\tilde{\eta}} = \dfrac{3EJ}{\ell^2} \cdot \tilde{\eta} = \dfrac{3EJ}{\ell^2} \cdot \dfrac{2q\ell^4}{EJ} = 6q\ell^2 \\ h_{0,\tilde{\eta}} = -\dfrac{3EJ}{\ell^3} \cdot \tilde{\eta} = -3\dfrac{EJ}{\ell^3} \cdot \dfrac{2q\ell^4}{EJ} = -6q\ell \end{cases}$$

⊙ $\Delta T \neq 0 \qquad \varphi_B = \eta = q = \tilde{\eta} = 0$





$$\begin{cases} m_{0,\Delta T} = \dfrac{3EJ}{\ell^2} \alpha \Delta T \ell - \dfrac{3EJ}{\ell^2} \alpha \Delta T \ell = 0 \\ h_{0,\Delta T} = \dfrac{3EJ}{\ell^3} \alpha \Delta T \ell + \dfrac{3EJ}{\ell^3} \alpha \Delta T \ell = 6 \dfrac{EJ}{\ell^3} \cdot \dfrac{q\ell^3}{EJ} \cdot \ell = 6q\ell \end{cases}$$

Il sistema risolvente risulta:

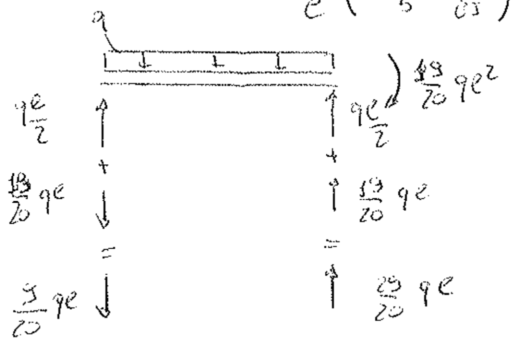
$$\begin{cases} 10\frac{EJ}{l}\cdot\varphi_B + 0\,\eta + 6ql^2 = 0 \\ 0\,\varphi_B + 10\frac{EJ}{l^2}\eta - \frac{5}{4}ql - 6ql + 6ql = 0 \end{cases}$$

Risolvendo si ottiene:

$$\begin{cases} \varphi_B = -\frac{3}{5}\frac{ql^3}{EJ} \\ \eta = \frac{1}{8}\frac{ql^4}{EJ} \end{cases}$$

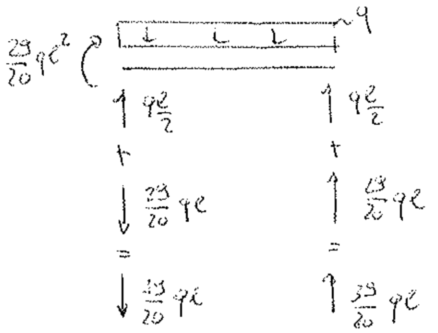
⊙ Asta AB

$$\frac{3EJ}{l}\left(-\frac{3}{5}\frac{ql^3}{EJ}\right) - \frac{3EJ}{l^2}\left(\frac{1}{8}\frac{ql^4}{EJ}\right) + \frac{ql^2}{8} + 6ql^2 - 3ql^2 = \frac{13}{20}ql^2$$



⊙ Asta BC

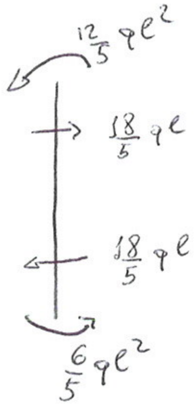
$$\frac{3EJ}{l}\left(-\frac{3}{5}\frac{ql^3}{EJ}\right) + \frac{3EJ}{l^2}\left(\frac{1}{8}\frac{ql^4}{EJ}\right) - \frac{ql^2}{8} + 3ql^2 = \frac{29}{20}ql^2$$



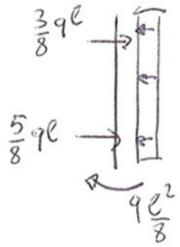
- Asta BD

$$\frac{4EJ}{\ell}\left(-\frac{3}{5}\, q\frac{\ell^3}{EJ}\right) = -\frac{12}{5} q\ell^2$$

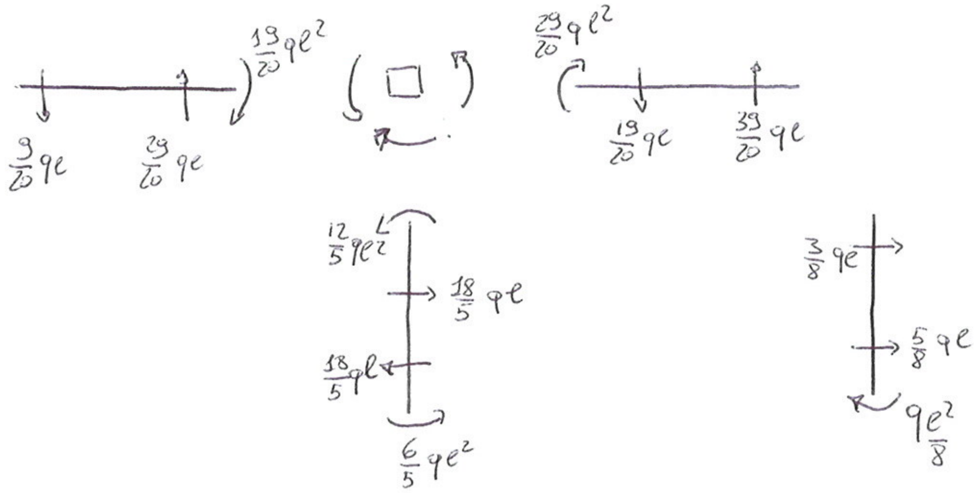
$$\frac{2EJ}{\ell}\left(-\frac{3}{5}\, \frac{q\ell^3}{EJ}\right) = -\frac{6}{5} q\ell^2$$



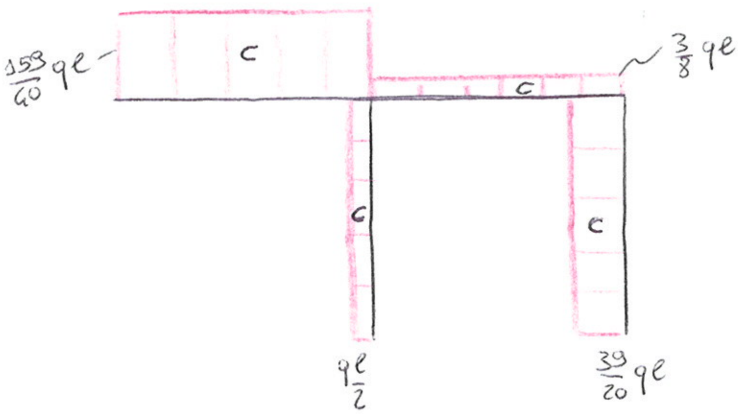
- Asta CE



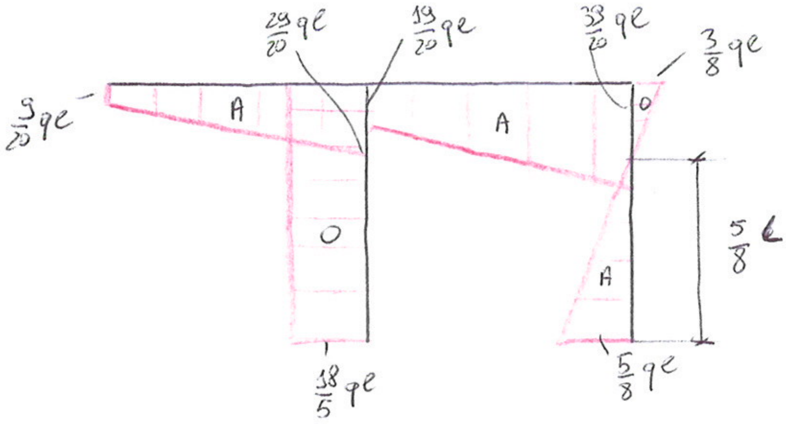
- Unendo le singole aste si ottiene



- Azione assiale N

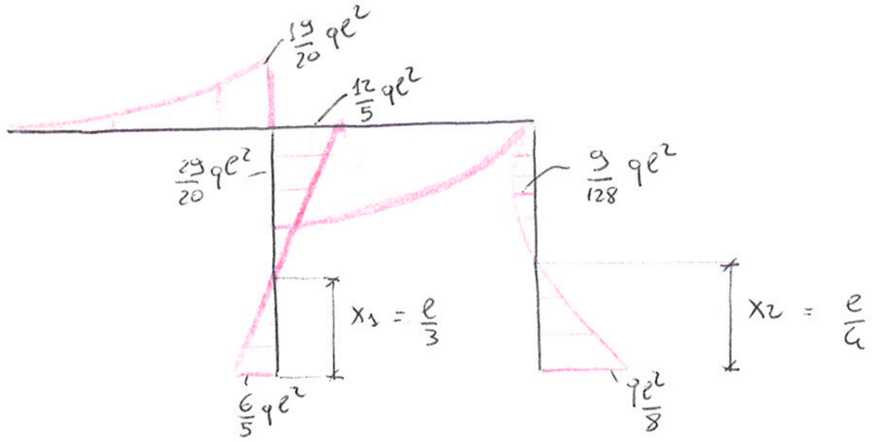


- Taglio ✓



$$M\left(\frac{5}{8}l\right) = \frac{5}{8}ql \cdot \frac{5}{8}l - \frac{ql^2}{8} - q \cdot \frac{5}{8}l \cdot \frac{5}{16}l = \frac{9}{128}ql^2$$

- Momento

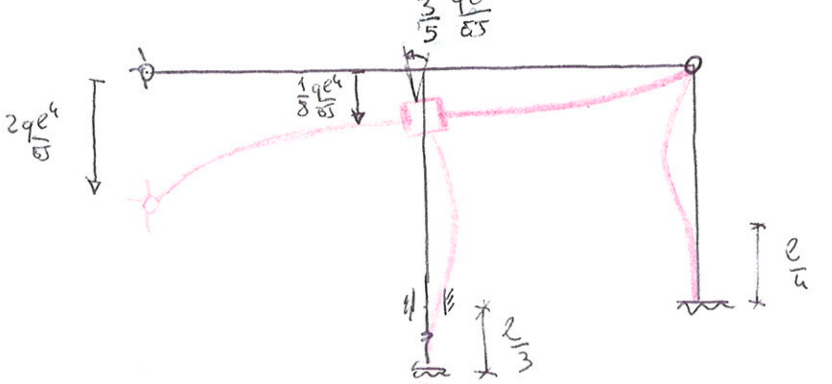


$$\frac{6}{5} : x_1 = \frac{18}{5} : l \Rightarrow x_1 = \frac{l}{3}$$

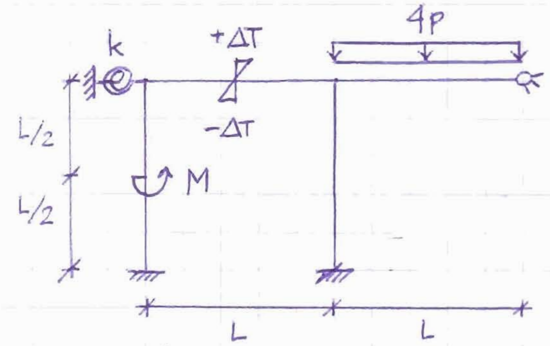
$$q\frac{l^2}{8} + q\frac{x_2^2}{2} - \frac{5}{8}ql\,x_2 = 0 \; ; \quad x_2^2 - \frac{5}{4}l\,x_2 + \frac{l^2}{4} = 0$$

$$x_2 = \frac{\frac{5}{4} \pm \sqrt{\frac{25}{16} - 1}}{2} \cdot l = \begin{cases} x_{2,1} = l & \text{NO} \\ x_{2,2} = \frac{l}{4} & \text{SI} \end{cases}$$

- Deformata qualitativa



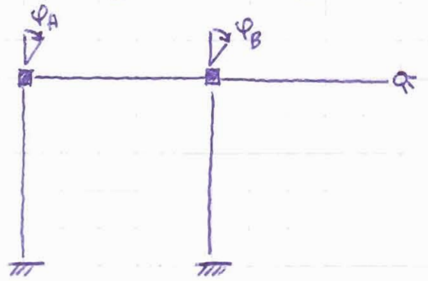
TELAIO TECNICA delle COSTRUZIONI



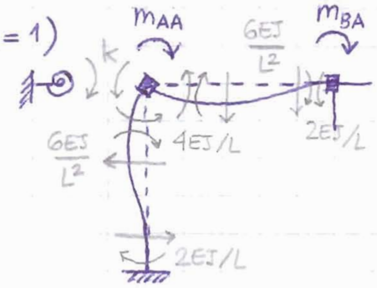
$$\frac{\alpha \Delta T EJ}{h} = pl^2 \qquad M = 4pL^2$$

$$k = 4\frac{EJ}{L}$$

Telaio 2 volte iperstatico.
⇒ BLOCCO I NODI A e B



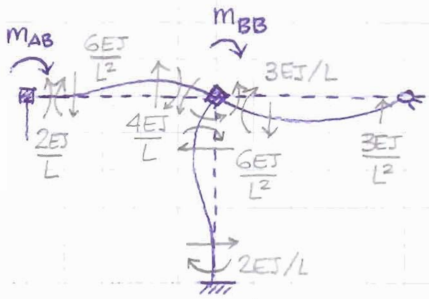
$\varphi_A = 1)$



$$m_{AA} = \frac{8EJ}{L} + k = \frac{12EJ}{L}$$

$$m_{BA} = \frac{2EJ}{L}$$

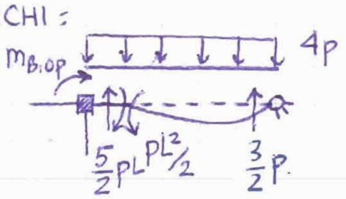
$\varphi_B = 1)$



$$m_{AB} = \frac{2EJ}{L} \qquad m_{BB} = \frac{11EJ}{L}$$

CARICHI :

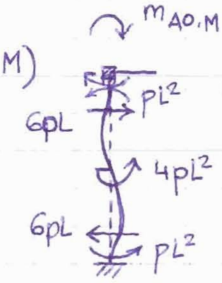
4p)



$$m_{A0,p} = 0$$

$$m_{B0,p} = -\frac{pL^2}{2}$$

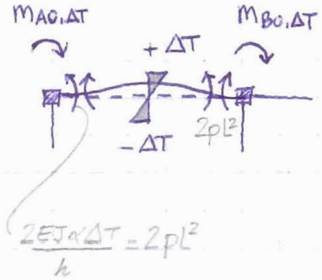
M)



$$m_{A0,M} = -pL^2$$

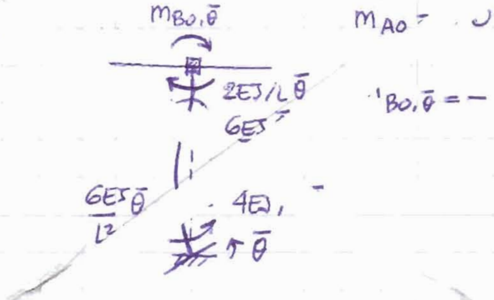
$$m_{B0,M} = 0$$

ΔT)



$$m_{A0,\Delta T} = 2pL^2$$

$$m_{B0,\Delta T} = -2pL^2$$



$$m_{A0} = 0$$

$$m_{B0,\bar{\theta}} = -4pL^2$$

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SISTEMA RISOLVENTE

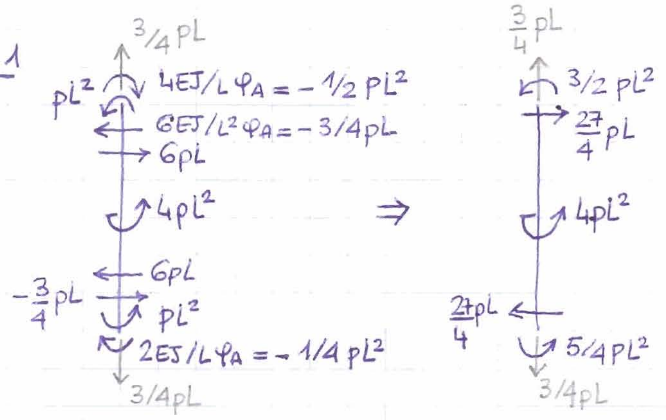
$$\begin{bmatrix} \frac{12EJ}{L} & \frac{2EJ}{L} \\ \frac{2EJ}{L} & \frac{11EJ}{L} \end{bmatrix} \begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = \begin{bmatrix} -pL^2 \\ +\frac{5}{2}pL^2 \end{bmatrix} \Rightarrow \varphi_B = -6\varphi_A - \frac{pL^3}{2EJ}$$

$$+\frac{2EJ}{L}\varphi_A - 66\frac{EJ}{L}\varphi_A - \frac{11}{2}pL^2 = \frac{5}{2}pL^2 \Rightarrow -64\frac{EJ}{L}\varphi_A = 8pL^2 \Rightarrow \boxed{\varphi_A = -\frac{1}{8}\cdot\frac{pL^3}{EJ}}$$

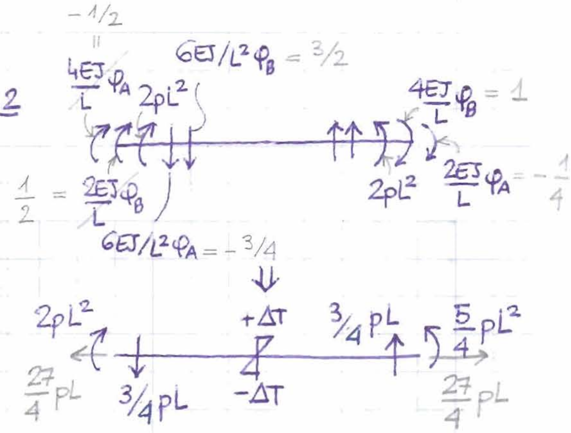
$$\boxed{\varphi_B = \frac{1}{4}\cdot\frac{pL^3}{EJ}}$$

AZIONI INTERNE ASTA PER ASTA

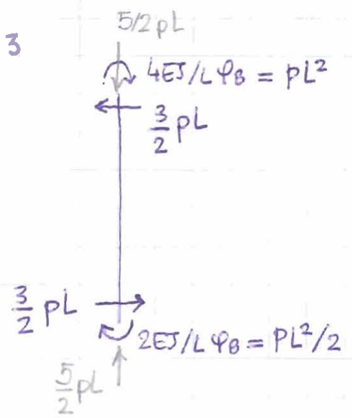
ASTA 1



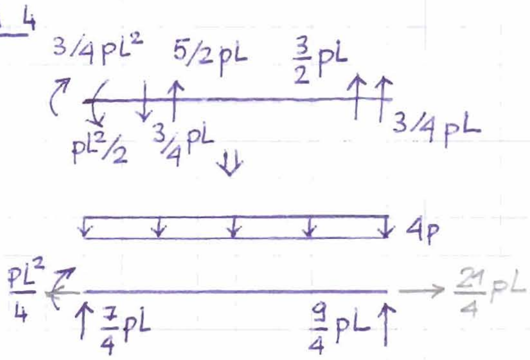
ASTA 2



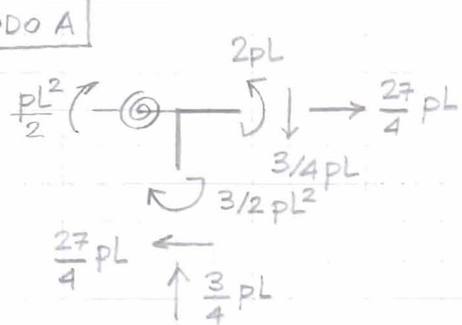
ASTA 3



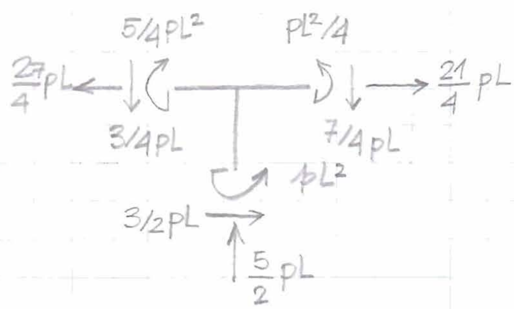
ASTA 4



NODO A



NODO B



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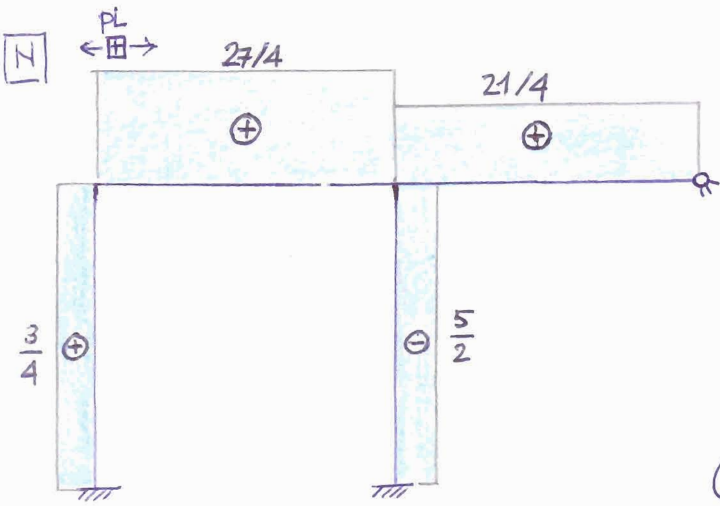
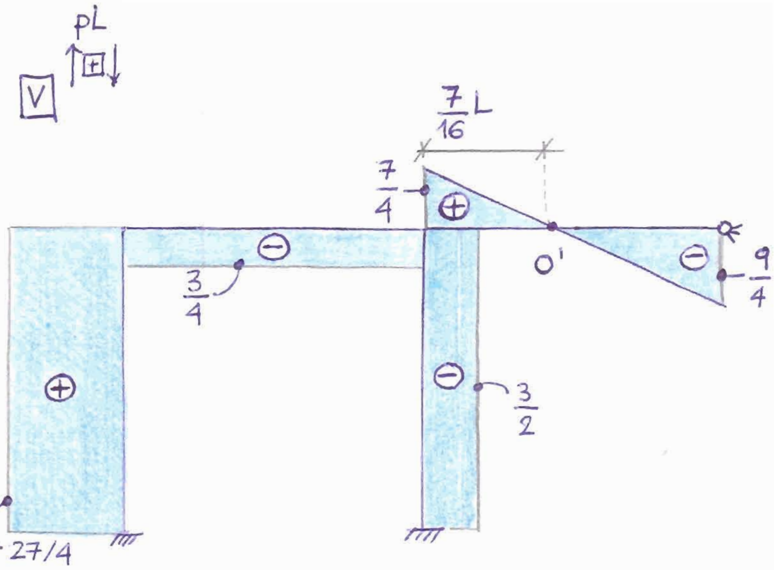
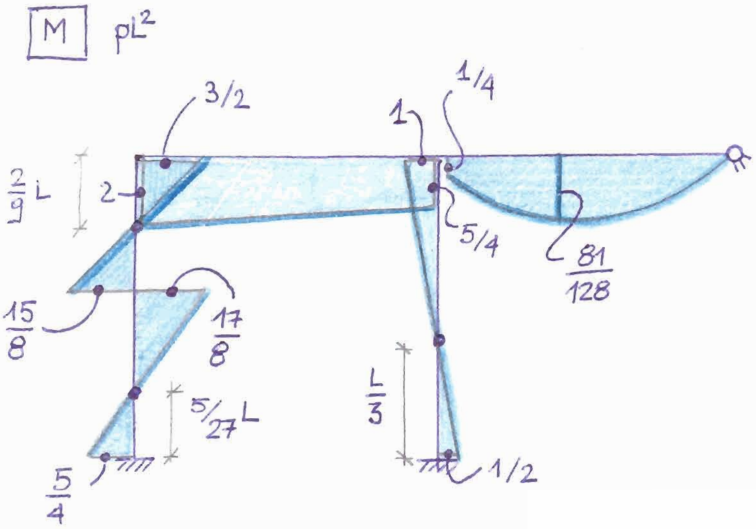
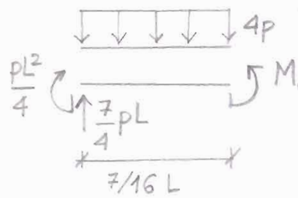
DIAGRAMMI DELLE AZIONI INTERNE







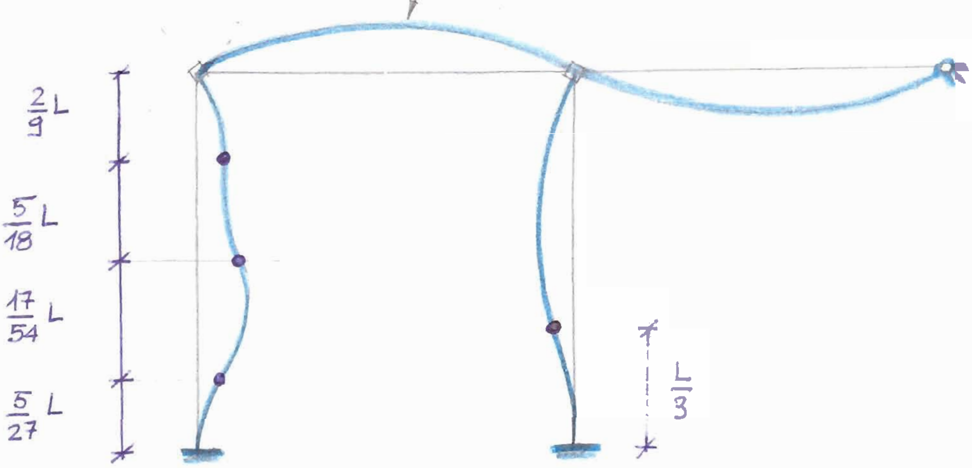


$$M_{O'} = \frac{pL^2}{4} + \frac{7}{4}pL \cdot \frac{7}{16}L - 4p\,\frac{7}{16}L \cdot \frac{7}{32}L =$$

$$= \frac{pL^2}{4} + \frac{49}{64}pL^2 - \frac{49}{128}pL^2 = \frac{81}{128}pL^2$$

DEFORMATA QUALITATIVA

LA CURVATURA È COSÌ A CAUSA DELLA DEFORMAZIONE TERMICA (NONOSTANTE IL MOMENTO TENDA ALLE FIBRE DI SOTTO)



$$y'' = \frac{-M}{EJ} + \frac{2\alpha\Delta T}{H} =$$

$$-\frac{1}{EJ}\left(-M + 2pL^2\right) > 0$$

$$\frac{5}{4}pL^2 < M < 2pL^2$$

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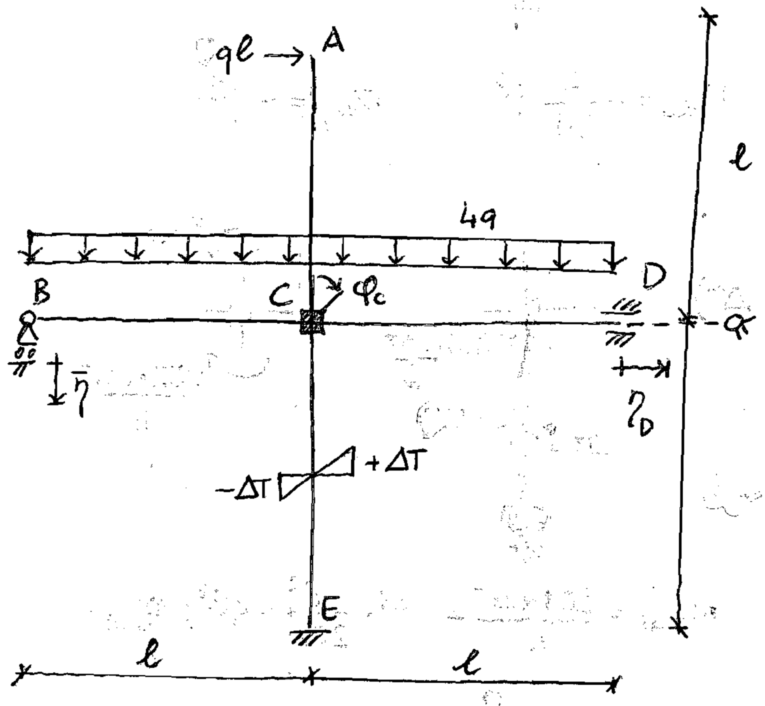
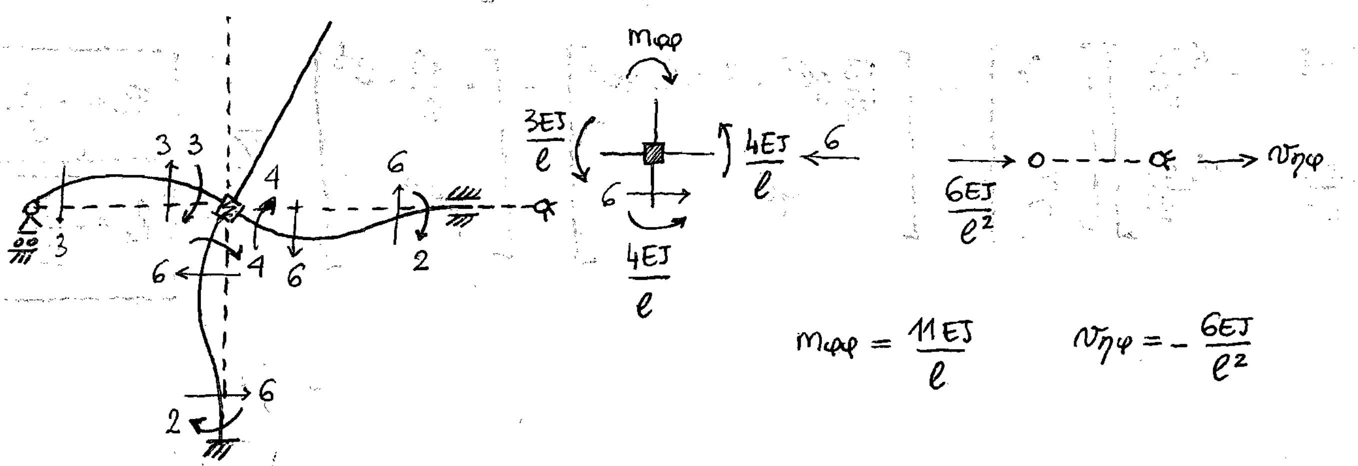
www.cte-it.org



$$\frac{\alpha \Delta T}{h} = \frac{q\ell^2}{2EJ}$$

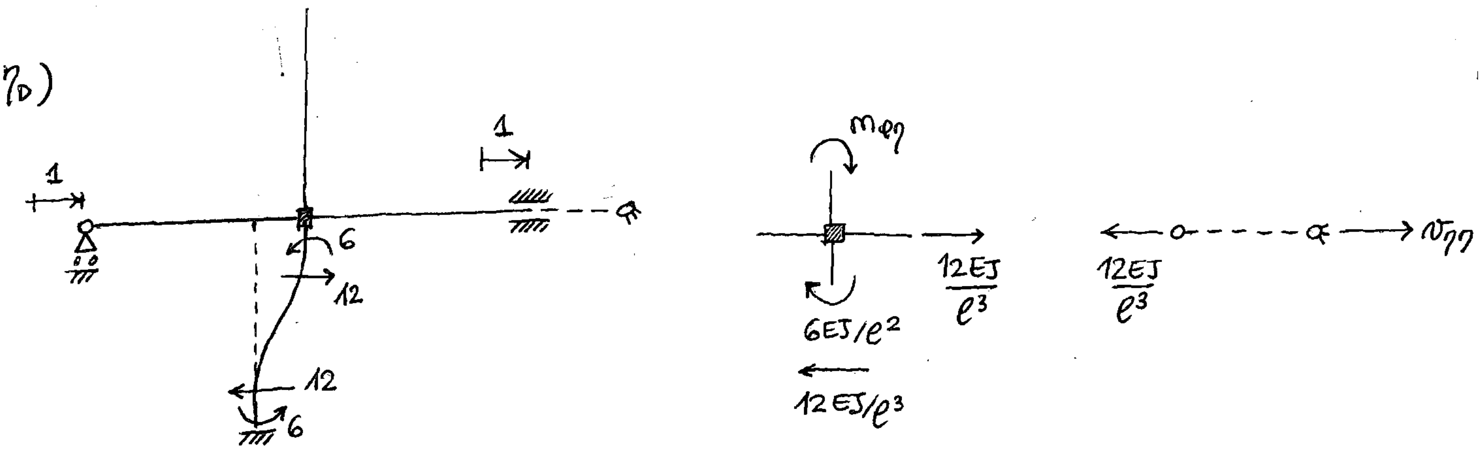
$$\bar{\eta} = \frac{q\ell^4}{EJ}$$

φc)



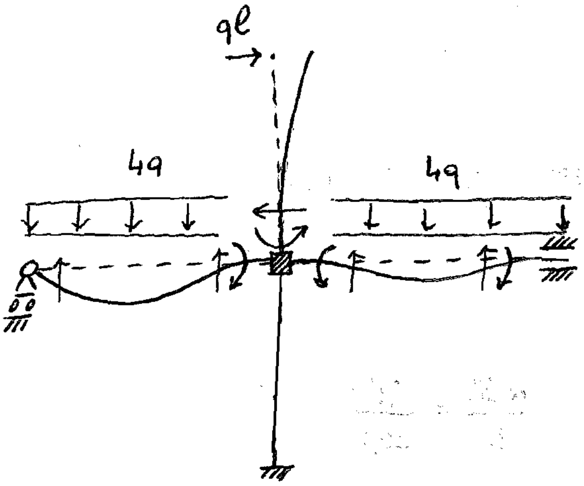
$$m_{\varphi\varphi} = \frac{11EJ}{\ell} \qquad v_{\eta\varphi} = -\frac{6EJ}{\ell^2}$$

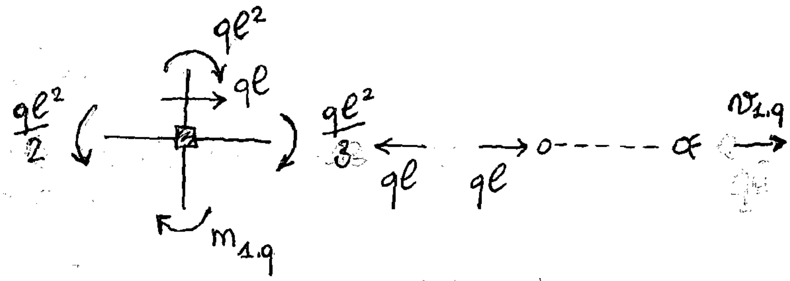
ηD)



$$m_{\varphi\eta} = -\frac{6EJ}{\ell^2} \qquad v_{\eta\eta} = \frac{12EJ}{\ell^3}$$

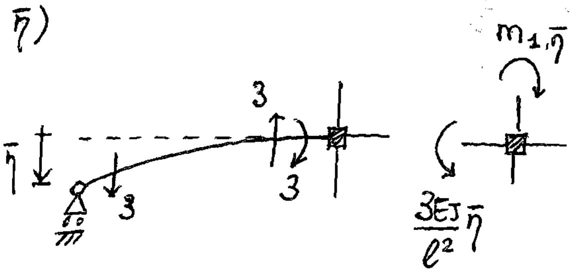
q)





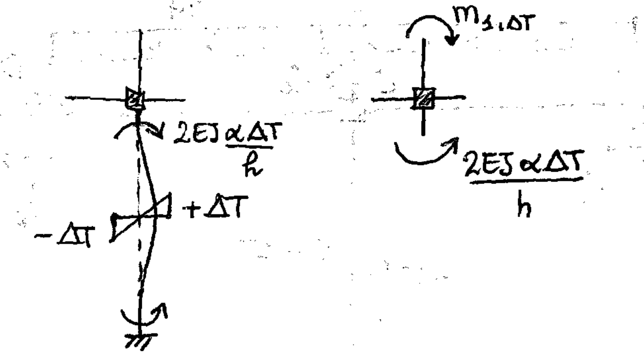
$$m_{1,q} = -\frac{5}{6}q\ell^2 \qquad v_{1,q} = -q\ell$$

$\bar{\eta}$)



$$m_{1,\bar{\eta}} = \frac{3EJ}{\ell^2}\cdot\frac{q\ell^4}{EJ} = 3q\ell^2 \qquad v_{1,\bar{\eta}} = 0$$

ΔT)

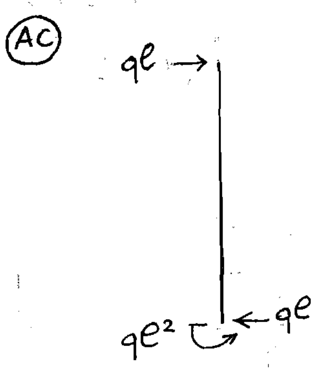


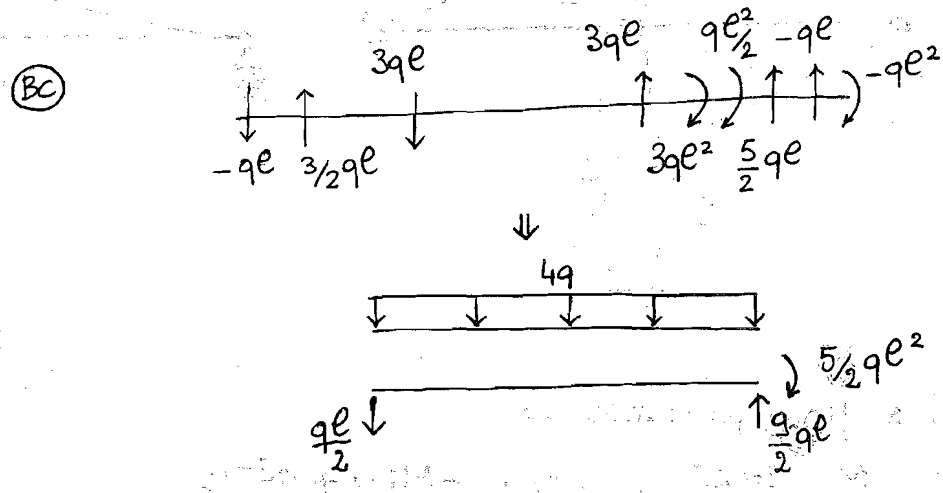
$$m_{1,\Delta T} = \frac{2EJ\alpha\Delta T}{h} = 2EJ\cdot\frac{q\ell^2}{2EJ} = q\ell^2; \quad v_{1,\Delta T} = 0$$

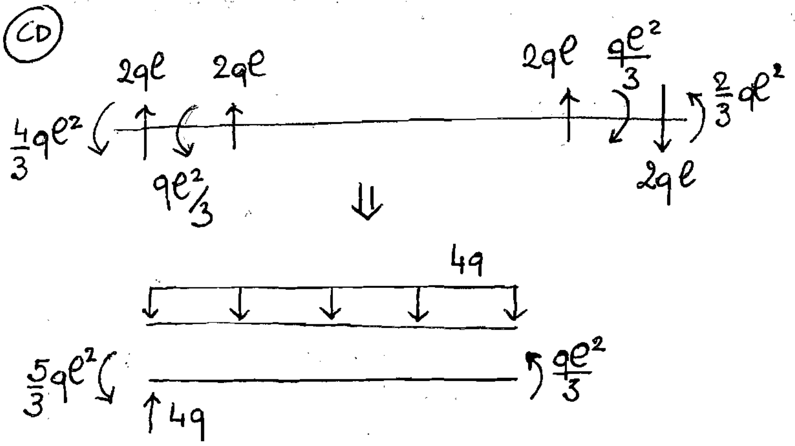
SISTEMA RISOLVENTE

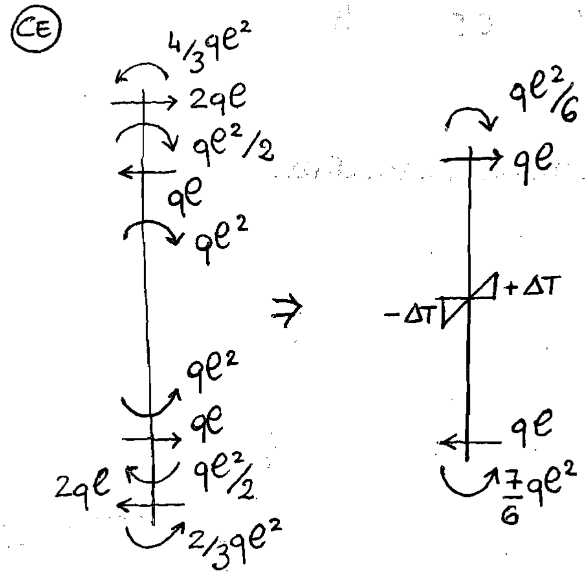
$$\begin{bmatrix} 11\frac{EJ}{\ell} & -\frac{6EJ}{\ell^2} \\ -\frac{6EJ}{\ell^2} & 12\frac{EJ}{\ell^3} \end{bmatrix}\begin{bmatrix} \varphi_c \\ \eta_D \end{bmatrix} = \begin{bmatrix} \frac{5}{6}q\ell^2 - 3q\ell^2 - q\ell^2 \\ q\ell \end{bmatrix} = \begin{bmatrix} -\frac{19}{6}q\ell^2 \\ q\ell \end{bmatrix} \Rightarrow \boxed{\varphi_c = -\frac{1}{3}\cdot\frac{q\ell^3}{EJ}} \quad \boxed{\eta_D = -\frac{1}{12}\cdot\frac{q\ell^4}{EJ}}$$

ASTA PER ASTA

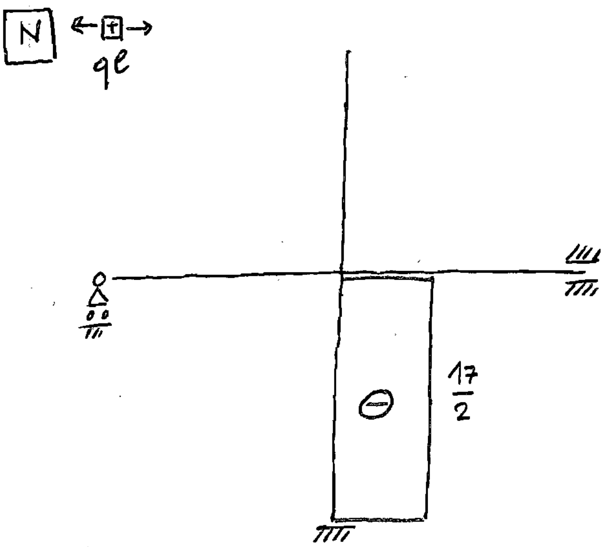




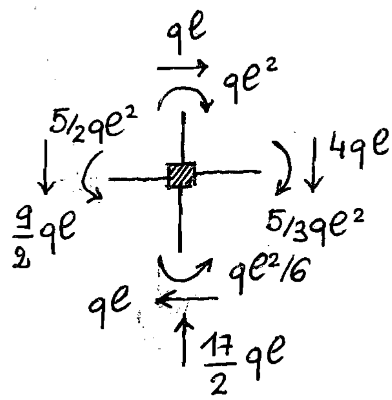


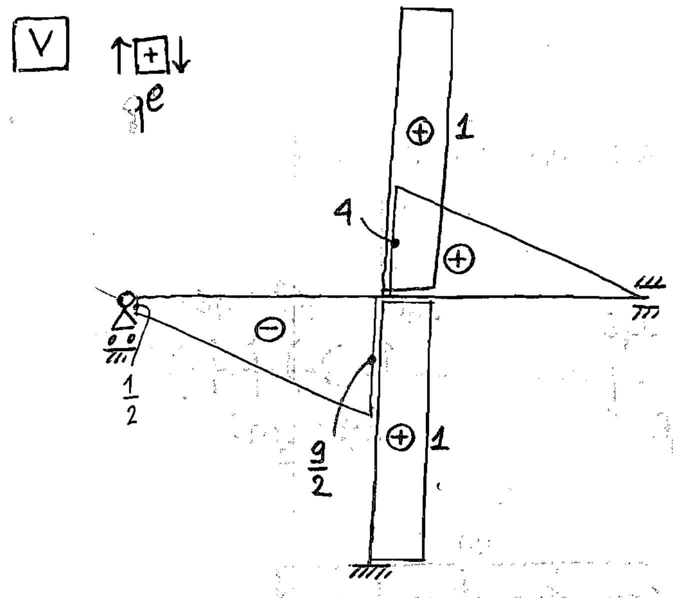


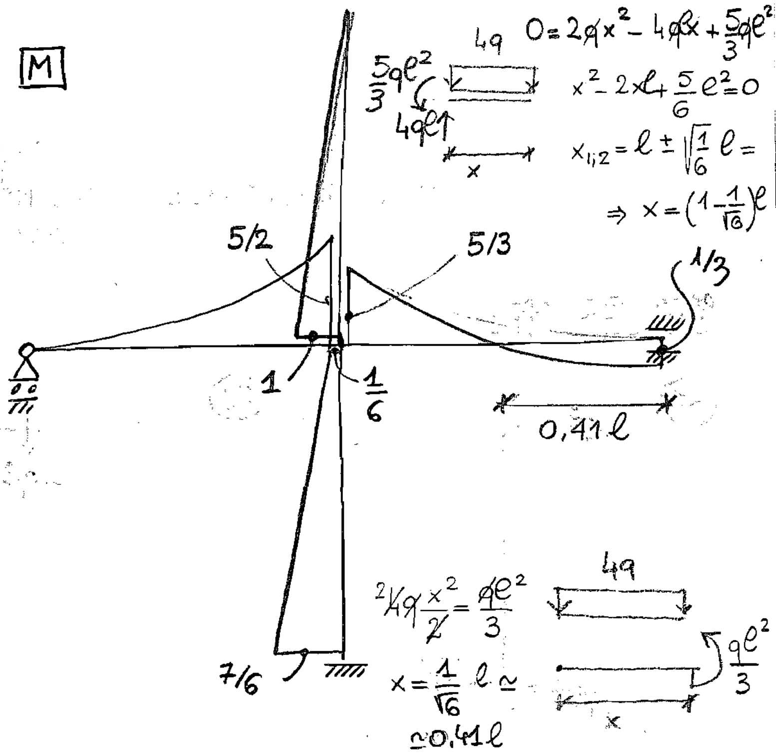
DIAGRAMMI AZIONI INTERNE



NOTA: EQUILIBRIO AL NODO C







Punti di flesso per l'asta CE

$$y'' = -\frac{M}{EJ} + \frac{2\alpha\Delta T}{h} \Rightarrow y''(x) = \frac{-M(x) + q\ell^2}{EJ} = -\frac{1}{EJ}\left(\frac{q\ell^2}{6} + q\ell x - q\ell^2\right) = \frac{1}{EJ}\left(\frac{5}{6}q\ell^2 - q\ell x\right)$$

$$\Rightarrow y'' = 0 \quad \text{per} \quad x = \frac{5}{6}\ell$$

Deformata qualitativa.